WIFO
ÖSTERREICHISCHES INSTITUT
FÜR WIRTSCHAFTSFORSCHUNG

WORKING PAPERS

Business Cycle Dating and Forecasting with Real-time Swiss GDP Data

Christian Glocker, Philipp Wegmüller

542/2017

WIFO

ÖSTERREICHISCHES INSTITUT FÜR WIRTSCHAFTSFORSCHUNG
AUSTRIAN INSTITUTE OF ECONOMIC RESEARCH

Business Cycle Dating and Forecasting with Real-time Swiss GDP Data

Christian Glocker, Philipp Wegmüller

WIFO Working Papers, No. 542

October 2017

Abstract

We develop a small-scale dynamic factor model for the Swiss economy based on an appropriately selected set of indicators. The resulting business cycle factor is in striking accordance with historical Swiss business cycle fluctuations. Our proposed model demonstrates a remarkable performance in short-term and medium-term forecasting. Using real-time GDP data since 2004, the model successfully anticipates the downturn of 2008-09 and responds in a timely manner to the recent sudden drop following the removal of the Swiss Franc lower bound. In a Markov-switching extension, we propose that our model could be used for Swiss recession dating. Our model does not indicate a regime-switch following the removal of the Swiss Franc lower bound.

E-mail addresess: christian.glocker@wifo.ac.at, Philipp.Wegmueller@seco.admin.ch
2017/255/W/0

© 2017 Österreichisches Institut für Wirtschaftsforschung
Medieninhaber (Verleger), Hersteller: Österreichisches Institut für Wirtschaftsforschung • 1030 Wien, Arsenal, Objekt 20 • Tel. (43 1) 798 26 01-0 • Fax (43 1) 798 93 86 • http://www.wifo.ac.at/ • Verlags- und Herstellungsort: Wien
Die Working Papers geben nicht notwendigerweise die Meinung des WIFO wieder
Kostenloser Download: http://www.wifo.ac.at/wwa/pubid/60645

Business Cycle Dating and Forecasting with Real-time Swiss GDP Data*

Christian Glocker † Philipp Wegmueller ‡

Abstract

We develop a small-scale dynamic factor model for the Swiss economy allowing for non-linearities by means of a two-state Markov-chain. The selection of an appropriate set of indicators utilizes a combinatorial algorithm. The model's forecasting performance is as good as that of peers with richer dynamics. It proofs particularly useful for a timely assessment of the business cycle stance, as the recessionary regime probabilities tend to have a leading property. The model successfully anticipated the downturn of the 2008-09 recession and promptly indicated a fall in GDP growth following the discontinuation of the exchange rate floor of the Swiss Franc.

JEL class: C32, C53, E37

Keywords: Dynamic Factor Model, Nowcasting, Real-Time Data, Markov-Switching, Business Cycle Dating.

*Thanks to Ferdy Adam, Lionel Fontagne, Alain Galli, Massimiliano Marcellino and Gabriel Perez-Quiros for valuable comments. P. Wegmueller would like to thank Ronald Indergand for support. Thanks also goes to seminar participants at SECO, WIFO, SNB, STATEC/Luxembourg, the SSES and ifo Dresden conferences for valuable comments. We are very grateful to Astrid Czaloun for research assistance. All errors are our own responsibility. The views expressed in this paper are those of the authors and do not necessarily represent those of the Swiss State Secretariat for Economic Affairs (SECO). This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

†Austrian Institute of Economic Research, Arsenal Objekt 20, 1030 Vienna, Austria. Phone: +43 (0) 1 789 26 01-303; E-mail: *Christian.Glocker@wifo.ac.at*

‡State Secretariat for Economic Affairs (SECO), Holzikofenweg 36, 3003 Bern, Switzerland. Phone: +41 (0) 58 465 95 38; E-mail: *Philipp.Wegmueller@seco.admin.ch*

1 Introduction

The accuracy in predicting GDP growth hinges critically on the statistical characterization of non-linearities inherent in its behaviour (DeJong, Liesenfeld, and Richard, 2005). Although linear models might be useful for forecasting GDP growth, they neglect that macroeconomic variables often behave differently according to fundamental changes in economic policy-making, or more generally, according to the business cycle phase the economy is in. Hence linear models might not reflect well the present situation. An illustrative example of such a fundamental economic policy realignment is the change in monetary policy performed by the Swiss National Bank (SNB) between 2011 and 2015.[1] Such kind of events add to the complexity of forecasting GDP growth and of assessing the current economic state in an already uncertain environment. In this context, policy realignments represent a particular form of non-linearity.

The objective of the present work is to construct a model which jointly allows for (i) assessing the business cycle stance, (ii) detecting business cycle turning points, and (iii) predicting GDP growth. Linear dynamic factor models (DFM) – pioneered by the work of Stock and Warson (1992) – have proven to be promising in achieving at least a subset of these goals. Following Mariano and Murasawa (2003), linear DFMs can handle indicators of quarterly and monthly frequency, ragged edges and missing observations, rendering them particularly well suited in monitoring day-to-day economic activity.[2] Concerning the forecasting performance, this class of models outperforms alternative standard univariate models as well as institutional forecasts based on expert judgement.[3] A shortcoming of linear DFMs is, however, their disability to provide information on the business cycle stance of an economy.

The introduction of a non-linear Markov-switching element fills this gap, as it allows for a probabilistic evaluation of an economy's position in the business cycle. Therefore, the Markov-switching dynamic factor model (MS-DFM) brought forward by Kim (1994), Kim and Yoo (1995), Diebold and Rudebusch (1996) and Chauvet (1998) is used as a starting point of our analysis. They proposed a fully non-linear DFM in which the common component is governed by an unobservable regime-switching variable controlling the business cycle dynamics.[4] The MS-DFM provides information on an economy's current position in the business cycle and can be used for now- and forecasting in a context in which non-linearities are allowed for explicitly. Camacho, Perez-Quiros, and Poncela (2018) account for the possibility of mixed-frequencies and ragged edges by integrating the methodology of Mariano and Murasawa (2003). By means of Monte-Carlo analysis they find substantial increases in the accuracy of business cycle identification compared to traditionally used MS-DFMs with balanced data. A number of studies assign only

[1] To the surprise of markets and institutions, the Swiss National Bank (SNB) decided for a discontinuation of the Euro-Swiss Franc (CHF) on January 15, 2015, which had been introduced on September 6, 2011.

[2] Applications of such linear models to countries include for instance Argentina, Canada, Czech Republic, Spain, Switzerland, etc. (Chernis and Sekkel, 2017; Camacho and Perez-Quiros, 2011; Camacho, Dal Bianco, and Martinez-Martin, 2015; Rusnák, 2016; Marcellino, Porqueddu, and Venditti, 2016; Galli, 2018).

[3] See for instance Giannone, Reichlin, and Small (2008), Rünstler, Barhoumi, Benk, Cristadoro, Reijer, Jakaitiene, Jelonek, Rua, Ruth, and Nieuwenhuyze (2009), Barhoumi, Darné, and Ferrara (2010), Marcellino and Schumacher (2010), Bańbura and Rünstler (2011), Aastveit and Trovik (2012), and D'Agostino, McQuinn, and O'Brien (2012); see Bańbura, Giannone, and Reichlin (2010) for a review.

[4] Camacho, Perez-Quiros, and Poncela (2015) show that this one-step estimation outperforms the estimation of a Markov-switching process on the factor in a second step.

a subordinate role to parametric non-linear models in macroeconomic forecasting.[5] Calhoun and Elliott (2012) identify a wrong choice of non-linearity for the weak out-of-sample forecasting performance of such models. In general, more complicated models tend to have a good in-sample fit, better than linear specifications, because of their greater flexibility. Yet, often the resulting models are too specific for the particular estimation sample, and their good in-sample performance is not replicated in an out-of-sample forecasting context.[6]

We propose a small-scale Markov-switching dynamic factor model applied to the Swiss economy. The application to the Swiss economy is interesting for various reasons. Specific to this country are the fundamental monetary policy realignments of 2011 and 2015 in an environment of increased macroeconomic uncertainty linked to the European debt crisis as well as several years of economic struggle in the 1990s caused by a crashing real estate market. Besides, the global financial crisis of 2008-09 lead to the greatest decline in quarterly GDP growth in three decades. These events render the country an illustrative example for potential non-linearities in the data. Further, contrary to survey indicators, published quarter-over-quarter (q-o-q) GDP growth rates of the Swiss economy show a rather low degree of persistence. As a consequence, a simple auto-regressive lag-structure for a factor extracted jointly from these data seems less promising. The Swiss case is also particular regarding data availability, as high-frequency business cycle indicators are available only to a limited extent.

Related literature for Switzerland is scarce. Galli (2018) constructs a business cycle index for the Swiss economy based on a broad set of indicators. While the resulting coincident index provides useful information on business cycle turning points, it does not provide inference on the particular business cycle stance. The same index is included in Galli, Hepenstrick, and Scheufele (2017), who perform a horse race of forecasting models. Among others, they also consider a small-scale linear DFM as competing model. In contrast to their analysis, we provide more transparency concerning the process of selecting the indicators. Moreover we include real-time GDP data to our model and compare the forecasting accuracy to alternative models containing the same set of indicators as well as to judgemental and consensus forecasts.

We follow the lines of Camacho, Perez-Quiros, and Poncela (2018) and extend their work in several directions. First, rather than focusing on a stylized experiment, we pay particular attention to the process of variable selection. According to Bai and Ng (2008), Bańbura, Giannone, and Reichlin (2010) and Camacho and Garcia-Serrador (2014) small-scale DFMs are well suited for now- and forecasting, especially when the indicators used are selected appropriately. By means of a combinatorial algorithm, we focus on a model specification with a limited number of indicators only. This algorithm serves to assess the contribution of an additional variable, taking into account different variable combinations. Second, we incorporate explicitly revisions to GDP and allow thereby to predict both the first and final release of our target variable. The *final*-model builds upon a single factor, extracted from a set of ten indicators and real GDP growth. Following Mariano and Murasawa (2003), the model deals easily with (i) ragged edges, occurring from a non-synchronous release of official data, and (ii) mixed frequencies of the indicators. We allow for both linear and non-linear elements – the dynamics within a particular state

[5] See for instance Faust and Wright (2013), Umer, Sevil, and Sevil (2018), among others.
[6] See for instance Stock and Watson (1999).

are characterized by a linear specification, whereas the switch across states depicts a non-linear element. The regime switch is specified by a two-state Markov chain whose two states capture expansionary and contractionary episodes.[7] Last, we test the model's performance in now- and forecasting against a set of competing models.

As regards the results, we find that the MS-DFM is able to capture past recessionary episodes of the Swiss economy surprisingly well. We compare the resulting recession probabilities of our model to both the business cycle dating of the Economic Cycle Research Institute (ECRI) and a technical recession classification. For either case, the recession probabilities spike at the beginning of a recessionary episode, or even prior to the actual beginning of a recession. The model estimates highlight the importance of taking non-linearities into account. The estimated Markov-switching parameters are all statistically different from zero. The expected duration of an expansion is around 50 months and that of a contraction is around 7 months; the average q-o-q growth rate of GDP in these states is around 0.6 %, and -1.0 %, respectively. Furthermore, the extracted business cycle factor tracks the business cycle dynamics accurately and is able to explain 74 % of the variation of Swiss GDP growth in real-time. It can be regarded a trustworthy indicator of Swiss economic developments over the last three decades.

In an out-of-sample exercise, we use real-time data from Indergand and Leist (2014) to construct 312 bi-weekly vintages starting in January 2004. We accurately account for the lag of synchronicity in data publication as observed in the real-time data flow. We show that our model was particularly well-suited in determining turning points of the Swiss business cycle and to timely assess recessionary episodes in real-time. During the outbreak of the global financial crisis of 2008-09, the model predicted a negative GDP growth rate for the fourth quarter 2008 already in August of that year. Moreover, the model correctly anticipated the negative GDP growth of the first quarter 2015 by the end of the first half of February, i.e., only one month after the removal of the Swiss Franc lower bound. The now- and forecasting performance of our model is not just more accurate in comparison to naive models; indeed, the forecasting performance of the MS-DFM turns out to be as good as peers that allow for richer dynamics. For instance, its forecasts are significantly more accurate than those of a Mixed Frequency-Factor Augmented Vector Autoregressive (MF-FAVAR) model. The results prove to be robust for different model specifications. For instance, adding more indicators to our model not necessarily improves the forecasting performance nor its ability in detecting recessionary episodes.

The paper is organized as follows: Section 2 outlines the econometric framework of the MS-DFM and motivates alternative model specifications. We describe the selection of indicators in Section 3. Section 4 presents the empirical evidence, first discussing the In-sample-properties, and second reporting the Out-of-sample performance. We check for robustness of our findings in Section 5. Section 6 concludes.

[7] Alternatively, one could also consider three states as for instance in Carstensen, Heinrich, Reif, and Wolters (2017), who distinguish between normal and severe recessions in an application to the German economy.

2 The econometric framework

In this section we describe the details of the dynamic factor model including the Markov-switching process for the factor. The main challenge hereby consists of finding an appropriate framework which takes the following into account: (1) the data displays different sampling frequencies (i.e. monthly business cycle indicators and quarterly National Accounts data); (2) the GDP data is subject to substantial revisions; (3) the data contains missing observations. The outline of the model closely follows the work of Camacho, Perez-Quiros, and Poncela (2018) and incorporates some ingredients of Camacho and Perez-Quiros (2010), Mariano and Murasawa (2003) and Chauvet (1998).

2.1 Mixing quarterly and monthly observations

Combining monthly observations with quarterly data requires to express the quarterly data as a function of monthly figures. Following Mariano and Murasawa (2003), if the sample mean of the three within quarter monthly observations can be approximated by the geometric mean, then the quarterly growth rates can be decomposed as weighted averages of monthly growth rates.[8]

Let y_t^q be the quarter-over-quarter growth rate of an observed quarterly indicator and y_t^m its latent month-over-month growth rate. Then, y_t^q, can be expressed as the weighted average of month-on-month growth rates by

$$y_t^q = \frac{1}{3}y_t^m + \frac{2}{3}y_{t-1}^m + y_{t-2}^m + \frac{2}{3}y_{t-3}^m + \frac{1}{3}y_{t-4}^m. \tag{1}$$

2.2 Accounting for GDP-revisions

A key drawback of simple ARIMA-models in forecasting is their high sensitivity to data revisions. Instead, the present model directly takes into account revisions to GDP. We get vintage GDP data from Indergand and Leist (2014), starting in 2002-Q3. The first quarterly estimate of real GDP growth is denoted by y_t^{1st}, while the latest available vintage – *final GDP* – is called y_t^f.[9] We follow the recommendations of Aruoba (2008) to test whether revisions to Swiss quarterly GDP growth are *news* or *noise*. In the former, the initial announcement is an efficient forecast for final GDP, reflecting all available information; revisions then only incorporate new information. In the latter, the initial announcement is an observation of the final series, measured with error.

The test results yield the following: The mean of revisions is not significantly different from zero; revisions are positively correlated with final estimates, but uncorrelated with initial estimates. For the forecast efficiency test on the regression $y_t^{1st} = \alpha_0 + \alpha_1 y_t^f + \epsilon_t$, the joint null hypothesis of $\alpha_0 = 0$ and $\alpha_1 = 1$ is rejected with a p-value of 0.001; contrary, the same null

[8] Since the evolution of macroeconomic series is smooth enough, such an approximation is appropriate. For instance, Proietti and Moauro (2006) avoid this approximation at the cost of moving to a complicated non-linear state-space model.

[9] In Switzerland, two distinct authorities are responsible for quarterly and yearly GDP estimates. Based on the yearly GDP measures from the Federal Statics Office (FSO), the State Secretariat of Economic Affairs (SECO) uses temporal disaggregation methods to estimate quarterly GDP figures, which are published periodically about 65 days after the end of a quarter. Revisions to the real-time measure from SECO can stem from different sources: (1) revisions to quarterly indicators; (2) revisions to annual base data; (3) changes in the methodology of national accounts (benchmark revisions); (4) minor changes in the quarterly estimation methods; (5) technical reasons like changes in seasonal adjustment.

hypothesis cannot be rejected for the regression $y_t^f = \alpha_0 + \alpha_1 y_t^{1st} + \epsilon_t$. Summarized, these results indicate that revisions are news, rather than noise.[10]

According to Mankiw and Shapiro (1986), this finding motivates to link the first estimates to the final GDP growth rates by

$$y_t^f = y_t^{1st} + \epsilon_t, \qquad \epsilon_t \sim NID(0, \sigma_\epsilon^2). \tag{2}$$

2.3 Model specification

The premise of dynamic factors models is that a vector of observed time series $\boldsymbol{X}_t$ of dimension $n_{\boldsymbol{X}}$ can be decomposed into two orthogonal components: common components, also called latent factors, denoted by f_t, which capture the co-movements among the observed variables in $\boldsymbol{X}_t$ and an idiosyncratic component $u_{t,i}$, $\forall\ i = 1, ..., n$. These idiosyncratic disturbances arise from measurement error and from special features of the data.

The vector of time series $\boldsymbol{X}_t$ consists of various different business cycle indicators. A key feature of these time series is that they describe the economic conditions prevailing at time t_0 in relation to some point in time in the recent past. In most cases this is either the previous month $(t_0 - 1)$ or at times it can be the same month of the last year $(t_0 - 11)$. Hence monthly business cycle variables either describe month-over-month patterns or on the other hand, year-over-year patterns on a monthly frequency. The econometric specification for the construction of the latent factor f_t has to take these peculiarities into account. In order to simplify the exposition, assume that all variables in $\boldsymbol{X}_t$ are observed at a monthly frequency. We construct the latent factor such that monthly growth rates of quarterly series and monthly growth rates of indicators of real economic activity (*hard*) exhibit a direct relation with the common factor f_t. The corresponding factor loadings measure the sensitivity of each hard indicator to movements in the latent factor directly.[11]

In contrast to hard indicators, the relation between the common factor and survey indicators (*soft*) has to be treated differently. Camacho and Perez-Quiros (2010) acknowledge, among others, that each confidence indicator is calculated as the simple arithmetic average of the balances of answers to specific questions chosen from the full set of questions in each individual survey. The selection of questions is guided by the aim of achieving an as high as possible coincident correlation of the confidence indicator with the reference series, such as year-over-year growth in GDP. This implies that the business cycle conditions prevailing at time t_0 are compared to the ones of the same month of the previous year. In order to account for this, we relate the level of soft indicators where necessary with the year-on-year common growth rate which can be written as the sum of current values of the common factor and its last eleven lagged values.

For this reason we decompose the variables in $\boldsymbol{X}_t$ into a set of n_h hard indicators $\boldsymbol{x}_t^h$ and n_s soft indicators in $\boldsymbol{x}_t^s$. Additionally, the vector of observed time series includes first and final estimates of the quarterly GDP growth rate: $\boldsymbol{X}_t = \left[\left(\boldsymbol{x}_t^h\right)', \left(\boldsymbol{x}_t^s\right)', y_t^{1st}, y_t^f \right]'$. We standardize the observed

[10] We start the revision analysis for the vintage 2004-Q1 up to the final vintage 2016-Q4. Our results are qualitatively robust for using different vintages of y_t^f.

[11] The terminology as regards *soft* vs. *hard* indicators is not necessarily restricted to truly soft or hard indicators. In fact, if a hard indicator, as for instance industrial production, were to be used in the model as year-over-year growth rate, then this transformed variable would have to be treated in the model as a *soft* indicator.

time series in $\boldsymbol{X}_t$ prior to using them in the dynamic factor model. Let $u^h_{t,i}$, $\forall\ i = 1, ..., n_h$ be the idiosyncratic (dynamic) error term for the hard indicators, $u^s_{t,i}$, $\forall\ i = 1, ..., n_s$ the equivalent for the soft indicators, and $u_{t,q}$ the idiosyncratic error term for the two measures of GDP growth.

The dynamic factor model can be specified $\forall\ t = 1, ..., T$ as follows:

System of static equations

$$\begin{aligned} \begin{pmatrix} \boldsymbol{x}^s_t \\ \boldsymbol{x}^h_t \end{pmatrix} &= \begin{pmatrix} \gamma_s \cdot \sum_{j=0}^{11} f_{t-j} \\ \boldsymbol{\gamma}_h \cdot f_t \end{pmatrix} + \begin{pmatrix} \boldsymbol{u}^s_t \\ \boldsymbol{u}^h_t \end{pmatrix}, && (3) \\ \begin{pmatrix} y^f_t \\ y^{1st}_t \end{pmatrix} &= \begin{pmatrix} \omega(L) \\ \omega(L) \end{pmatrix} \cdot [\gamma_q f_t + u_{t,q}] + \begin{pmatrix} 0 \\ \epsilon_t \end{pmatrix}, && (4) \end{aligned}$$

where $\boldsymbol{u}^h_t = \left(u^h_{t,1}, ..., u^h_{t,n_h}\right)'$, $\boldsymbol{u}^s_t = \left(u^s_{t,1}, ..., u^s_{t,n_s}\right)'$ with $n = n_h + n_s$, $n_{\boldsymbol{X}} = n + 2$, and $\omega(L) := \frac{1}{3} + \frac{2}{3} \cdot L + L^2 + \frac{2}{3} \cdot L^3 + \frac{1}{3} \cdot L^4$, where L is the lag operator. The stochastic properties of ϵ_t are defined in equation (2). The vector of factor loadings $\boldsymbol{\gamma} = (\gamma_q, \boldsymbol{\gamma}'_h, \boldsymbol{\gamma}'_s)'$ captures the relation between the latent factor f_t and the observed variables $\boldsymbol{X}_t$ modified by the scalar ω. This takes into account that y^{1st}_t and y^f_t are not observed on a monthly basis and are hence interpolated using the relation given in equation (1).

System of dynamic equations

$$\begin{aligned} f_t &= \mu(\zeta_t) + \nu^f_t, && (5) \\ (1 - \phi_q(L)) \cdot u_{t,q} &= \nu^q_t \quad \text{with} \quad \nu^q_t \sim NID\left(0, \sigma^2_q\right), && (6) \\ (\boldsymbol{I} - \boldsymbol{\Phi}_u(L)) \begin{pmatrix} \boldsymbol{u}^s_t \\ \boldsymbol{u}^h_t \end{pmatrix} &= \boldsymbol{\nu}_t, && (7) \\ \begin{pmatrix} \nu^f_t \\ \boldsymbol{\nu}_t \end{pmatrix} &\sim NID\left(\boldsymbol{0}, \begin{bmatrix} \sigma^2_f & \boldsymbol{0} \\ \boldsymbol{0} & \Sigma_\nu \end{bmatrix}\right), && (8) \end{aligned}$$

where $\phi_q(L)$ and $\boldsymbol{\Phi}_u(l)$ are in each case second-order lag-polynomials.[12] We assume that $\boldsymbol{\Phi}_u(L)$ and Σ_ν are diagonal, implying that all covariances are zero by construction. For identification reasons we impose that σ^2_f is unity.

Markov-switching factor

The dynamic behavior of the latent factor f_t is governed by an unobserved regime-switching state variable ζ_t which follows a two-state Markov-chain whose transition probabilities are given by:

$$p(\zeta_t = i|\zeta_{t-1} = j, \zeta_{t-2} = h, ...) = p(\zeta_t = i|\zeta_{t-1} = j) = p_{ij}. \quad (9)$$

The state variable ζ_t interacts with the common factor according to equation (5). Knowledge of ζ_t characterizes the population parameter $\mu(\zeta_t)$, though it still leaves some uncertainty about the common factor that comes from the shock ν^f_t.[13] The non-linearity of the observed time series is

[12]The AR(2) assumption can be considered as a parsimonious specification: (1) It only requires the estimation of two parameters; (2) it allows a rich dynamic pattern since the roots of $\phi_q(L)$ and $\boldsymbol{\Phi}_u(L)$ can be complex.

[13]We omitted autoregressive terms in equation (5) as they turned out to be not significantly different from zero.

captured by $\mu(\zeta_t)$, which is allowed to change across the two distinct states. We define two states at time t: expansion ($\zeta_t = 0$) and recession ($\zeta_t = 1$). Further technical details can be found in an Appendix and in Camacho, Perez-Quiros, and Poncela (2018).

2.4 Estimation

As regards the estimation of the Markov-switching dynamic factor model (MS-DFM), we follow Camacho, Perez-Quiros, and Poncela (2018). The idea is to work with the highest frequency of the data. Those series, which are, however, available only at a lower frequency, are treated as time series with periodically missing observations. The models are then cast into a state-space representation and estimated using the Kalman filter; within this approach unobserved cells in the time series can be treated as missing observations and the maximum likelihood estimation remains valid. The above system of static and dynamic equations extended with a Markov-switching element can be cast into the state-space representation as follows:

$$\boldsymbol{y}_t = H\boldsymbol{s}_t + \boldsymbol{w}_t, \tag{10}$$

$$\boldsymbol{s}_t = \boldsymbol{\mu}(\zeta_t) + F\boldsymbol{s}_{t-1} + \boldsymbol{v}_t, \tag{11}$$

We provide details of the matrices H, F, R, Q, the vectors $\boldsymbol{y}_t$, $\boldsymbol{s}_t$, $\boldsymbol{w}_t$ and $\boldsymbol{v}_t$ and their relation to the equation system of the models in an Appendix.

If all series in the model were observable at a monthly frequency and the data panel was balanced, then the estimation of the dynamic factor model could be implemented using standard maximum likelihood methods in conjunction with the Kalman filter. This assumption is, however, rather unrealistic, since in our empirical application we have to deal with mixing quarterly and monthly frequencies and with time series which are published at different time lags and which start at different points in time. Moreover, our context requires projections of the indicators, which can be considered as missing data for a certain date as well.

According to Mariano and Murasawa (2003), with the subtle transformation of replacing missing observations by random draws $r_t \sim NID(0, \sigma_r^2)$, the system of equations remains valid.[14] Importantly, if the distribution of r_t does not depend on the parameter space that characterizes the Kalman filter, then the matrices in the state-space representation are conformable and do not have an impact on the model estimation since the missing observations just add a constant term in the likelihood function to be estimated.

Let $y_{i,t}$ be the ith element of vector $\boldsymbol{y_t}$ and let R_{ii} be its variance. Let $H_{i,t}$ be the ith row of matrix H_t, which has z columns. The measurement equation can then be replaced by the following expressions

$$y_{i,t}^* = \begin{cases} y_{i,t} & \text{if } y_{i,t} \text{ is observable} \\ r_t & \text{otherwise,} \end{cases} \tag{12}$$

$$H_{i,t}^* = \begin{cases} H_i & \text{if } y_{i,t} \text{ is observable} \\ 0_{1\times z} & \text{otherwise,} \end{cases} \tag{13}$$

$$w_{i,t}^* = \begin{cases} 0 & \text{if } y_{i,t} \text{ is observable} \\ r_t & \text{otherwise,} \end{cases} \tag{14}$$

[14] Means, medians or zeros are valid alternatives.

$$R^*_{ii,t} = \begin{cases} 0 & \text{if } y_{i,t} \text{ is observable} \\ \sigma_r^2 & \text{otherwise.} \end{cases} \tag{15}$$

With this transformation the time-varying state-space model can be treated as having no missing observations and the Kalman filter can be directly applied to $\boldsymbol{y}^*_t$, H^*_t, $\boldsymbol{w}^*_t$ and R^*_t. The implementation of this algorithm corresponds to expanding $\boldsymbol{y}_t$, H, $\boldsymbol{w}_t$ and R in equation (A.1) in the Appendix by means of an indicator function which takes into account if $y_{i,t} \in \boldsymbol{y}_t$ is observed or not. The estimation of the models' parameters is done by maximizing the log-likelihood of $\{\boldsymbol{y}^*_t\}_{t=1}^{t=T}$ numerically with respect to the unknown parameter matrices.

The Kalman filter algorithm calculates recursively one-step-ahead prediction and updating equations of the dynamic factor and the mean squared error matrices, given the parameters of the model and starting values for the state vector, the mean squared error and, additionally, the probabilities of the Markov states. The updating equations are computed as averages weighted by the probabilities of the Markov states. The maximum likelihood estimators and the sample data are then used in a final application of the filter to draw inferences about the dynamic factor and the state probabilities.[15]

2.5 Competing models

To assess the performance of the MS-DFM, we compare our framework to several competing models. Importantly, we lay our focus on models that share some of the main characteristics of the MS-DFM. They contain the same set of indicators, allow for mixed-frequencies of the indicator series, account for missing observations and revisions to GDP data. The assumptions regarding lag-polynomials and identification restrictions remain the same.

Linear dynamic single-factor model (DSFM)

The specification of a Markov-switching process for the common factor, f_t, has introduced a non-linearity into an otherwise standard linear dynamic factor model. Given its simplicity and parsimoniousness, the DSFM is the most natural model of reference. This applies in particular to the computational complexity within the Kalman filter iterations.

To obtain the DSFM, the model presented in Section 2.3 changes only in equation (5). The equation has to be adjusted to read

$$(1 - \phi_f(L)) \cdot f_t = \nu^f_t, \tag{16}$$

in which $\phi_f(L)$ is a second-order lag-polynomial. Details on the model specification and solution can be found in the Appendix.

Mixed Frequency-Factor Augmented Vector Autoregressive (MF-FAVAR) model

The single-factor model presented above only allows for the dynamics to occur in the equation of the factor as well as in the error terms; this might at times be too restrictive. For this reason we consider the MF-FAVAR model as an alternative. It allows for a dynamic interaction between the factor f_t and some observed variables of interest. This extension increases the overall dynamic

[15] Technical details are explained in the Appendix.

potential of the model noticeably. The single-factor model can be extended to a MF-FAVAR model by modifying two equations only. In particular, equation (4) changes to the following

$$\begin{pmatrix} y_t^f \\ y_t^{1st} \end{pmatrix} = \begin{pmatrix} \omega(L) \\ \omega(L) \end{pmatrix} \cdot \left[\gamma_q \cdot y_t^M\right] + \begin{pmatrix} 0 \\ \epsilon_t \end{pmatrix} \tag{17}$$

where y_t^M constitutes the monthly GDP growth rate extrapolated from the quarterly growth rates given of y_t^f and y_t^{1st}. Hence y_t^M can be treated as an observed variable. Finally, equation (5) changes to

$$(\boldsymbol{I} - \phi_f(L)) \cdot \begin{bmatrix} y_t^M \\ f_t \end{bmatrix} = \boldsymbol{\nu}_t^f \tag{18}$$

where ν_t^f is now a 2×1 vector, and $E\left[\boldsymbol{\nu}_t^f \cdot (\boldsymbol{\nu}_t^f)'\right] = \Sigma_f$ is a 2×2 matrix and $\phi_f(L)$ a matrix lag-polynomial. We identify the model by imposing that the $(2, 2)$ element in Σ_f is unity and $\gamma_q = 1$. The MF-FAVAR model is discussed in detail in the Appendix.

The main differences between the DSFM presented above and the MF-FAVAR specification are that (i) the factor of the MF-FAVAR model is constructed without the information in $[y_t^f,\ y_t^{1st}]'$; and (ii) the stochastic characteristics of the two models differ; in particular, since $\phi_f(L)$ is a matrix lag-polynomial of order two, the MF-FAVAR allows for much richer dynamic properties than the DSFM.

3 Data

This section elaborates on the indicator selection process and describes the final data set used. To select an appropriate set of indicators, we use the simple dynamic single-factor model (DSFM) as benchmark model instead of the MS-DFM. The reason for this choice is mainly driven by the computational complexity introduced by the Markov-switching process.

3.1 Selection of indicators

A meaningful starting point for selecting indicators entering a factor model is the approach of Stock and Warson (1992). Accordingly, monthly data for production, expenditure and income as well as employment form the basis.[16] One could then add other specific indicators to potentially improve the model fit. As data availability in the case of Switzerland is rather scarce, we are however forced to take a modified approach.

First, we collect as many monthly indicators as possible. Key criteria to keep a variable in our sample are (i) timely publication and (ii) the length of a series. For instance, industrial production does not fulfil these criteria. The series begins in 2014 only and has a publication lag of 60 days. Employment is another series that we have to discard. It is only available on a quarterly frequency and is released with a significant publication lag. We were able to collect a set of 31 variables, covering a wide range of economic data.[17] To these monthly variables we add the two quarterly series for GDP (first and final releases).

[16] In the case of Stock and Warson (1992), they chose the four monthly coincident variables comprised in the Index of Coincident Economic Indicators (CEI) compiled by the US Department of Commerce (DOC). In particular industrial production, total personal income less transfer payments, total manufacturing and trade sales and employees on non-agricultural payrolls.

[17] Imports, exports, overnight stays, retail sales, new car registrations, energy consumption, term spread, Swiss market index (SMI), Swiss performance index (SPI), oil price, real and nominal effective exchange rate, bank

Second, we implement a procedure based on a combinatorial algorithm to identify an appropriate subset out of the 31 monthly indicators. The selection process is as follows:

1. Identification of a common factor based on a combinatorial algorithm to choose the best combination of variables within the dynamic factor model. The number of variables included is pre-determined. Let k be the number of variables being included in the dynamic factor model and $n > k$ be the number of variables in the sample, we then obtain a total of $b_{n,k} := k!/(k! \cdot (n-k)!)$ different combinations. For each combination, we compute the share of variance in GDP growth explained by the common factor.

2. Exclusion of all variable combinations whose R^2 is lower than some threshold value $\tilde{\epsilon}$. This leaves us with a smaller subset of variable combinations relative to the original $b_{n,k}$ combinations.

3. Use of economic judgement to restrict the subset of variable combinations of the second step to only one final combination of variables. The selection pays attention in particular to economic phenomena related to financial market developments and aggregate demand, among others; and to data specific issues as for instance time delay in publication, etc.

The motivation for choosing a combinatorial algorithm within the selection process stems from Boivin and Ng (2006) and Bańbura and Rünstler (2011). Accordingly, the inclusion of an additional variable to the model does not necessarily improve its performance. Rather, the resulting factor might explain less of the variance in GDP, even when the new variable displays a decent correlation with GDP. When the additional variable is correlated with a subset of variables already in the model, the factor has a bias towards this subset of variables. As a consequence, the resulting factor explains a large fraction of variation in each variable of this subgroup, but less of the variance in GDP. We account for this potential problem by considering the contribution of an additional variable subject to the variables already present in the model. In principle, our procedure for indicator selection is similar to Camacho and Perez-Quiros (2010). They only consider a variable relevant for their model if by adding a new variable, the model fit improves – they too rely on the R^2 between the factor and GDP. By this, however, they ignore the potential of the combination of different variables already being part of the model in having an effect on the model fit when adding a new variable. This is relevant once the variables of a small subset are highly correlated among each other, possibly worsening the overall model fit, even though each variable of this subset might be useful individually (see Boivin and Ng, 2006). Our methodology comprises a more formal statistical approach for selecting the variables. Though computationally more complex, it allows for more flexibility within the selection of an appropriate set of variables.

With the objective to select a number of variables similar to Camacho and Perez-Quiros (2010) who consider 10 indicators, applying simply our combinatorial algorithm – the first step of the selection procedure – would amount in selecting a combination of 10 variables out of 31, resulting in nearly 85 million possible indicator combinations. To reduce the computational complexities,

assets, loans, KOF industrial orders, PMI, UBS consumer survey, KOF industry and construction surveys, vacancy postings, unemployment rate, social security contributions, CPI, EPI, IFO survey, ZEW survey.

we apply the algorithm twice. First, we identify a *Core*-model with $k = 4$.[18] Second, by relying on the same selection procedure, we extend the *Core*-model to a more comprehensive, i.e., *final*-model set-up. This approach allows us to reduce the number of variable combinations being considered within the process of variable selection drastically and make the selection process tractable.[19]

As regards the *Core*-model, we strive to select indicators possibly similar to the ones used in Stock and Warson (1992).[20] For this purpose, we restrict the selection procedure for the *Core*-model to the subset of *hard* indicators only. We rest upon the combinatorial algorithm only and neither rely on a threshold value for the R^2 nor on economic judgement to choose among candidate *Core*-models. Among all possible variable combinations, we then consider the one which exhibits the highest R^2. The variables that have not been selected for the *Core*-model are then part of the selection process for the *final*-model. Out of the remaining 27 indicators we then choose six variables, using the selection procedure described above for a second time. The combinatorial algorithm yields a set of different variable combinations from which we have to decide upon a final set. To this purpose, we now use the R^2 from the *Core*-model as threshold value to discard sets of variable combinations having a fit worse than the *Core*-model. Among the remaining variable combinations, we apply economic judgement to obtain a final set of variables for the MS-DFM. In fact, this involves only a few variable combinations and the discrimination among them is difficult as they all show a comparably high value of the R^2. It turns out that these variable combinations share the same set of hard indicators, however, they contain fairly different soft indicators. Many of the soft indicators are highly correlated with each other, implying an overall similar model fit. We choose the particular variable combination with soft indicators that are (i) earliest available, and (ii) comprise the longest time series.

The *Core*-model consists of four monthly indicators (total imports of goods, total retail sales, the term spread and bank assets) and two quarterly GDP series. With this specification the MS-DFM achieves a correlation of 0.74 of the factor with the first release of GDP (0.69 with the final vintage). The *Core*-model comprises variables that are generally perceived as important in the context of forecasting GDP (see for instance Camacho and Garcia-Serrador, 2014). Imports are an indicator for internal demand, and in the case of Switzerland – a small open economy – also for external demand since the import share in exports is extraordinarily high. Total retail sales cover a broad range of private consumption on the expenditure side. The term spread is well known as a leading business cycle indicator capturing both expectations as well as the monetary policy stance of the economy (see for instance Ang, Piazzesi, and Wei, 2006; Wheelock and Wohar, 2009). It enters the model in first-differences. Finally, bank assets contain information on liquidity in the financial sector. As highlighted in Adrian and Shin (2010), this variable captures swings in financial markets and correlates highly with the value added in the financial

[18] In principle, we could also choose $k = 3$ or $k = 5$. We chose $k = 4$ keeping in mind that the DSFM of Stock and Warson (1992) was built on the same number of indicators.

[19] With our approach, the maximum number of variables being considered in the selection algorithm is fixed – this can indeed weigh on the fit of the model. In Section 5.2 we show in how far the inclusion of further variables changes the fit of our *final*-model.

[20] Optimally, we would choose the same variables as in Stock and Warson (1992). For Switzerland, however, such data does not exist, either because of the frequency (e.g., employment is only available on a quarterly frequency) or lack of data (e.g., industrial production).

Table 1: Indicators selected for Swiss DSFM

Name	Definition	First obs.	R.	F.	S.A.	T.	D.	M.	S.
GDP 1^{st}	Quarterly real GDP (real time)	2002-Q3	Y	Q	1	1	5	2	SECO
GDP^{f}	Quarterly real GDP (final estimate)	1980-Q1	Y	Q	1	1	5	2	SECO
Imports	Total imports of goods, volume	1980-M1	Y	M	1	1	20	1	FCA
Sales	Total retail sales, volume	1980-M1	Y	M	1	1	5	2	FSO
Spread	10y-govt bond yield minus 3-month rate	1980-M1	N	M	0	0	1	1	SNB
REER	Swiss Franc real effective exchange rate (broad)	1980-M1	N	M	0	1	15	1	BIS
Orders	KOF manufacturing order (to previous month)	1980-M1	Y	M	1	0	5	1	KOF
Loans	Loans of private households (without mortgages)	1980-M1	Y	M	0	1	20	2	SNB
Assets	Total assets of commercial banks	1987-M12	Y	M	0	1	20	2	SNB
VSMI	Swiss equity market volatility index	1999-M1	N	M	0	0	1	1	SIX
PMI	Total purchasing managers' index	1995-M1	N	M	1	0	1	1	Markit
UBSc	UBS consumption indicator	1996-M2	Y	M	1	0	30	1	UBS

Note: From left to right: *Name* reports the acronym for the variable; *Definition* describes the respective indicator series; *First obs.* specifies since when data are available (the format is either year-quarter or year-month); *R.* indicates whether the series are permanently subject to revisions; *F.* determines the frequency of the series (M: monthly; Q: quarterly); *S.A.* specifies whether the variable is seasonally adjusted; *T.* specifies whether a variable has been transformed to growth rates; *D.* reports the approximate day of release of each variable; *M.* indicates how many months after the end of the reference period the data are released; and finally *S.*: BIS - Bank of International Settlements; FCA - Federal Customs Administration; FSO - Swiss Federal Statistical Office; KOF - Swiss Economic Institute; SECO - State Secretariat for Economic Affairs; SIX - Swiss Stock Exchange; SNB - Swiss National Bank; UBS - United Bank of Switzerland.

sector. As the financial sector constitutes an important industry of the Swiss economy, we hence consider bank assets jointly with the term spread as important variables for the model. In other words, to the extent that the combinatorial algorithm attaches great importance to these two variables, this pure statistical result can be underpinned by economic theory.

3.2 Final set of indicators

The *final*-model consists of a total of 10 monthly indicator variables. Details are summarized in Table 1. For the MS-DFM the correlation of the factor with GDP is 0.86 for the first release (0.83 for the final vintage). The enlarged model features two soft indicators, the PMI and the consumption indicator. The former asks managers about their economic sentiment with respect to the previous month. The latter contains information on private consumption trends, in particular it is based on credit card transactions made via the bank UBS. [21] Furthermore, the model contains the trade-weighted (broad) real exchange rate, the stock of orders compared to the previous month out of the KOF business survey (in levels), loans of private households without mortgages and the Swiss stock market volatility index VSMI.[22]

Out of the initial 31 variables the selected combination of ten monthly indicators was the one which performed best and is economically meaningful.[23] The final set proved to be robust

[21]Both indicators exhibit higher correlation with the year-on-year GDP growth rate than with quarter-on-quarter rate, which is why they load with 11 lags on the common factor. The model outcome does not change qualitatively when they are specified as hard indicators, i.e., loading contemporaneously on the factor.

[22]An interesting alternative to stock market volatility would be a general measure of financial market stress (Duprey, Klaus, and Peltonen, 2017; Glocker and Kaniovski, 2014) or a measure of business uncertainty (Glocker and Hölzl, 2019); unfortunately data for these measures are not available.

[23]The information used in the model stems entirely from business cycle indicators. Economic policy as such does not enter. However, our model is flexible enough so that it could be extended to combine the following two pieces of information on economic policy in real-time: (i) the ex-ante path of policy as published/announced by policy makers; (ii) incoming, observed data on the actual degree of implementation of ongoing plans. In this context Pérez Quirós, Pérez, and Paredes (2015); Riguzzi and Wegmueller (2015); Glocker (2013, 2012), among others, show that government (consumption) spending conveys useful information about ex-post policy developments relevant for GDP.

Table 2: Factor loadings

Model	GDP^f	Imports	Sales	Spread	Assets	REER	Loans	Orders	VSMI	PMI	UBSc
Core	0.37	0.11	0.09	-0.26	0.16						
	(0.12)	(0.09)	(0.02)	(0.10)	(0.08)						
DSFM	0.06	0.02	0.02	-0.06	0.02	-0.02	0.03	0.14	-0.15	0.27	0.23
	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.05)	(0.07)	(0.11)	(0.12)
MS-DFM	0.10	0.07	0.06	-0.15	0.05	-0.13	0.07	0.08	-0.15	0.25	0.22
	(0.03)	(0.01)	(0.00)	(0.05)	(0.02)	(0.08)	(0.04)	(0.03)	(0.02)	(0.05)	(0.05)

Note: For each model specification, the first row reports estimated factor loadings and (in brackets) their standard errors. See Table 1 for a description of the indicators, and Table 7 in the Appendix for a complete list of parameters estimated.

to enlargements of the model in various directions. We tested our model using disaggregated versions of the variables already included in the model. For instance, we used retail sales without oil related products instead of total retail sales. We failed at improving our model in all cases. Further we tested whether exchanging specific variables could lead to equal or similar results. In that case, we included vacancies as a labour market indicator and exports as an indicator for external demand; however, the factor loading of vacancies turned out to be not significantly different from zero, although the correlation between the factor and GDP would have been comparably high. For exports, the correlation decreased substantially. As a final test, we simulated the model including the complete set of 31 variables. The resulting correlation for this large sample factor model decreased to as low as 0.15.

4 Results

Section 4.1 presents the in-sample properties of the MS-DFM and contrasts them to the outcomes of the linear DSFM. In Section 4.2, we study the out-of-sample forecasting performance of the models outlined in Section 2 based on real-time GDP data. We compare the performance both for quarterly (short-term) and annual (medium-term) GDP growth. Further, we show the usefulness of our preferred model to detect turning points and to identify recessions in the Swiss business cycle.

4.1 In-sample properties

We present the estimated factor loadings in Table 2; with standard errors in parenthesis (the complete list of estimated parameters can be found in the Appendix). The estimates reflect the degree to which variations in each observed variable are correlated with the latent factor. The first two rows display the loadings estimated with the DSFM and the third row reports the loadings estimated with the MS-DFM. All variables in the model show statistically significant factor loadings. Qualitatively, there is no difference in factor loadings between the linear DSFM and the MS-DFM. Quantitatively, the difference is overall quite small. In the MS-DFM, the term spread and REER load more negatively on the factor, imports and sales slightly more positively. As a consequence of the autoregressive structure characterising the factor in the DSFM, it captures less of the short run volatility contained in these indicators, leading to slightly lower factor loadings (in absolute terms).

Table 3: Cumulative weights (in%)

Month	$GDP1^{st}$	GDP^f	**PMI**	**UBSc**	**Imports**	**Sales**	**Spread**	**REER**	**Loans**	**Orders**	**Assets**	**VSMI**
2016.06	100	0	0	0	0	0	0	0	0	0	0	0
2016.07	0	0	7.6	7.0	18.7	10.2	12.4	2.5	11.6	14.5	5.2	10.3
2016.08	0	0	8.0	7.4	19.4	10.6	12.5	2.5	11.9	13.3	5.2	9.2
2016.09	100	0	0	0	0	0	0	0	0	0	0	0
2016.10	0	0	7.6	7.0	18.7	10.2	12.4	2.5	11.6	14.5	5.2	10.3
2016.11	0	0	8.0	7.4	19.4	10.6	12.5	2.5	11.9	13.3	5.2	9.2
2016.12	100	0	0	0	0	0	0	0	0	0	0	0
2017.01	0	0	7.6	7.0	18.7	10.2	12.4	2.5	11.6	14.5	5.2	10.3
2017.02	0	0	8.0	7.4	19.4	10.6	12.5	2.5	11.9	13.3	5.2	9.2
2017.03	0	0	9.9	9.2	23.6	12.9	15.1	3.0	0	15.6	0	10.7
2017.04	0	0	19.5	0	0	0	29.2	0	0	30.3	0	21.0

Note: See Table 1 for acronyms, data transformation and a description of the indicators.

Table 3 reports the evolution of the forecast weights (standardized to sum up to 100) over the latest months.[24] Whenever the GDP figures are published (March, June, September and December), the cumulative forecast weights of all other indicators are zero. The series have weights different from zero only in those periods where GDP data are not available. The weights change according to the information set available at each point in time. In the first half of the month, the term spread (29.2%) and orders (30.3%) carry the largest weights; however, with more data becoming available, their weights decline. Although the real exchange rate and bank assets carry relatively low weights, they still incorporate non-negligible information about the stance of the Swiss business cycle and are therefore important for the performance of the model.

The maximum likelihood estimates of the Markov-switching elements are given in Table 4. The complete list of estimated parameters of the MS-DFM can be found in the Appendix. The maximum likelihood estimates imply that, as concerns the regime represented by $\zeta_t = 0$, the intercept is positive and statistically significant; while the regime represented by $\zeta_t = 1$, has a statistically significant negative intercept. Hence we can associate the first regime with economic expansions and the second regime with recessions. Table 4 also shows the mean values for the two states once they have been de-standardized ($\mu^*(\zeta_t = 0)$ and $\mu^*(\zeta_t = 1)$); these values imply that the average quarter-over-quarter growth rate of final GDP in the expansionary regime is around 0.6%, and -1.0% in the recessionary regime.

Our estimates for the transition probabilities are 0.98 for p_{00} and 0.86 for p_{11}, respectively. These estimates are in line with the well-known fact that expansions are longer than contractions, on average. In this context, p_{00} describes the probability that an expansion is followed by an expansion, and p_{11} captures the probability that a recession follows a recession. These estimates imply that the expected duration of an expansion is around 50 months ($= 1/(1-p_{00})$) and that of a recession of around 7 months ($= 1/(1-p_{11})$).

Based on these estimates, the upper subplot in Figure 1 shows the business cycle factors from the MS-DFM (orange bars). The factor based on the MS-DFM displays a high degree of non-linearity. The non-linearities appear to be particularly pronounced during recessionary episodes. In periods of an economic downturn, both business cycle factors tend to lead GDP growth slightly.

The upper subplot in Figure 1 also shows the recession probabilities. In particular, the regime

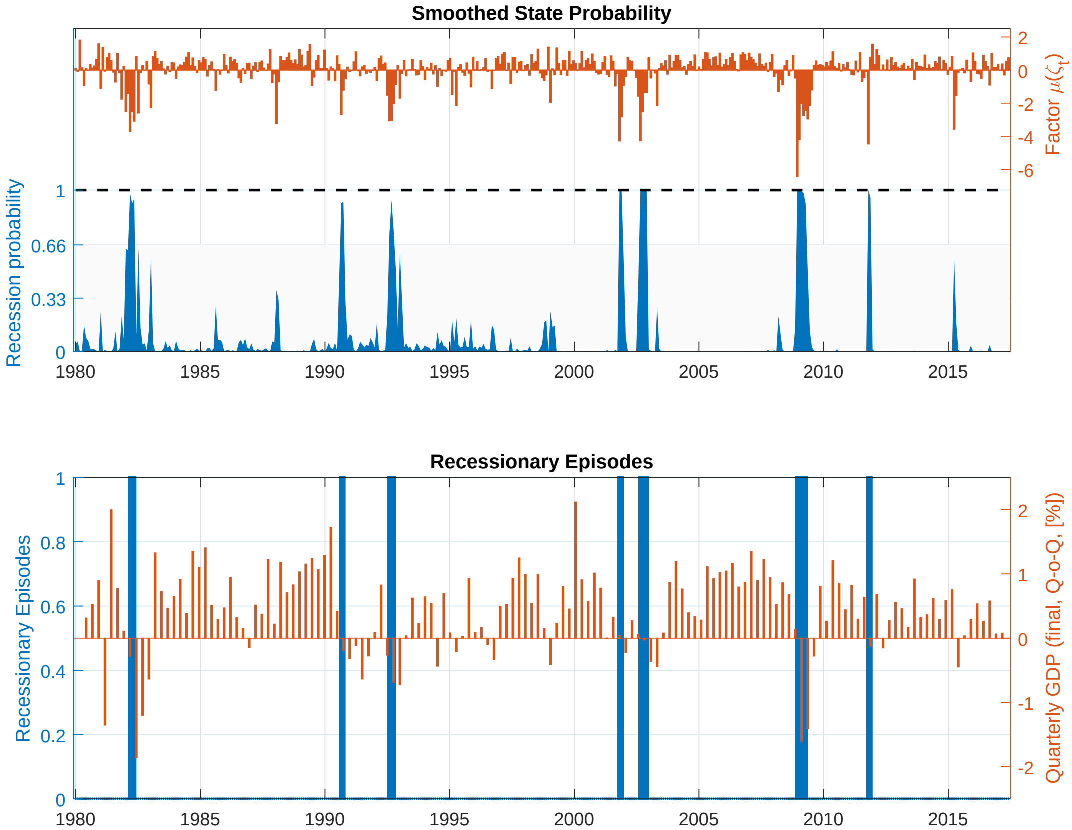
[24] The calculations are explained in detail in the Appendix.

Table 4: MS-DFM - Estimates

Parameters	$\mu(\zeta_t=0)$	$\mu^*(\zeta_t=0)$	$\mu(\zeta_t=1)$	$\mu^*(\zeta_t=1)$	p_{00}	p_{11}
Estimates	0.28 (0.06)	0.61%	-2.94 (0.46)	-1.04%	0.98 (0.17)	0.86 (0.20)

Note: $\mu^*(\zeta_t=0)$ and $\mu^*(\zeta_t=1)$ refer to the de-standardized values of $\mu(\zeta_t=0)$ and $\mu(\zeta_t=1)$, respectively. They refer to the conditional mean quarter-over-quarter growth rate of final GDP in either state. The values in parentheses are p-values for the null-hypothesis that the corresponding point estimate is zero.

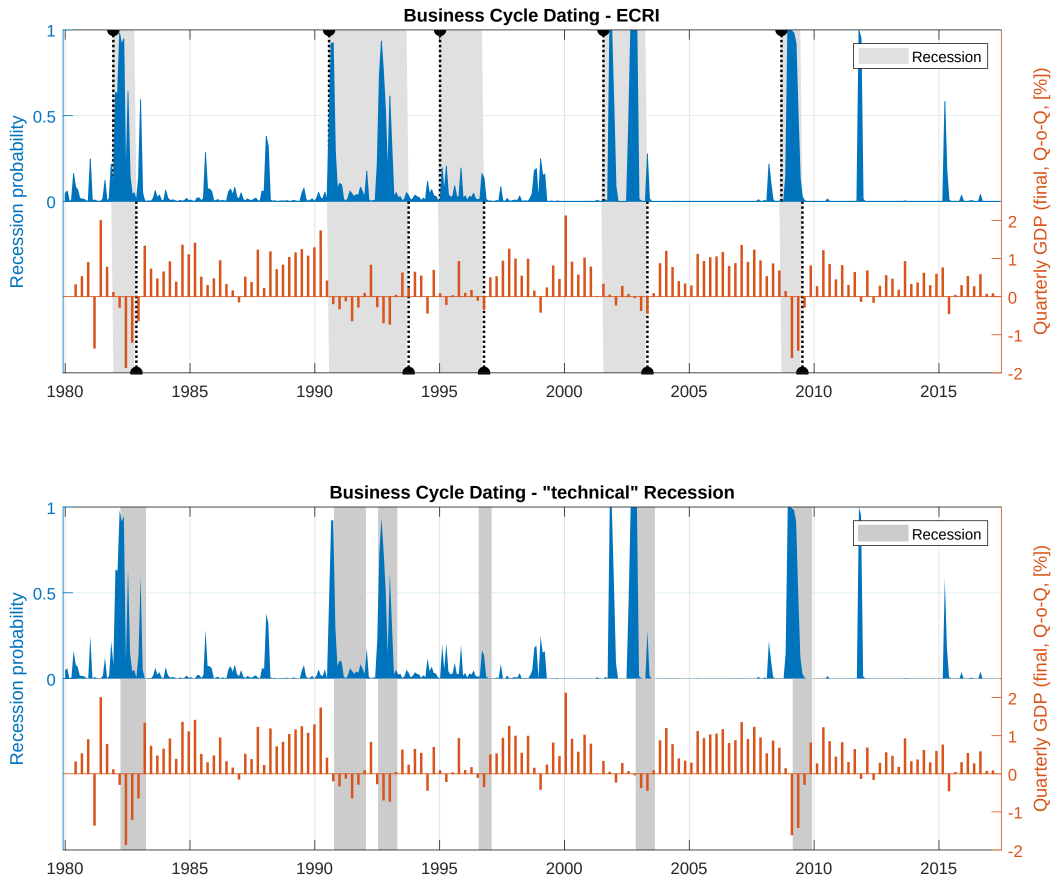
Figure 1: Recessionary Episodes - in sample estimates of state probability



Note: The upper subplot of the figure displays the business cycle factor from the MS-DFM, together with the smoothed state probability based on an in-sample estimation. The lower subplot shows the quarterly growth rate of final GDP and the recessionary episodes based on values of the smoothed state probability above a threshold value of 0.66.

probabilities refer to $prob(\zeta_t = 1|I_T;\boldsymbol{\vartheta})$ where $I_t = (\boldsymbol{X}_1, ..., \boldsymbol{X}_t)$ is the information set up to and including period t, and $t = 1, ..., T$ where T is the sample length and $\boldsymbol{\vartheta}$ is the vector comprising all estimated parameters of the MS-DFM. The regime probability $prob(\zeta_t = 1|I_T;\boldsymbol{\vartheta})$ allows to make inference about what regime was more likely to have been responsible for producing the date t observation of $\boldsymbol{X}_t$. It provides clear advice concerning several recessionary episodes. There are, however, also a few episodes where the regime probability is at a value of around 0.5 and hence the evidence concerning the prevailing state is uncertain. Following Nierhaus and Abberger (2015), we associate only those values of the regime probability with a recession which are above

Figure 2: Business Cycle Dating - Comparison



Note: The upper figure displays business cycle dating for the Swiss economy based on the Economic Cycle Research Institute (ECRI). The lower figure shows technical recessions, defined by two consecutive negative quarters of GDP growth. The grey bars characterize recessionary episodes. Additionally, the figures show our measure for the state probability for recessionary episodes from the MS-DFM and the quarterly final GDP growth rate.

0.66.[25] This leaves us with seven recessionary episodes, which are depicted in the lower subplot in Figure 1. These recessions are characterized by differences concerning the duration of the economic downswing as well as the deepness of the recession. Apparently, the removal of the Swiss Franc lower bound in January 2015 did not induce a regime shift. However, the period of strong appreciation of the currency between end-2007 and 2011 caused a regime switch. This provides an explanation why the SNB at that time decided to introduce the currency floor in the first place. Other recessions can be attributed to the global slump triggered by the second oil crisis in 1982, the domestic housing crisis in the mid-1990s, the DotCom-bubble of 2003 and the financial crisis of 2008-09.

In Figure 2 we compare the recession classification of our model with two alternative business cycle dating approaches. The grey bars in the upper subplot of the figure display recessionary

[25] We consider our decision rule as rather agnostic – a commonly used alternative threshold is a value of 0.5 (see for instance (Hamilton, 1989; Carstensen, Heinrich, Reif, and Wolters, 2017)). According to our results, the choice of threshold is of second order importance, as the smoothed state probabilities quickly jumps to one whenever a technical recession materialized.

episodes as proposed by the Economic Cycle Research Institute (ECRI).[26] Alternatively, the lower subplot of the figure displays recessionary episodes based on the technical definition for recessions. In that case, a recession is defined by two consecutive quarters of negative GDP growth. In the absence of an official recession dating committee, an advantage of the technical definition over alternative business cycle dating measures is its simplicity and timeliness. But, as the period 1995 to 1997 for Switzerland illustrates, the technical definition does not capture accurately recessionary phases in which GDP growth rates are close to zero but not necessarily negative. In that case it is instructive to consider a more judgemental measure like the one from ECRI.[27]

We find a high overlap of our recessionary regime probabilities (displayed in blue) with both the recession dating approach of ECRI and the technical recessions. Remarkably, the recession probabilities have a spike either at the beginning of a technical recession episode, or show a strong increase prior to the actual beginning of a recession. Our findings highlight the reliability of our model to identify recessionary episodes in a timely manner. Another observation is remarkable. In the case of severe monetary policy intervention in 2011 (introduction of exchange rate floor) and 2015 (abolition of exchange rate floor), the recession probability shortly increased to indicate a recession. In the latter case, it did not pass the threshold of 0.66, but was shortly greater than 0.5. Although in both cases a recession did not materialize, the Swiss economy experienced substantial downswings and periods of GDP growth rates below average.

4.2 Out-of-sample properties

The starting point of the out-of-sample exercise is the construction of a real-time data set. To do so, we follow the principle of putting the data available at a specific point in time into its corresponding cell within the so-called real-time data set. Thereby we ensure that at each point in time when a forecast is made, only the information available at that specific day is used. This allows to assess the models' forecasting performance in real-time.[28]

We construct our real-time data set on bi-weekly vintages. For each month within the period 2004-2016 we collect the whole set of time series available at the following two vintages: h1/mm/yy and h2/mm/yy; where h1 refers to the end of the first half of a month and h2 to the end of the same month. These vintages are kept fixed until the point in time when a new series was updated. Our analysis is truly real-time in the sense that we use the genuine real-time GDP

[26]ECRI classifies an episode as a *recession* in which companies dismiss employees, incomes fall, spending goes down, and output declines – the co-movement of all four variables is key. According to Lakshman and Banerji (2004), this definition provides clarity when it comes to determining if a recession has begun, unlike the popular "two quarters of negative GDP growth" rule of thumb, according to which, if GDP falls for two straight quarters, we have met the "technical" definition of a recession. GDP is just a measure of an economy's output. But if employment, income, and sales do not fall at the same time, the temporary period of negative-output growth will not catch on and spread, and no recession will occur.

[27]For completeness, we have also compared our recession estimates to the business cycle dates published by the OECD. Rather than recessions, this approach identifies the time between a business cycle peak and trough. The OECD business cycle phases are based on the growth-cycle approach, where cycles and turning points are measured and identified in the deviation from trend-series. Against this background, the OECD business cycle phases comprise only a vague basis of comparison. Nevertheless, our recession probabilities are well in line with the identified downswings – we would like to thank an anonymous referee who pointed this out.

[28]An evaluation of forecast errors by using the ex-post data for a specific point in time is questionable since measures of forecast errors – as root-mean-squared error (RMSE) – can be deceptively lower when using ex-post data for GDP rather than real-time data (Stark and Croushore, 2002).

series from Indergand and Leist (2014). With respect to the monthly indicators the exercise is pseudo real-time, i.e., we use the latest available data vintage.[29] This data set allows us to closely mimic the forecasting procedure a practitioner would have performed at any time during the last few years when computing model forecasts. Table 1 shows which of the time series got updated at which of the two vintages (h1 or h2). The first vintage for which we collect data for all indicators is vint-h1/01/04. We end up with 312 different vintages for the period h1/01/04 to h2/12/16.

4.2.1 Short-run forecasting

Our real-time data set allows to assess the gain in prediction precision of the models once further prompt observations are added. We calculate predictions for (i) forecasts, (ii) nowcasts, and (iii) backcasts of quarterly Swiss real GDP growth. For the nowcast of quarter t_q, we use all information up to and including the middle of quarter t_q, that is, the middle of the second month of quarter t_q. Similarly, we compute backcasts based on information up to and including one month after quarter t_q ended. Finally, forecasts are made with information available six months before the end of t_q. We do so for all quarters from Q1:2004 until Q4:2016.

Table 5 reports in the first row the mean-squared error (MSE) statistics of the MS-DFM. Next, rows 2 to 6 report the MSE of the MS-DFM relative to the MSE of DSFM, the MF-FAVAR, two benchmark models – a random walk (RW) and an autoregressive model of order two (AR) – and the Bloomberg Consensus forecasts (available only for the period 2014Q4-2016Q4). Both benchmark models are estimated with real-time data to produce predictions. The last five rows display the p-values resulting from the modified Diebold-Mariano test.[30]

Overall, the linear DSFM displays a slightly smaller MSE than the MS-DFM for backcasts, whereas the MS-DFM tends to have a slightly a smaller MSE for nowcasts. Their performance is very much comparable and differences are in no case significantly different from zero. Compared to the other models and the consensus forecast, the gain in using the MS-DFM in forecasting GDP essentially depends on the horizon. As concerns backcasts, the MS-DFM significantly outperforms both benchmark models and the MF-FAVAR. It performs also significantly better than the consensus forecast when considering GDP^{1st}. For nowcasts, the performance of the MS-DFM is still remarkable. When it comes to forecasts, however, the MSE of the MS-DFM is only significantly lower compared to the random walk forecast. Compared to the rest, the forecasting performance of the MS-DFM is not significantly better, though its MSE statistics are smaller for GDP^{1st}.

4.2.2 Medium-run forecasting

We extend the forecasting horizon to evaluate the models' performance in predicting annual growth rates of GDP. Particularly among policy institutions and practitioners, the prediction of

[29]To the best of our knowledge, there is no real-time data of monthly Swiss economic indicators publicly available. Of the ten monthly indicators, only imports and sales might have undergone substantial revisions. Financial variables are not revised, and revisions to survey data are seldom and at most marginal.

[30]Diebold and Mariano (1995) provide a pairwise test to analyse whether the differences between two or more competing models are statistically significant. As there is potentially a short-sample problem, we apply the modified version of the Diebold-Mariano test according to Harvey, Leybourne, and Newbold (1997).

Table 5: Predictive accuracy

	GDP^{1st}			GDP^{f}		
	Backcasts	**Nowcasts**	**Forecasts**	**Backcasts**	**Nowcasts**	**Forecasts**
Mean squared errors						
MS-DFM	0.06	0.06	0.08	0.11	0.11	0.20
Relative performance of MS-DFM to						
RW	0.36	0.29	0.29	0.15	0.15	0.28
AR	0.48	0.39	0.54	0.25	0.30	0.60
DSFM	1.02	0.96	0.98	1.03	0.97	1.08
MF-FAVAR	0.63	0.50	0.62	0.38	0.33	0.66
Consensus	0.79	0.51	0.85	1.07	0.74	1.32
Equal predictive accuracy tests (p-values)						
RW	0.00	0.02	0.03	0.00	0.04	0.06
AR	0.00	0.07	0.14	0.01	0.11	0.21
DSFM	0.96	0.92	0.95	0.91	0.86	0.93
MF-FAVAR	0.03	0.06	0.17	0.03	0.14	0.26
Consensus	0.03	0.32	0.56	0.68	0.38	0.80

Note: Totally 52 quarters in the period 2004Q1-2016Q4 are evaluated. Entries in rows 2-6 are mean squared errors (MSE) of MS-DFM relative to a random walk (RW), an autoregressive process of order two (AR), the linear dynamic single-factor model (DSFM), the Mixed Frequency-Factor Augmented Vector Autoregressive (MF-FAVAR) model and the Bloomberg Consensus forecast. The last five rows display the p-values of the *modified* Diebold-Mariano test of equal forecast accuracy according to Harvey, Leybourne, and Newbold (1997). In contrast to the forecasts of all models, the Consensus-forecasts used here rest on a rather small number of observations (2014Q4-2016Q4); hence the MSE-statistics should be interpreted with care.

annual growth rates is of importance (for instance for the budgetary process of the government). In the case of Switzerland, every quarter the Federal Government's Expert Group publishes the official forecast for annual Swiss GDP growth. This *judgemental* forecast is the outcome of a discussion among several federal agencies.[31] This forecast is usually published around 10 days after the release of the quarterly estimate of Swiss GDP. While the judgement of experts might be helpful in increasing the precision of forecasts, the disadvantage of such judgemental forecasts is that they might be blurred by individual optimism or pessimism. For our analysis, we collect forecasts from the Federal Government's Expert Group starting in 2002Q4 up to 2016Q4. The forecast horizon varies from 1 to 8 quarters.

Apart from the official forecast, a variety of institutions provide forecasts of Swiss GDP growth.[32] A combination of such forecasts might also provide a more accurate prediction of GDP growth as if only one institution is considered. For a series of countries, the Economist Poll of Forecasters provides GDP forecasts by averaging the predictions of several major banks. This procedure is similar to a *consensus* forecast. These forecasts are available on a monthly basis. We collect these data starting in March 2003. The first year to be predicted is 2004 and the forecast horizon is between 1 and 24 months.

Table 6 reports the results from comparing the performance of the MS-DFM with the judgemental and consensus forecast as well as with the other competing models. We use our real-time analysis and generate forecasts for GDP^{1st} for as many quarters as necessary to complete the current and the following year. The Expert Group's and Consensus forecasts are based on the vintage of GDP available at the time of the forecast, i.e., they include revisions to GDP. To make the exercise comparable, we therefore appended our model forecasts to the corresponding GDP

[31]Participants of the meeting are the State Secretariat for Economic Affairs (SECO), the Federal Customs Administration (FCA), the Swiss Federal Statistical Office (FSO), the Federal Finance Administration (FFA) and the Swiss National Bank (SNB).

[32]For instance major banks or economic research institutes, among others.

Table 6: Predictive accuracy for annual growth rates

Horizon [quarters]	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**
Mean squared errors								
MS-DFM	1.727	1.524	0.971	0.352	0.360	0.269	0.117	0.013
Relative performance of MS-DFM to								
RW	0.30	0.32	0.35	0.24	0.25	0.44	0.65	0.57
AR	1.10	0.94	0.62	0.31	0.45	0.75	0.80	0.50
DSFM	1.01	1.01	1.02	0.98	1.08	1.00	0.98	0.93
MF-FAVAR	1.08	0.95	0.62	0.43	0.60	0.82	0.84	0.46
Consensus	1.03	0.98	0.97	0.77	1.14	1.07	0.90	0.17
Judgmental	0.98	0.81	0.68	0.49	1.06	0.81	1.86	0.57
Equal predictive accuracy tests (p-values)								
RW	0.22	0.29	0.26	0.14	0.04	0.04	0.03	0.18
AR	0.56	0.72	0.39	0.32	0.03	0.06	0.06	0.27
DSFM	0.93	0.96	0.93	0.91	0.82	0.82	0.74	0.81
MF-FAVAR	0.61	0.75	0.44	0.43	0.06	0.09	0.09	0.10
Consensus	0.82	0.62	0.62	0.33	0.89	0.88	0.47	0.28
Judgmental	0.71	0.33	0.35	0.16	0.94	0.71	0.10	0.04

Note: Totally 13 years in the period 2004-2016 are evaluated. Consensus refers to the Economist Poll of Forecasters. Entries in rows 2-6 are mean squared error (MSE) of the MS-DFM relative to a random walk (RW), an autoregressive process of order two (AR), the linear dynamic single-factor model (DSFM), the Mixed Frequency-Factor Augmented Vector Autoregressive (MF-FAVAR) model, the Consensus forecast and the forecast from the Expert Group. The last six rows display the p-values of the *modified* Diebold-Mariano test of equal forecast accuracy according to Harvey, Leybourne, and Newbold (1997).

vintage and calculate annual growth rates.[33] Again, we compute relative mean squared errors (MSE) and test for significant differences via the modified Diebold-Mariano test.

Several results emerge: (i) the MS-DFM outperforms the random walk model at all horizons; (ii) in predicting growth of the following year (horizon quarter 8-5), the MS-DFM outperforms the judgemental forecast; (iii) the MF-FAVAR performs significantly worse than the MS-DFM in predicting current years growth (iv) the DSFM tends to perform slightly better than the MS-DFM for longer horizons, but the performance is nearly indistinguishable as regards their MSE statistics and hence their forecast accuracy.

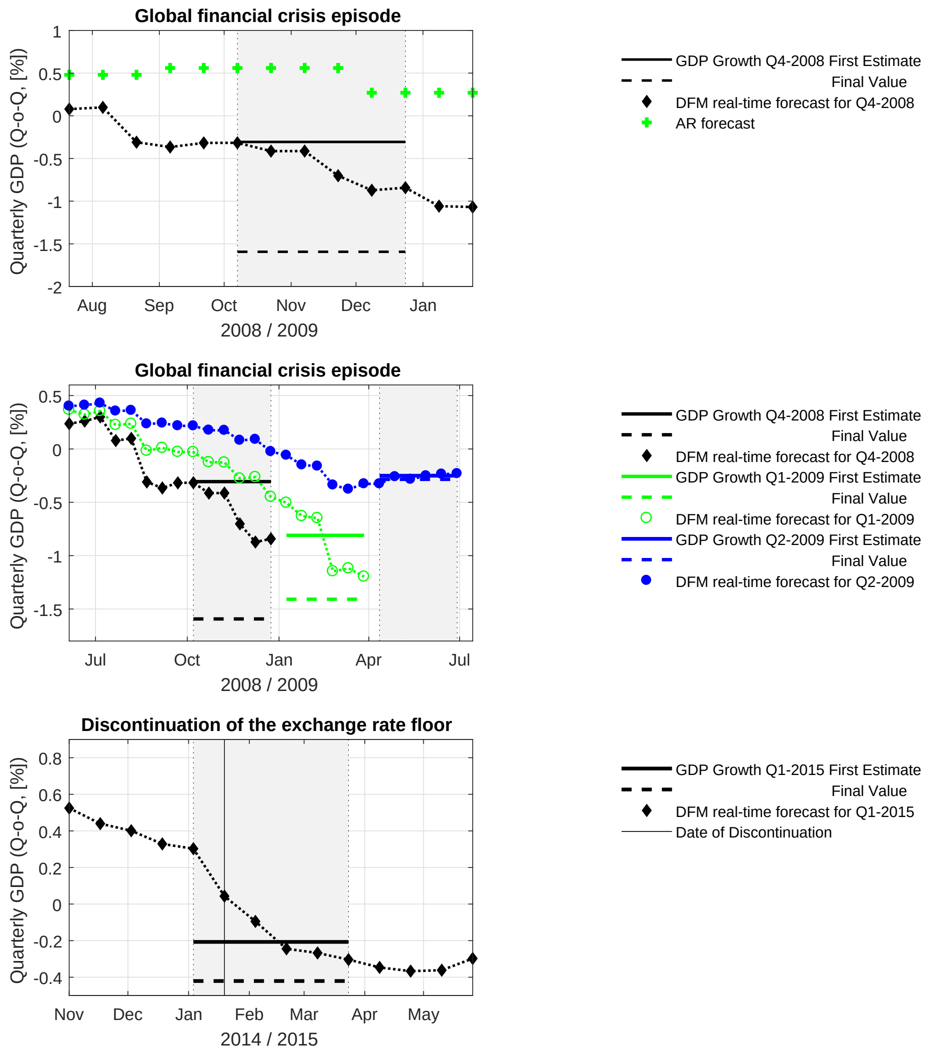
Following the methodology of Harvey, Leybourne, and Newbold (1997), we report results for the equal predictive accuracy tests in the last six rows in Table 6. Notably, at the 10% significance level the MS-DFM outperforms the RW benchmark, the AR-model and the MF-FAVAR at the two, three, and four quarters horizon. Compared to consensus forecasts, the performance of the MS-DFM is not significantly better at any horizon. In comparison to the judgemental predictions, the MS-DFM has better predictive power in the very short horizon of one quarter. This is in line with the results presented in Section 4.2.1 and confirms the fact that dynamic factor models are particularly well suited for short-term forecasting. There is no evidence that the forecasts of the MS-DFM have a higher predictive accuracy than those of the DSFM.

4.2.3 Detecting turning points

Besides investigating the models' forecasting performance over the sample, of particular interest is to study how the model performs during specific historic episodes. Our real-time data-set allows for such an assessment. We focus our analysis on two distinct episodes: (1) the global

[33] Consider the Appendix for the technical details on the calculation of annual GDP growth rates from quarterly growth rates.

Figure 3: Forecasting in real-time - different episodes



Note: The figure plots real-time forecasts of the Markov-switching Dynamic Factor Model (MS-DFM) for various episodes. The first subplot shows real-time forecasts for Q4:2008 (black diamonds) jointly with a forecast based on an AR(2)-model (green line) and the GDP growth rate for Q4:2008 (first estimate and final value; black dashed and solid lines). The second subplot shows the real-time forecasts for Q4:2008, Q1:2009 and Q2:2009 jointly with the corresponding GDP growth rates for each quarter. Finally, the third subplot shows real-time forecasts for Q1:2015 and the GDP growth rate for Q1:2015 (first estimate and final value; black dashed and solid lines).

financial crisis of 2008-09; and (2) the first quarter of 2015 when the Swiss economy was exposed to a significant monetary policy shock. We focus on the MS-DFM here.

The global financial crisis

The upper subplot in Figure 3 shows the forecast trajectory of the model for the fourth quarter 2008 which was made at different points in time.[34] This plot is useful in order to address one important question: When did the authorities realize that the downturn had started?

The model's forecasts for GDP growth for 2008-Q4 (black line-dotted path) were positive

[34] We have omitted the confidence bands for better visibility of the point estimates.

until the end of July. The model predicted a negative growth rate for 2008-Q4 the first time by mid-August 2008 (vint-h1/08/08). There are two variables at work, which drive this significant drop; these are on the one hand the PMI – it dropped from 48.5 (July) down to 41.4 (August) – as well as the consumption indicator – it dropped from 2.0 (June) down to 1.78 (July). Noticeably, the model's forecast for 2008-Q4 GDP growth made at mid-August is already very close to the first estimate (black horizontal solid line), which in turn was published at the beginning of March 2009. We conclude that the model gave a rather precise projection for the first estimate of 2008-Q4 GDP growth, seven months ahead of the first official GDP release.

The second subplot of Figure 3 displays the forecasts for some quarters from the outbreak of the global financial crisis onwards. The episode was marked by substantial financial turbulence and Switzerland recorded the first recession in several years (see also Figure 2). We produce forecasts for the quarters 2008-Q4 – 2009-Q2. Next to each quarter's projections, the figure also displays the first estimate (solid horizontal line) and the final value (dashed horizontal line) of the growth rate of GDP for the aforementioned quarters.

The black dotted line is the same as in the upper subplot. The projections for 2009-Q1 and 2009-Q2 follow a similar trajectory as the one for 2008-Q4. The predictions for the growth rate of GDP drop in mid-summer 2008 and decline further the lower the forecasting horizon. For each quarter depicted in the subplot, the model's projections match the first estimate of the GDP growth rate quite well.

Severe monetary policy interventions

Between end-2007 and 2011, the Swiss Franc (CHF) appreciated sharply against the Euro (EUR) and against the US-Dollar (USD). This appreciation made the Swiss National Bank (SNB) believe that it posed "an acute threat to the Swiss economy". On September 6, 2011, the SNB put in place an exchange rate floor of 1.20 CHF/EUR to avoid a further appreciation of the Franc against the Euro. To the surprise of markets and institutions, on January 15, 2015, SNB removed the exchange rate floor of 1.20 CHF/EUR. Within one day, the CHF appreciated more than 19% to 1.01 CHF/EUR.

In what follows we analyse the extent to which the MS-DFM was able to capture the effect of the monetary policy shock in January 2015. The empirical model allows for two transmission channels of the monetary policy intervention: (1) the news channel[35] and (2) the financial market channel. Severe policy changes are usually associated with a change in economic agents' perception of the future economic outlook. In our MS-DFM, news are considered as changes in economic sentiment, which in turn are captured by soft indicators. The MS-DFM contains two such indicators (PMI, UBSc). They allow for an immediate effect, as they are likely to be affected already at the time of the announcement of the new policies. The latter channel is captured by means of the real effective exchange rate, the term structure and the implicit stock market volatility index. They are promptly available and characterise the effect of the monetary policy interventions that operates by means of financial markets.

[35]In this context, news does not refer to data revisions as in Section 2.2, but rather to economic sentiment and the surprises therein.

In the bottom subplot of Figure 3, we show the model's projections for 2015-Q1. The monetary policy shock triggered some immediate contractionary effects for real economic activity with the result of a negative GDP growth rate. The first official release of GDP data for this quarter was in June 2015, more than five months after the monetary policy shock. We report the growth projections for 2015-Q1 at different points in time. Up to and including the information until mid-January 2015, the growth projections remain between 0.3% and 0.5%. However, at the end of January (vint-h2/01/15) they drop significantly owing to a strong decline in the PMI. In mid-February, the most recent values for the real effective exchange rate and the implicit stock market volatility index for January are added to the information set, rendering the nowcast for GDP growth negative. At the end of February, the model's predictions are basically indistinguishable from the first official GDP estimate.

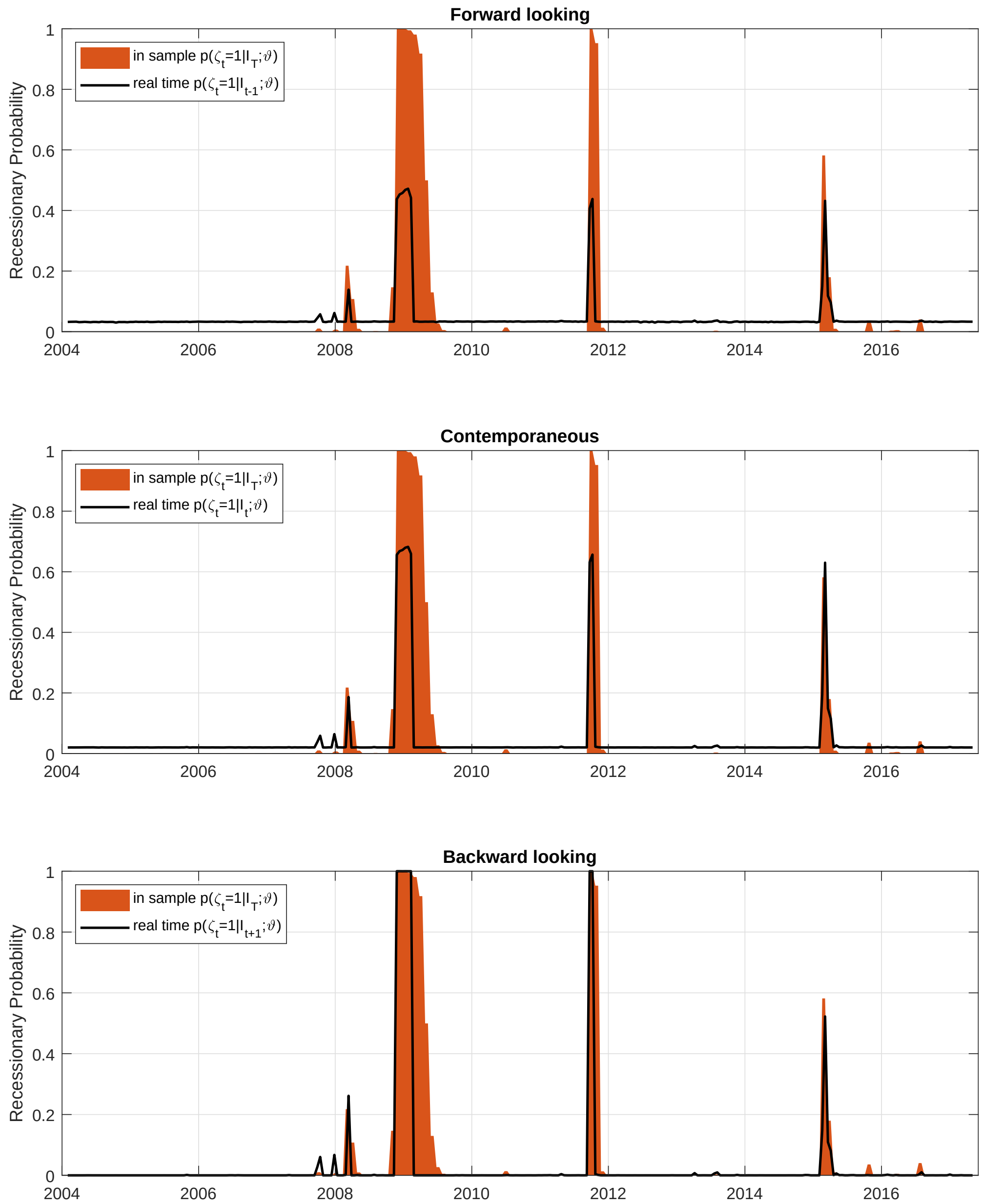
4.2.4 Assessing recessionary episodes in real-time

The regime probabilities presented in Section 4.1 were based on information on the whole sample. However, due to different information sets and data revisions, the real-time data can be deceptively less helpful in monitoring real activity. Against this background, we evaluate the performance of our MS-DFM in tracking past Swiss business cycles shifts in real time by means of the real-time data set.

We compute out-of-sample state probabilities of recessionary episodes for different information sets to evaluate in how far new information changes the model's assessment of the current regime. We consider the following three measures for the recessionary regime probability: (i) $prob(\zeta_t = 1|I_{t-1}; \boldsymbol{\vartheta})$, (ii) $prob(\zeta_t = 1|I_t; \boldsymbol{\vartheta})$, and (iii) $prob(\zeta_t = 1|I_{t+1}; \boldsymbol{\vartheta})$. The first case estimates the regime probability at time t with information up to and including time $t-1$, the estimate for the regime probability at time t is hence a forecast. The second probability measure captures the contemporaneous scenario; it evaluates the regime probability at time t considering all information up to and including time t. Finally, the third probability measure considers the backward looking scenario: it evaluates the regime probability at time t considering all information up to and including time $t + 1$.

Figure 4 shows the corresponding path of the regime probabilities for each scenario (black solid line). It contrasts the estimates of the real-time regime probabilities with the in-sample estimates (orange bars). In the forward looking scenario, the model gives only a vague hint for each recessionary episode. The estimated regime probabilities for a recession always remain below 0.5. This applies also to the episode surrounding the global financial crisis of 2008-09. In the contemporaneous scenario, the model's real-time estimates for the recessionary regime probabilities match their in-sample counterpart already to a large extent. The probabilities for the 2008-09 recession and the one in 2011-Q3/Q4 are already larger than 2/3 and hence would be classified as recessionary episodes by our classification. Finally, in the backward looking case, the recessionary regime probabilities from the real-time estimation can hardly be distinguished from the in-sample estimates. However, there is still one noteworthy difference: the in-sample estimates for the recessionary regime probabilities point towards a longer recession duration; in other words, the real-time estimates of the recessionary regime probabilities tend to underestimate the

Figure 4: Recessionary Episodes - real-time estimates of state probability



Note: The figure shows different estimates for recessionary episodes based on real-time out-of-sample estimations of the state probability at time t. The subplots compare the real-time estimates of the recessionary episodes with the in-sample estimates for the recessionary probabilities based on the whole sample.

duration of the recessionary episodes considered in Figure 4 to some extent.

5 Robustness analysis

We check the robustness of the MS-DFM across various dimensions. First, there is the possibility that the combinatorial algorithm for indicator selection is subject to sample-dependency.

Second, the MS-DFM features only a limited number of variables. It could still be the case that introducing at least one further variable could lead to a significant model improvement. Finally, we evaluate our assumption of the lag-length of the autoregressive processes and the number of latent factors.

5.1 Sub-sample instability

The combinatorial algorithm used in Section 3.1 for selecting an appropriate set of variables has been applied to the whole sample. However, structural changes in the underlying economic dynamics could influence the choice of indicators over time or lead to time-variation in the estimated parameters, rendering the MS-DFM less reliable for forecasting purposes. Against this background, we analyse the extent to which the fit of our MS-DFM varies over time. We consider time-variation in the factor loadings γ_q, γ_s and γ_h, in the variances σ_q^2 and $diag(\Sigma_\nu)$ and in the model fit measured by the R^2 from a regression of the factor on the (final) GDP growth rate. We consider a time-span starting in January 2004 and ending in December 2016 and apply a rolling regression approach: we estimate the model based on a sample starting in 01:1980 until 01:2004 and recursively extend the window by adding an additional month until we end up in 12:2016. For each step we fully estimate the model.

We observe that some degree of time-variation is present, however, it is fairly small. The time-variation of the factor loadings is smaller than their estimate of the in-sample standard deviation. The same finding also applies to the variances and to the overall model fit.[36] We conclude that problems related to sub-sample instabilities are negligible.

5.2 Additional variables

Our proposed dynamic factor model features only ten monthly indicators. Although applications of this approach for other countries use a similar number of variables, it could still be possible that further variables improve the model fit. For this reason, we estimate the MS-DFM again, but now with an additional eleventh variable. For each eleventh variable we evaluate the model fit by means of the R^2. We find that the inclusion of an eleventh variable generally renders the model fit worse. We could not find a single variable yielding a noticeably higher R^2 than the Benchmark model.

5.3 Lag-length of the autoregressive processes

The combinatorial algorithm discussed in Section 2.4 is based on the prior assumption of a lag-order equal to two of the autoregressive processes in the system of dynamic equations (6)-(7). The lag-length is, of course, a testable assumption and should preferably be checked for each set of variables within the combinatorial algorithm. This is, however, computationally not tractable. Instead, we evaluate the plausibility of this assumption by using information criteria. We calculate Akaike's information criterion (AIC) and Schwartz's Bayesian information criterion (SBIC). AIC selects a lag-order of three and SBIC selects a lag-order of two. Given these results, we consider a lag-order of two as appropriate for at least two reasons: first of all, simplier models

[36]The results for this are available upon request.

should be preferred, and secondly, the AIC information criterion is fairly indifferent between a model with two and/or three lags.

5.4 Number of factors in DSFM

We consider the DSFM, however, with two factors.[37] We assess the performance of this model by means of MSE. Considering the Mariano-Diebold test statistics, there is no statistical evidence that the model with two factors performs better than the single-factor model. Moreover, the MSE error statistics of the two models are fairly similar. Hence preference is given to single-factor models as they comprise a more parsimonious specification.

6 Conclusions

Fundamental policy changes have the potential of introducing a regime switch. The prevalence of a regime switch, or of non-linearities in general, pose a challenge for linear forecasting models. The Swiss economy has witnessed several incidents of severe policy realignments recently. Against this background, we propose a dynamic factor model (DFM) with a non-linear element for the Swiss economy. We assess non-linearities by using a two-state Markov-chain. The dynamics within a particular state are characterized by a linear specification, whereas the switch across states depicts a non-linear element.

Our model estimates confirm the importance of considering non-linear elements. Additionally, the Markov-switching specification allows for inference on the business cycle stance of the Swiss economy. In an out-of-sample exercise, we show that our model provides an effective tool for recession dating in real-time.

Finally, we find that the forecasting performance of the MS-DFM turns out to be as good as peers that allow for richer dynamics. Moreover, in the short-run its predictions are significantly more accurate than those of expert judgement and alternative baseline models.

[37] This implies that equation (16) comprises two independent autoregressive processes. We maintain a lag order of two for all lag polynomials. The state-space representation of the two-factor model comprises a straight forward extension to the one of the single-factor model outlined in the Appendix. Details on this and on the estimated coefficients of the model are available upon request.

References

Aastveit, K., and T. Trovik (2012): "Nowcasting norwegian GDP: the role of asset prices in a small open economy," *Empirical Economics*, 42(1), 95–119.

Adrian, T., and H. S. Shin (2010): "Liquidity and leverage," *Journal of Financial Intermediation*, 19(3), 418–437.

Ang, A., M. Piazzesi, and M. Wei (2006): "What does the yield curve tell us about GDP growth?," *Journal of Econometrics*, 131(1-2), 359–403.

Aruoba, S. B. (2008): "Data revisions are not well behaved," *Journal of Money, Credit and Banking*, 40(2-3), 319–340.

Bai, J., and S. Ng (2008): "Forecasting economic time series using targeted predictors," *Journal of Econometrics*, 146(2), 304–317.

Bańbura, M., D. Giannone, and L. Reichlin (2010): "Large Bayesian vector auto regressions," *Journal of Applied Econometrics*, 25(1), 71–92.

Bańbura, M., and G. Rünstler (2011): "A look into the factor model black box: Publication lags and the role of hard and soft data in forecasting GDP," *International Journal of Forecasting*, 27(2), 333–346.

Barhoumi, K., O. Darné, and L. Ferrara (2010): "Are disaggregate data useful for factor analysis in forecasting French GDP?," *Journal of Forecasting*, 29(1-2), 132–144.

Boivin, J., and S. Ng (2006): "Are more data always better for factor analysis?," *Journal of Econometrics*, 132(1), 169–194.

Calhoun, G., and G. Elliott (2012): "Why Do Nonlinear Models Provide Poor Macroeconomic Forecasts?," *Seventh ECB Workshop on Forecasting Techniques-New Directions for Forecasting, Frankfurt am Main, Germany.*

Camacho, M., M. Dal Bianco, and J. Martinez-Martin (2015): "Toward a more reliable picture of the economic activity: An application to Argentina," *Economics Letters*, 132, 129–132.

Camacho, M., and A. Garcia-Serrador (2014): "The Euro-Sting Revisited: The Usefulness of Financial Indicators to Obtain Euro Area GDP Forecasts," *Journal of Forecasting*, 33(3), 186–197.

Camacho, M., and G. Perez-Quiros (2010): "Introducing the euro-sting: Short-term indicator of euro area growth," *Journal of Applied Econometrics*, 25(4), 663–694.

——— (2011): "Spain-Sting: Spain Short-Term Indicator Of Growth," *The Manchester School*, 79(s1), 594–616.

Camacho, M., G. Perez-Quiros, and P. Poncela (2015): "Extracting nonlinear signals from several economic indicators," *Journal of Applied Econometrics*, 30(7), 1073–1089.

——— (2018): "Markov-switching dynamic factor models in real time," *International Journal of Forecasting*, 34(4), 598–611.

Carstensen, K., M. Heinrich, M. Reif, and M. H. Wolters (2017): "Predicting Ordinary and Severe Recessions with a Three-State Markov-Switching Dynamic Factor Model. An Application to the German Business Cycle," CESifo Working Paper Series 6457, CESifo Group Munich.

Chauvet, M. (1998): "An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching," *International Economic Review*, 39(4), 969–996.

Chernis, T., and R. Sekkel (2017): "A dynamic factor model for nowcasting Canadian GDP growth," *Empirical Economics*, 53(1), 217–234.

D'Agostino, A., K. McQuinn, and D. O'Brien (2012): "Nowcasting Irish GDP," *OECD Journal: Journal of Business Cycle Measurement and Analysis*, 2012(2), 21–31.

DeJong, D. N., R. Liesenfeld, and J.-F. Richard (2005): "A non-linear forecasting model of GDP growth," *The Review of Economics and Statistics*, 87(4), 697–708.

Diebold, F. X., and R. S. Mariano (1995): "Comparing Predictive Accuracy," *Journal of Business & Economic Statistics*, 13(3), 253–263.

Diebold, F. X., and G. D. Rudebusch (1996): "Measuring business cycles: a modern perspective," *The Review of Economics and Statistics*, 78(1), 67–77.

Duprey, T., B. Klaus, and T. Peltonen (2017): "Dating systemic financial stress episodes in the EU countries," *Journal of Financial Stability*, 32(C), 30–56.

Faust, J., and J. H. Wright (2013): "Forecasting Inflation," *Part A of Handbook of Economic Forecasting*, 2, 2–56.

Galli, A. (2018): "Which Indicators Matter? Analyzing the Swiss Business Cycle Using a Large-Scale Mixed-Frequency Dynamic Factor Model," *Journal of Business Cycle Research*, 14(2), 179–218.

Galli, A., C. Hepenstrick, and R. Scheufele (2017): "Mixed-frequency models for tracking short-term economic developments in Switzerland," Working Papers 2017-02, Swiss National Bank.

Giannone, D., L. Reichlin, and D. Small (2008): "Nowcasting: The real-time informational content of macroeconomic data," *Journal of Monetary Economics*, 55(4), 665–676.

Glocker, C. (2012): "Unemployment compensation and aggregate fluctuations," *International Review of Economics*, 59(1), 21–39.

——— (2013): "Government expenditures and business cycles - policy reaction and surprise shocks," *The Journal of Applied Economic Research*, 7(3), 215–254.

Glocker, C., and W. Hölzl (2019): "Assessing the economic content of direct and indirect business uncertainty measures," WIFO Working Papers 576, Austrian Institute of Economic Research.

Glocker, C., and S. Kaniovski (2014): "A financial market stress indicator for Austria," *Empirica*, 41(3), 481–504.

Hamilton, J. D. (1989): "A new approach to the economic analysis of non-stationary time series and the business cycle," *Econometrica*, 57(2), 357–384.

Harvey, D., S. Leybourne, and P. Newbold (1997): "Testing the equality of prediction mean squared errors," *International Journal of Forecasting*, 13(2), 281–291.

Indergand, R., and S. Leist (2014): "A Real-Time Data Set for Switzerland," *Swiss Journal of Economics and Statistics*, 150(4), 331–352.

Kim, C.-J. (1994): "Dynamic linear models with Markov-switching," *Journal of Econometrics*, 60(1-2), 1–22.

Kim, C.-J., and J.-S. Yoo (1995): "New index of coincident indicators: A multivariate Markov switching factor model approach," *Journal of Monetary Economics*, 36(3), 607–630.

Lakshman, A., and A. Banerji (2004): *Beating the Business Cycle : How to Predict and Profit from Turning Points in the Economy*. Crown Business.

Mankiw, N. G., and M. D. Shapiro (1986): "News or Noise? An Analysis of GNP Revisions," Discussion Paper 1939, National Bureau of Economic Research, Inc.

Marcellino, M., M. Porqueddu, and F. Venditti (2016): "Short-Term GDP Forecasting With a Mixed-Frequency Dynamic Factor Model With Stochastic Volatility," *Journal of Business & Economic Statistics*, 34(1), 118–127.

Marcellino, M., and C. Schumacher (2010): "Factor MIDAS for nowcasting and forecasting with ragged-edge data: a model comparison for German GDP*," *Oxford Bulletin of Economics and Statistics*, 72(4), 518–550.

Mariano, R. S., and Y. Murasawa (2003): "A new coincident index of business cycles based on monthly and quarterly series," *Journal of Applied Econometrics*, 18(4), 427–443.

Nierhaus, W., and K. Abberger (2015): "ifo Konjunkturampel revisited," *Ifo Schnelldienst*, 68(05), 27–32.

Pérez Quirós, G., J. J. Pérez, and J. Paredes (2015): "Fiscal targets. A guide to forecasters?," Working Paper Series 1834, European Central Bank.

Proietti, T., and F. Moauro (2006): "Dynamic factor analysis with non-linear temporal aggregation constraints," *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 55(2), 281–300.

Riguzzi, M., and P. Wegmueller (2015): "Economic Openness and Fiscal Multipliers," Diskussionsschriften dp1504, Universitaet Bern, Departement Volkswirtschaft.

Rünstler, G., K. Barhoumi, S. Benk, R. Cristadoro, A. D. Reijer, A. Jakaitiene, P. Jelonek, A. Rua, K. Ruth, and C. V. Nieuwenhuyze (2009): "Short-term forecasting of GDP using large datasets: a pseudo real-time forecast evaluation exercise," *Journal of Forecasting*, 28(7), 595–611.

Rusnák, M. (2016): "Nowcasting Czech GDP in real time," *Economic Modelling*, 54(C), 26–39.

Stark, T., and D. Croushore (2002): "Forecasting with a real-time data set for macroeconomists," *Journal of Macroeconomics*, 24(4), 507–531.

Stock, J. H., and M. W. Warson (1992): *A probability model of the coincident economic indicators*p. 63–90. Cambridge University Press.

Stock, J. H., and M. W. Watson (1999): *A Comparison of Linear and Nonlinear Univariate Models for Forecasting Macroeconomic Time Series*pp. 1–44. Oxford University Press, Oxford.

Umer, U. M., T. Sevil, and G. Sevil (2018): "Forecasting performance of smooth transition autoregressive (STAR) model on travel and leisure stock index," *The Journal of Finance and Data Science*, 4(2), 90–100.

Wheelock, D. C., and M. E. Wohar (2009): "Can the term spread predict output growth and recessions? A survey of the literature," *Federal Reserve Bank of St. Louis Review*, 91(5 Part 1), 419–440.

A Appendix

The Sections A.1 - A.3 provide details on how the respective dynamic factor models can be cast into a state-space representation.

A.1 State-space representation of the dynamic single-factor model (DSFM)

For simplicity, we begin with the dynamic single-factor model. To illustrate how the corresponding matrices of the transition and measurement equation look like, we use the following: $0_{(i,j)}$ is a matrix of zeros of dimension $i \times j$, I_r is an identity matrix of dimension r. We note that $\boldsymbol{x}_t^s$ is a n_s dimensional vector of soft indicators, $\boldsymbol{x}_t^h$ is a n_h dimensional vector of hard indicators. We assume here that the variables are all observed at a monthly frequency and that there are no missing observations. Relaxing this assumption would require to extend $\boldsymbol{y}_t$, $\boldsymbol{w}_t$, R and H by means of an indicator function.

Transition equation: This equation relates the observed variables to the factor and can be expressed as:

$$\boldsymbol{y}_t = H\boldsymbol{s}_t + \boldsymbol{w}_t, \qquad \boldsymbol{w}_t \sim NID\left(\mathbf{0}, R\right) \tag{19}$$

We use the following defintion of the vectors $\boldsymbol{y}_t$, $\boldsymbol{s}_t$ $\boldsymbol{w}_t$ and variance co-variance matrix R:

$$\begin{aligned}
\boldsymbol{y}_t &= \left[y_t^f, y_t^{1st}, (\boldsymbol{x}_t^s)', (\boldsymbol{x}_t^h)'\right]' &(20)\\
\boldsymbol{w}_t &= 0_{(n_{\boldsymbol{X}},1)} &(21)\\
R &= 0_{(n_{\boldsymbol{X}},n_{\boldsymbol{X}})} &(22)
\end{aligned}$$

with $n = n_h + n_s$, $n_{\boldsymbol{X}} = n + 2$ and[38]

$$\begin{aligned}
\boldsymbol{s}_t = \ &[f_t, ..., f_{t-11},\ u_{t,q}, ..., u_{t-4,q},\ \epsilon_t, ...\\
&u_{t,1}^s, u_{t-1,1}^s, ..., u_{t,n_s}^s, u_{t-1,n_s}^s, ...\\
&u_{t,1}^h, u_{t-1,1}^h, ..., u_{t,n_h}^h, u_{t-1,n_h}^h\Big]'
\end{aligned} \tag{23}$$

Given these definitions, the matrix H will be the following:

$$H = \begin{pmatrix} \overbrace{\eta_{11} \quad 0_{(1,6)}}^{f_t,...,f_{t-11}} & \overbrace{\eta_{12}}^{u_t,...,u_{t-4}} & \overbrace{0}^{\epsilon_t} & \overbrace{0_{(1,2\cdot n_s)}}^{\tilde{\boldsymbol{u}}_t^s} & \overbrace{0_{(1,2\cdot n_h)}}^{\tilde{\boldsymbol{u}}_t^h} \\ \eta_{11} \quad 0_{(1,6)} & \eta_{12} & 1 & 0_{(1,2\cdot n_s)} & 0_{(1,2\cdot n_h)} \\ \eta_{31} \quad \eta_{31} & 0_{(n_s,5)} & 0_{(n_s,1)} & \eta_{32} & 0_{(n_s,2\cdot n_h)} \\ \eta_{41} \quad 0_{(n_h,6)} & 0_{(n_h,5)} & 0_{(n_h,1)} & 0_{(n_h,2\cdot n_s)} & \eta_{42} \end{pmatrix} \tag{24}$$

with $\tilde{\boldsymbol{u}}_t^s = \left(u_{t,1}^s, u_{t-1,1}^s,\ u_{t,2}^s, u_{t-1,2}^s, ..., u_{t,n_s}^s, u_{t-1,n_s}^s\right)$; $\tilde{\boldsymbol{u}}_t^h = \left(u_{t,1}^h, u_{t-1,1}^h,\ u_{t,2}^h, u_{t-1,2}^h, ..., u_{t,n_s}^h, u_{t-1,n_h}^s\right)$, and

$$\begin{aligned}
\eta_{11} &= \begin{pmatrix} \frac{\gamma_q}{3} & \frac{2\gamma_q}{3} & \gamma_q & \frac{2\gamma_q}{3} & \frac{\gamma_q}{3} & 0 \end{pmatrix} &(25)\\
\eta_{12} &= \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} \end{pmatrix} &(26)\\
\eta_{32} &= I_{n_s} \otimes \begin{pmatrix} 1 & 0 \end{pmatrix} &(27)\\
\eta_{42} &= I_{n_h} \otimes \begin{pmatrix} 1 & 0 \end{pmatrix} &(28)
\end{aligned}$$

and η_{31} is a $(n_s \times 6)$ matrix whose columns are $\boldsymbol{\gamma}_s$ and η_{41} is a $(n_h \times 6)$ matrix of zeros whose first column is $\boldsymbol{\gamma}_h$.

State equation: Using the previous definitions of the vectors, the state equation can be expressed as:

$$\boldsymbol{s}_t = F\boldsymbol{s}_{t-1} + \boldsymbol{v}_t, \qquad \boldsymbol{v}_t \sim NID\left(\mathbf{0}, Q\right) \tag{29}$$

[38] In our particular application, $n_h = 8$, $n_s = 2$ implying that $n = 10$ and $n_{\boldsymbol{X}} = 12$.

where Q is a matrix whose off-diagonal elements are all zero and its diagonal is given by:

$$diag(Q) \quad = \quad \left[\sigma_f^2, 0_{(1,11)}, \sigma_q^2, 0_{(1,4)}, \sigma_\epsilon^2, diag\left(\Sigma_\nu\right)' \otimes \left(\begin{array}{cc} 1 & 0 \end{array}\right)\right]' \tag{30}$$

where $diag\left(\Sigma_\nu\right) = \left(\sigma^2_{\nu_s,1}, ..., \sigma^2_{\nu_s,n_s}, \sigma^2_{\nu_h,1}, ..., \sigma^2_{\nu_h,n_h}\right)'$ and the error term $\boldsymbol{v}_t$ is given by:

$$\boldsymbol{v}_t = \left[\nu_t^f, 0_{(1,11)}, \nu_t^q, 0_{(1,4)}, \epsilon_t, [\nu^s_{t,1}, 0], ..., [\nu^s_{t,n_s}, 0], [\nu^h_{t,1}, 0], ..., [\nu^h_{t,n_h}, 0]\right]' \tag{31}$$

The matrix F becomes:

$$F = \begin{array}{c}
\begin{array}{cccccccc}
\overbrace{\qquad\qquad}^{f_{t-1},...,f_{t-12}} & \overbrace{\qquad}^{u_{t-1},...,u_{t-5}} & \overbrace{\quad}^{\epsilon_{t-1}} & \overbrace{\qquad}^{u^s_{t-1,1},u^s_{t-2,1}} & \overbrace{\quad}^{[\cdots]} & \overbrace{\qquad}^{u^s_{t-1,n_s},u^s_{t-2,n_s}} & \overbrace{\qquad}^{u^h_{t-1,1},u^h_{t-2,1}} & \overbrace{\quad}^{[\cdots]} & \overbrace{\qquad}^{u^h_{t-1,n_h},u^h_{t-2,n_h}}
\end{array} \\
\left(\begin{array}{cccccccccc}
f_{11} & 0 & 0_{(1,5)} & 0 & 0_{(1,2)} & \cdots & 0_{(1,2)} & 0_{(1,2)} & \cdots & 0_{(1,2)} \\
I_{11} & 0 & 0_{(11,5)} & 0 & 0_{(11,2)} & \cdots & 0_{(11,2)} & 0_{(11,2)} & \cdots & 0_{(11,2)} \\
0_{(5,11)} & 0 & f_{\gamma_q} & 0 & 0_{(5,2)} & \cdots & 0_{(5,2)} & 0_{(5,2)} & \cdots & 0_{(5,2)} \\
0_{(1,11)} & 0 & 0_{(1,5)} & 0 & 0_{(1,2)} & \cdots & 0_{(1,2)} & 0_{(1,2)} & \cdots & 0_{(1,2)} \\
0_{(2,11)} & 0 & 0_{(2,5)} & 0 & f_1^s & 0_{(2,\cdot)} & 0_{(2,2)} & 0_{(2,2)} & \cdots & 0_{(2,2)} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\
0_{(2,11)} & 0 & 0_{(2,5)} & 0 & 0_{(2,2)} & \cdots & f^s_{n_s} & 0_{(2,2)} & \cdots & 0_{(2,2)} \\
0_{(2,11)} & 0 & 0_{(2,5)} & 0 & 0_{(2,2)} & \cdots & 0_{(2,2)} & f^h_1 & 0_{(2,\cdot)} & 0_{(2,2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\
0_{(2,11)} & 0 & 0_{(2,5)} & 0 & 0_{(2,2)} & \cdots & 0_{(2,2)} & 0_{(2,2)} & \cdots & f^h_{n_h}
\end{array}\right)
\end{array} \tag{32}$$

$$f_{11} = \left(\begin{array}{ccc} \phi_{f,1} & \phi_{f,2} & 0_{(1,9)} \end{array}\right) \tag{33}$$

$$f_{\gamma_q} = \left(\begin{array}{ccccc} \phi_{q,1} & \phi_{q,2} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \tag{34}$$

$$f_1^s = \left(\begin{array}{cc} \phi^s_{1,1} & \phi^s_{1,2} \\ 1 & 0 \end{array}\right) \tag{35}$$

$$f^s_{n_s} = \left(\begin{array}{cc} \phi^s_{n_s,1} & \phi^s_{n_s,2} \\ 1 & 0 \end{array}\right) \tag{36}$$

$$f_1^h = \left(\begin{array}{cc} \phi^h_{1,1} & \phi^h_{1,2} \\ 1 & 0 \end{array}\right) \tag{37}$$

$$f^h_{n_h} = \left(\begin{array}{cc} \phi^h_{n_h,1} & \phi^h_{n_h,2} \\ 1 & 0 \end{array}\right) \tag{38}$$

Kalman filter recursion: Let $\hat{s}_{t|t-1}$ be the estimate of $\boldsymbol{s}_t$ based on information up to period $t-1$. Let $P_{t|t-1}$ be its covariance matrix. The prediction equations are:

$$\hat{s}_{t|t-1} = F\hat{s}_{t-1|t-1}, \tag{39}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q. \tag{40}$$

The predicted value of $\boldsymbol{y}_t$ with information up to $t-1$, denoted $\hat{y}_{t|t-1}$ is $\hat{y}_{t|t-1} = H_t^*\hat{s}_{t|t-1}$, such that the prediction error is $\eta_{t|t-1} = \boldsymbol{y}_t^* - \hat{y}_{t|t-1} = \boldsymbol{y}_t^* - H_t^*\hat{s}_{t|t-1}$ with covariance matrix $\xi_{t|t-1} = H_t^*P_{t|t-1}H_t^* + R_t^*$. In each iteration, the log-likelihood can therefor be computed as

$$\log L_{t|t-1} = -\frac{1}{2}\ln\left(2\pi\left|\xi_{t|t-1}\right|\right) - \frac{1}{2}\eta'_{t|t-1}\left(\xi_{t|t-1}\right)^{-1}\eta_{t|t-1} \tag{41}$$

The updating equations are:

$$\hat{s}_{t|t} = \hat{s}_{t|t-1} + K_t^* \eta_{t|t-1} \tag{42}$$
$$P_{t|t} = P_{t|t-1} - K_t^* H_t^* P_{t|t-1} \tag{43}$$

in which K_t^* is the Kalman gain defined as $K_t^* = P_{t|t-1} H_t^{*'} (\xi_{t|t-1})^{-1}$. The initial values $s_{0|0} = 0$ and $P_{0|0} = I$, used to start the filter, are a vector of zeros and the identity matrix, respectively.

A.2 Markov-switching Dynamic Factor Model (MS-DFM)

We consider equations of the MS-DFM and cast them into the following state-space representation:

$$\boldsymbol{y}_t = H\boldsymbol{s}_t + \boldsymbol{w}_t, \tag{44}$$
$$\boldsymbol{s}_t = \boldsymbol{\mu}(\zeta_t) + F\boldsymbol{s}_{t-1} + \boldsymbol{v}_t, \tag{45}$$

and

$$\begin{bmatrix} \boldsymbol{w}_t \\ \boldsymbol{v}_t \end{bmatrix} \sim i.i.d.N\left(\mathbf{0}, \begin{bmatrix} R & \mathbf{0} \\ \mathbf{0} & Q \end{bmatrix}\right) \tag{46}$$

The Markov-switching term $\boldsymbol{\mu}(\zeta_t)$ in turn is related to $\mu(\zeta_t)$ as follows:

$$\boldsymbol{\mu}(\zeta_t) := \begin{bmatrix} \mu(\zeta_t) \\ \mathbf{0}_{\varsigma-1,1} \end{bmatrix} \tag{47}$$

where ς is the length of the state vector $\boldsymbol{s}_t$. The definition of the vectors ($\boldsymbol{y}_t$, $\boldsymbol{s}_t$, $\boldsymbol{w}_t$ and $\boldsymbol{v}_t$) and matrices (H, R, F and Q) is the same as in equations (19) and (29) with the exception that now expression (33), which is part of the matrix F, changes to the following:

$$f_{11} = \begin{pmatrix} 0 & 0 & 0_{(1,9)} \end{pmatrix} \tag{48}$$

that is, a vector of zeros, since equation (2.5) does not feature any autoregressive terms. Further details can be found in Section A.1. The computational complexity within the estimation of the MS-DFM model arises once we maximize the likelihood function, as the combination of the Kalman filter with a Markov-switching element produces a 2-fold increase in the number of cases to be considered. The problem here can be illustrated as follows:

Assume that we have some initial values for the parameters to be estimated; this gives us an initial value $\boldsymbol{s}_{0|0}$ and $P_{0|0}$ which is the unconditional variance of the state equation and captures the uncertainty of $\boldsymbol{s}_{0|0}$. The algorithm behind the Kalman filter implies the following forecasting step within the first iteration:

$$\boldsymbol{s}_{1|0}^{(j)} = \boldsymbol{\mu}(\zeta_0 = j) + F\boldsymbol{s}_{0|0} \tag{49}$$
$$P_{1|0} = FP_{0|0}F' + Q \tag{50}$$
$$\boldsymbol{\eta}_{1|0}^{(j)} = \boldsymbol{y}_1 - \hat{\boldsymbol{y}}_{1|0}^{(j)} = \boldsymbol{y}_1 - H\boldsymbol{s}_{1|0}^{(j)} \tag{51}$$
$$\boldsymbol{\Phi}_{1|0} = E\left[\left(\boldsymbol{y}_1 - \hat{\boldsymbol{y}}_{1|0}^{(j)}\right)\left(\boldsymbol{y}_1 - \hat{\boldsymbol{y}}_{1|0}^{(j)}\right)'\right] = HP_{1|0}H' + R \tag{52}$$

and equivalent expressions for: $\boldsymbol{s}_{1|0}^{(i)}$, and $\boldsymbol{\eta}_{1|0}^{(i)}$. Once having obtained expressions for the error terms $\boldsymbol{\eta}_{1|0}^{(j)}$ and $\boldsymbol{\eta}_{1|0}^{(i)}$, we can proceed and compute the log-likelihood functions for each of the two states: $\zeta_0 = j$ and $\zeta_0 = i$:

$$\lambda_1^{(j)} = -\frac{1}{2}\ln\left(2\pi\left|\boldsymbol{\Phi}_{1|0}\right|\right) - \frac{1}{2}\boldsymbol{\eta}_{1|0}^{(j)}\left(\boldsymbol{\Phi}_{1|0}\right)^{-1}\left(\boldsymbol{\eta}_{1|0}^{(j)}\right)' \tag{53}$$

and an equivalent expression for $\lambda_1^{(i)}$. The unconditional density of $\boldsymbol{y}_1$ can be found by summing $\lambda_1^{(\zeta_0=\{i,j\})}$ over all values of the states $\{i, j\}$:

$$
\begin{aligned}
f(\boldsymbol{y}_1) &= \tilde{p}\cdot p(\zeta_0=j|I_0)e^{\lambda_1^{(j)}} + (1-\tilde{q})\cdot p(\zeta_0=i|I_0)e^{\lambda_1^{(j)}} + \\
&\quad \tilde{q}\cdot p(\zeta_0=i|I_0)e^{\lambda_1^{(i)}} + (1-\tilde{p})\cdot p(\zeta_0=i|I_0)e^{\lambda_1^{(i)}} \qquad (54)
\end{aligned}
$$

The updating step within the Kalman filter implies:

$$
\begin{aligned}
\boldsymbol{s}_{1|1}^{(j)} &= \boldsymbol{s}_{1|0}^{(j)} + K_{1|0}\boldsymbol{\eta}_{1|0}^{(j)} \qquad (55)\\
P_{1|1} &= \left(\boldsymbol{I} - K_{1|0}P_{1|0}H\right)\cdot P_{1|0} \qquad (56)
\end{aligned}
$$

and an equivalent expression for $\boldsymbol{s}_{1|1}^{(i)}$. The Kalman gain $K_{1|0}$ is given by: $K_{1|0} = P_{1|0}H\left(\boldsymbol{\Phi}_{1|0}\right)^{-1}$. Having obtained values for $\boldsymbol{s}_{1|1}^{(j)}$, and $\boldsymbol{s}_{1|1}^{(i)}$, the first iteration ends and we would move on with the second one; however, this is where the complexity arises: The first iteration yields two expressions for the state vector: $\boldsymbol{s}_{1|1}^{(j)}$ and $\boldsymbol{s}_{1|1}^{(i)}$. Considering now the second iteration, equation (55) yields *four* expressions for the state vector: $\boldsymbol{s}_{1|1}^{(j,j)}, \boldsymbol{s}_{1|1}^{(i,j)}, \boldsymbol{s}_{1|1}^{(j,i)}$ and $\boldsymbol{s}_{1|1}^{(i,i)}$. The third iteration in turn would yield eight expressions, and so on. Hence the iterations would quickly produce too many different state vectors rendering the estimation intractable.

The increase in complexity is not only due to the state vector $\boldsymbol{s}_{1|1}^{\{i,j\}}$; in fact for the first iteration we have that $P_{1|0} = P_{1|0}^{(i)} = P_{1|0}^{(j)}$, however, from the second iteration onwards we also have a Markov-switching state dependency in $P_{1|0}^{\{i,j\}}$, which adds an additional degree of complexity to the maximization of the log-likelihood function.

Viewed in more general terms, for each point in time t, the filter produces a 2-fold increase in the number of cases to be considered, since at each t, the variable $\boldsymbol{s}_t^{\{i,j\}}$ and in turn $P_t^{\{i,j\}}$, $\boldsymbol{\Phi}_t^{\{i,j\}}$, $K_t^{\{i,j\}}$, etc. can take two new values and therefore, at each t we have 2^t possible paths to consider when evaluating the likelihood.

As a solution to this problem, Kim (1994) proposed a modification which results in getting rid of the state dependency of the Markov-switching elements in the state vector; this approach was used by Kim and Yoo (1995), Chauvet (1998) and Camacho et al. (2018). In particular, to collapse the means and variances in order to apply equation (49) and (50) in a tractable form within all iterations, Camacho et al. (2018) approximate $\boldsymbol{s}_{t|t}^{(j)}$, $\boldsymbol{s}_{t|t}^{(i)}$ and $\boldsymbol{P}_{t|t}^{(j)}$, $\boldsymbol{P}_{t|t}^{(i)}$ by a weighted average of the updating equations, where the weights are given by the probabilities of the Markov state; this implies for the first iteration:

$$
\begin{aligned}
\boldsymbol{s}_{1|1} &= \boldsymbol{s}_{1|1}^{(j)}\cdot p(\zeta_0=j|I_1) + \boldsymbol{s}_{1|1}^{(i)}\cdot p(\zeta_0=i|I_1) \qquad (57)\\
P_{1|1} &= p(\zeta_0=j|I_1)\left(P_{1|0} + \left(\boldsymbol{s}_{1|1}^{(j)} - \boldsymbol{s}_{1|1}\right)\cdot\left(\boldsymbol{s}_{1|1}^{(j)} - \boldsymbol{s}_{1|1}\right)'\right) + \\
&\quad p(\zeta_0=i|I_1)\left(P_{1|0} + \left(\boldsymbol{s}_{1|1}^{(i)} - \boldsymbol{s}_{1|1}\right)\cdot\left(\boldsymbol{s}_{1|1}^{(i)} - \boldsymbol{s}_{1|1}\right)'\right) \qquad (58)
\end{aligned}
$$

or in more general terms:

$$
\begin{aligned}
\boldsymbol{s}_{t|t}^{(j)} &= \frac{\sum_{\zeta_{t-1}=0}^{1} p(\zeta_t=j,\zeta_{t-1}=i|I_t)\boldsymbol{s}_{t|t}^{(i,j)}}{p(\zeta_t=j|I_t)} \qquad (59)\\
P_{t|t}^{(j)} &= \frac{\sum_{\zeta_{t-1}=0}^{1} p(\zeta_t=j,\zeta_{t-1}=i|I_t)\left(P_{t|t}^{(i,j)} + \left(\boldsymbol{s}_{t|t}^{(j)} - \boldsymbol{s}_{t|t}^{(i,j)}\right)\left(\boldsymbol{s}_{t|t}^{(j)} - \boldsymbol{s}_{t|t}^{(i,j)}\right)'\right)}{p(\zeta_t=j|I_t)} \qquad (60)
\end{aligned}
$$

and equivalent expressions for $\boldsymbol{s}_{t|t}^{(i)}$ and $P_{t|t}^{(i)}$; and finally

$$
\begin{aligned}
\boldsymbol{s}_{t|t} &= \boldsymbol{s}_{t|t}^{(j)} \cdot p(\zeta_t = j|I_t) + \boldsymbol{s}_{t|t}^{(i)} \cdot p(\zeta_t = i|I_t) && (61)\\
P_{t|t} &= p(\zeta_t = j|I_t)\left(P_{t|t-1}^{(j)} + \left(\boldsymbol{s}_{t|t}^{(j)} - \boldsymbol{s}_{t|t}\right)\cdot\left(\boldsymbol{s}_{t|t}^{(j)} - \boldsymbol{s}_{t|t}\right)'\right) + \\
&\quad p(\zeta_t = i|I_t)\left(P_{t|t-1}^{(i)} + \left(\boldsymbol{s}_{t|t}^{(i)} - \boldsymbol{s}_{t|t}\right)\cdot\left(\boldsymbol{s}_{t|t}^{(i)} - \boldsymbol{s}_{t|t}\right)'\right) && (62)
\end{aligned}
$$

This approach eliminates the Markov-switching state dependency in the state vector within the Kalman filter iterations and allows in turn to run the Kalman filter in the standard form. Moreover, this set-up is nested in the dynamic factor model which allows for mixed frequencies and missing observations. It is worth noting that including a missing observation in the data set, the model will automatically replace the missing value by a forecast. Following the same reasoning, forecasts for longer horizons and forecasts for other indicators can be automatically computed.

A.3 The MF-FAVAR model

The Mixed Frequency-Factor Augmented Vector Autoregressive (MF-FAVAR) model is given by the following system of equations $\forall\ t = 1, ..., T$:

System of static equations

$$
\begin{aligned}
\begin{pmatrix} \boldsymbol{x}_t^s \\ \boldsymbol{x}_t^h \end{pmatrix} &= \begin{pmatrix} \boldsymbol{\gamma}_s \cdot \sum_{j=0}^{11} f_{t-j} \\ \boldsymbol{\gamma}_h \cdot f_t \end{pmatrix} + \begin{pmatrix} \boldsymbol{u}_t^s \\ \boldsymbol{u}_t^h \end{pmatrix} && (63)\\
\begin{pmatrix} y_t^f \\ y_t^{1st} \end{pmatrix} &= \begin{pmatrix} \omega(L) \\ \omega(L) \end{pmatrix} \cdot \left[\gamma_q \cdot y_t^M\right] + \begin{pmatrix} 0 \\ \epsilon_t \end{pmatrix} && (64)
\end{aligned}
$$

where $\boldsymbol{u}_t^h = \left(u_{t,1}^h, ..., u_{t,n_h}^h\right)'$, $\boldsymbol{u}_t^s = \left(u_{t,1}^s, ..., u_{t,n_s}^s\right)'$ with $n = n_h + n_s$, $n_{\boldsymbol{X}} = n + 2$, and $\omega(L) := \frac{1}{3} + \frac{2}{3} \cdot L + L^2 + \frac{2}{3} \cdot L^3 + \frac{1}{3} \cdot L^4$, where L is the lag operator.

System of dynamic equations

$$
\begin{aligned}
(\boldsymbol{I} - \phi_f(L)) \cdot \begin{bmatrix} y_t^M \\ f_t \end{bmatrix} &= \boldsymbol{\nu}_t^f && (65)\\
(\boldsymbol{I} - \boldsymbol{\Phi}_u(L)) \begin{pmatrix} \boldsymbol{u}_t^s \\ \boldsymbol{u}_t^h \end{pmatrix} &= \boldsymbol{\nu}_t && (66)\\
\begin{pmatrix} \boldsymbol{\nu}_t^f \\ \boldsymbol{\nu}_t \end{pmatrix} &\sim NID\left(\boldsymbol{0}, \begin{bmatrix} \Sigma_f & \boldsymbol{0} \\ \boldsymbol{0} & \Sigma_\nu \end{bmatrix}\right) && (67)
\end{aligned}
$$

where $\phi_f(L)$ and $\boldsymbol{\Phi}_u(l)$ are in each case second-order polynomials. We assume that $\boldsymbol{\Phi}_u(L)$ and Σ_ν are diagonal, implying that all covariances are zero by construction. For identification reasons we impose that the $(2,2)$ element in Σ_f is unity and $\gamma_q = [1,1]'$. The state-space representation of the MF-FAVAR model reads as follows:

Transition equation: $\quad \boldsymbol{y}_t = H\boldsymbol{s}_t + \boldsymbol{w}_t, \qquad \boldsymbol{w}_t \sim NID\,(\boldsymbol{0}, R)$ with

$$
\begin{aligned}
\boldsymbol{y}_t &= \left[y_t^f, y_t^{1st}, (\boldsymbol{x}_t^s)', (\boldsymbol{x}_t^h)'\right]' && (68)\\
\boldsymbol{w}_t &= 0_{(n_{\boldsymbol{X}},1)} && (69)\\
R &= 0_{(n_{\boldsymbol{X}},n_{\boldsymbol{X}})} && (70)
\end{aligned}
$$

and

$$
\begin{aligned}
\boldsymbol{s}_t = \Big[&y_t^M, ..., y_{t-4}^M,\ f_t, ..., f_{t-11},\ (\boldsymbol{\nu}_t^f)',\ \epsilon_t, ... \\
&u_{t,1}^s, u_{t-1,1}^s, ..., u_{t,n_s}^s, u_{t-1,n_s}^s, ... \\
&u_{t,1}^h, u_{t-1,1}^h, ..., u_{t,n_h}^h, u_{t-1,n_h}^h\Big]' && (71)
\end{aligned}
$$

Given these definitions, the matrix H will be the following:

$$H = \begin{array}{c} \begin{array}{ccccccc} \overbrace{\qquad}^{y_t^M,...,y_{t-4}^M} & \multicolumn{2}{c}{\overbrace{\qquad\qquad}^{f_t,...,f_{t-11}}} & \overbrace{\qquad}^{\boldsymbol{\nu}_t^f} & \overbrace{\quad}^{\epsilon_t} & \overbrace{\qquad}^{\tilde{\boldsymbol{u}}_t^s} & \overbrace{\qquad}^{\tilde{\boldsymbol{u}}_t^h} \end{array} \\ \left(\begin{array}{ccccccc} \eta_{12} & 0_{(1,6)} & 0_{(1,6)} & 0_{(1,2)} & 0 & 0_{(1,2\cdot n_s)} & 0_{(1,2\cdot n_h)} \\ \eta_{12} & 0_{(1,6)} & 0_{(1,6)} & 0_{(1,2)} & 1 & 0_{(1,2\cdot n_s)} & 0_{(1,2\cdot n_h)} \\ 0_{(n_s,5)} & \eta_{31} & \eta_{31} & 0_{(n_s,2)} & 0_{(n_s,1)} & \eta_{32} & 0_{(n_s,2\cdot n_h)} \\ 0_{(n_h,5)} & \eta_{41} & 0_{(n_h,6)} & 0_{(n_h,2)} & 0_{(n_h,1)} & 0_{(n_h,2\cdot n_s)} & \eta_{42} \end{array}\right) \end{array} \tag{72}$$

where $\tilde{\boldsymbol{u}}_t^s$, $\tilde{\boldsymbol{u}}_t^h$ and the parameter matrices/vectors η_{12}, η_{31}, η_{32}, η_{41} and η_{42} are defined in Section A.1.

State equation: $\quad \boldsymbol{s}_t = F\boldsymbol{s}_{t-1} + \boldsymbol{v}_t, \qquad \boldsymbol{v}_t \sim NID\left(\boldsymbol{0}, Q\right)$ where Q is a matrix whose diagonal is given by:

$$diag(Q) \;=\; \left[0_{(1,5)}, 0_{(1,12)}, diag\left(\Sigma_f\right), \sigma_\epsilon^2, diag\left(\Sigma_\nu\right)' \otimes \left(\begin{array}{cc} 1 & 0 \end{array}\right)\right]' \tag{73}$$

where $diag\left(\Sigma_\nu\right) = \left(\sigma^2_{\nu_s,1}, ..., \sigma^2_{\nu_s,n_s}, \sigma^2_{\nu_h,1}, ..., \sigma^2_{\nu_h,n_h}\right)'$ and the following two off-diagonal elements are different from zero: $Q_{(18,19)} = Q_{(19,18)} = E\left[\nu^f_{t,1} \cdot \nu^f_{t,2}\right] = \sigma_{\nu^f_{12}}$. The error term $\boldsymbol{v}_t$ is given by:

$$\boldsymbol{v}_t = \left[0_{(1,5)}, 0_{(1,12)}, \left(\boldsymbol{\nu}_t^f\right)', \epsilon_t, [\nu^s_{t,1}, 0], ..., [\nu^s_{t,n_s}, 0], [\nu^h_{t,1}, 0], ..., [\nu^h_{t,n_h}, 0]\right]' \tag{74}$$

Finally, the matrix F reads:

$$F = \begin{array}{c} \begin{array}{ccccccccc} \multicolumn{2}{c}{\overbrace{\qquad\qquad}^{\left[\tilde{y}^M_{t-1}, \tilde{f}_{t-1}\right]}} & \overbrace{\qquad}^{\boldsymbol{\nu}^f_{t-1}, \epsilon_{t-1}} & \overbrace{\qquad}^{u^s_{t-1,1}, u^s_{t-2,1}} & \overbrace{\quad}^{[\cdots]} & \overbrace{\qquad}^{u^s_{t-1,n_s}, u^s_{t-2,n_s}} & \overbrace{\qquad}^{u^h_{t-1,1}, u^h_{t-2,1}} & \overbrace{\quad}^{[\cdots]} & \overbrace{\qquad}^{u^h_{t-1,n_h}, u^h_{t-2,n_h}} \end{array} \\ \left(\begin{array}{ccccccccc} \tilde{f}_{11} & \tilde{f}_{12} & 0_{(5,3)} & 0_{(5,2)} & \cdots & 0_{(5,2)} & 0_{(5,2)} & \cdots & 0_{(5,2)} \\ \tilde{f}_{21} & \tilde{f}_{22} & 0_{(12,3)} & 0_{(12,2)} & \cdots & 0_{(12,2)} & 0_{(12,2)} & \cdots & 0_{(12,2)} \\ 0_{(3,5)} & 0_{(3,12)} & 0_{(3,3)} & 0_{(3,2)} & \cdots & 0_{(3,2)} & 0_{(3,2)} & \cdots & 0_{(3,2)} \\ 0_{(2,5)} & 0_{(2,12)} & 0_{(2,3)} & f^s_1 & 0_{(2,\cdot)} & 0_{(2,2)} & 0_{(2,2)} & \cdots & 0_{(2,2)} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\ 0_{(2,5)} & 0_{(2,12)} & 0_{(2,3)} & 0_{(2,2)} & \cdots & f^s_{n_s} & 0_{(2,2)} & \cdots & 0_{(2,2)} \\ 0_{(2,5)} & 0_{(2,12)} & 0_{(2,3)} & 0_{(2,2)} & \cdots & 0_{(2,2)} & f^h_1 & 0_{(2,\cdot)} & 0_{(2,2)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\ 0_{(2,5)} & 0_{(2,12)} & 0_{(2,3)} & 0_{(2,2)} & \cdots & 0_{(2,2)} & 0_{(2,2)} & \cdots & f^h_{n_h} \end{array}\right) \end{array} \tag{75}$$

where $\tilde{y}_t^M = [y_t^M, ..., y_{t-4}^M]$, and $\tilde{f}_t = [f_t, ..., f_{t-11}]$.

$$\tilde{f}_{11} \;=\; \left(\begin{array}{ccc} \phi^f_{(1,1),1} & \phi^f_{(1,1),2} & 0_{(1,3)} \\ 0 & 1 & 0_{(1,3)} \\ 0_{(3,1)} & 0_{(3,1)} & I_3 \end{array}\right)_{(5,5)} \tag{76}$$

$$\tilde{f}_{12} \;=\; \left(\begin{array}{ccc} \phi^f_{(1,2),1} & \phi^f_{(1,2),2} & 0_{(1,10)} \\ 0 & 0 & 0_{(1,10)} \\ 0_{(3,1)} & 0_{(3,1)} & 0_{(3,10)} \end{array}\right)_{(5,12)} \tag{77}$$

$$\tilde{f}_{21} \;=\; \left(\begin{array}{ccc} \phi^f_{(2,1),1} & \phi^f_{(2,1),2} & 0_{(1,3)} \\ 0 & 0 & 0_{(1,3)} \\ 0_{(10,1)} & 0_{(10,1)} & 0_{(10,3)} \end{array}\right)_{(12,5)} \tag{78}$$

$$\tilde{f}_{22} \;=\; \left(\begin{array}{ccc} \phi^f_{(2,2),1} & \phi^f_{(2,2),2} & 0_{(1,10)} \\ 0 & 1 & 0_{(1,10)} \\ 0_{(10,1)} & 0_{(10,1)} & I_{10} \end{array}\right)_{(12,12)} \tag{79}$$

$$\tag{80}$$

where $\phi_{f,1} = \left[\begin{array}{cc} \phi^f_{(1,1),1} & \phi^f_{(1,2),1} \\ \phi^f_{(2,1),1} & \phi^f_{(2,2),1} \end{array}\right]$ is the matrix coefficient of the first lag of equation (65), and $\phi_{f,2} = \begin{bmatrix} \phi^f_{(1,1),2} & \phi^f_{(1,2),2} \\ \phi^f_{(2,1),2} & \phi^f_{(2,2),2} \end{bmatrix}$ is the matrix of coefficients for the second lag. The matrices $f^s_1, ..., f^s_{n_s}$ and $f^h_1, ..., f^h_{n_h}$ are defined as in Section A.1.

A.4 Computing forecasts and weights

Computing short-term forecasts in real-time from these models is straightforward. The future values of the time series can be regarded as missing observations at the end of the sample periods. The Kalman filter accounts for the missing data, which are replaced by forecasts. Particularly, the k-period ahead forecasts are

$$\hat{y}_{t+k|t} = H^*_t \hat{s}_{t+k|t} \tag{81}$$

with $\hat{s}_{t+k|t} = F^k \hat{s}_{t|t}$.

The Kalman filter allows to compute the weights or cumulative impacts of each indicator to the forecast of GDP growth. The state vector $\boldsymbol{s}_t$ can be expressed as the weighted sum of available observations in the past. Given a large enough t such that the Kalman filter has approached its steady state, it holds that h-period ahead forecasts of GDP growth are approximately

$$y_{t+h} = \sum_{j=0}^{\infty} W'_j \boldsymbol{y}_t \tag{82}$$

in which W_j is a vector of weights to compute the cumulative weights of series i in forecasting GDP growth as $\sum_{j=0}^{\infty} W_j(i)$, where $W_j(i)$ is the ith element of W_j.

A.5 Calculating annual GDP growth rates

While the forecasts from the Economist Poll of Forecasters and the Federal Government's Expert Group are already reported in *annual* GDP growth rates, the forecasts from the benchmark models and the DFM have to be converted from quarterly to annual rates.

Suppose the quarterly levels of variable $X_{q,y}$ in year 1 are $X_{1,1}, ..., X_{4,1}$ and similarly in year 2 $X_{5,1}, ..., X_{8,1}$. Then the annual average growth rate $g_{X,annual}$ calculated with levels is given by

$$g_{X,annual} = \frac{X_{5,1} + X_{6,1} + X_{7,1} + X_{8,1}}{X_{1,1} + X_{2,1} + X_{3,1} + X_{4,1}} - 1 \tag{83}$$

Each quarterly level can be expressed in terms of quarterly growth rates multiplied by the level in the base quarter $X_{0,0}$, for instance, $X_{1,1} = g_1 \times X_{0,0}$ and $X_{2,1} = g_2 \times g_1 \times X_{0,0}$. After some algebra, the relationship between quarterly growth rates and annual average growth rates is expressed by

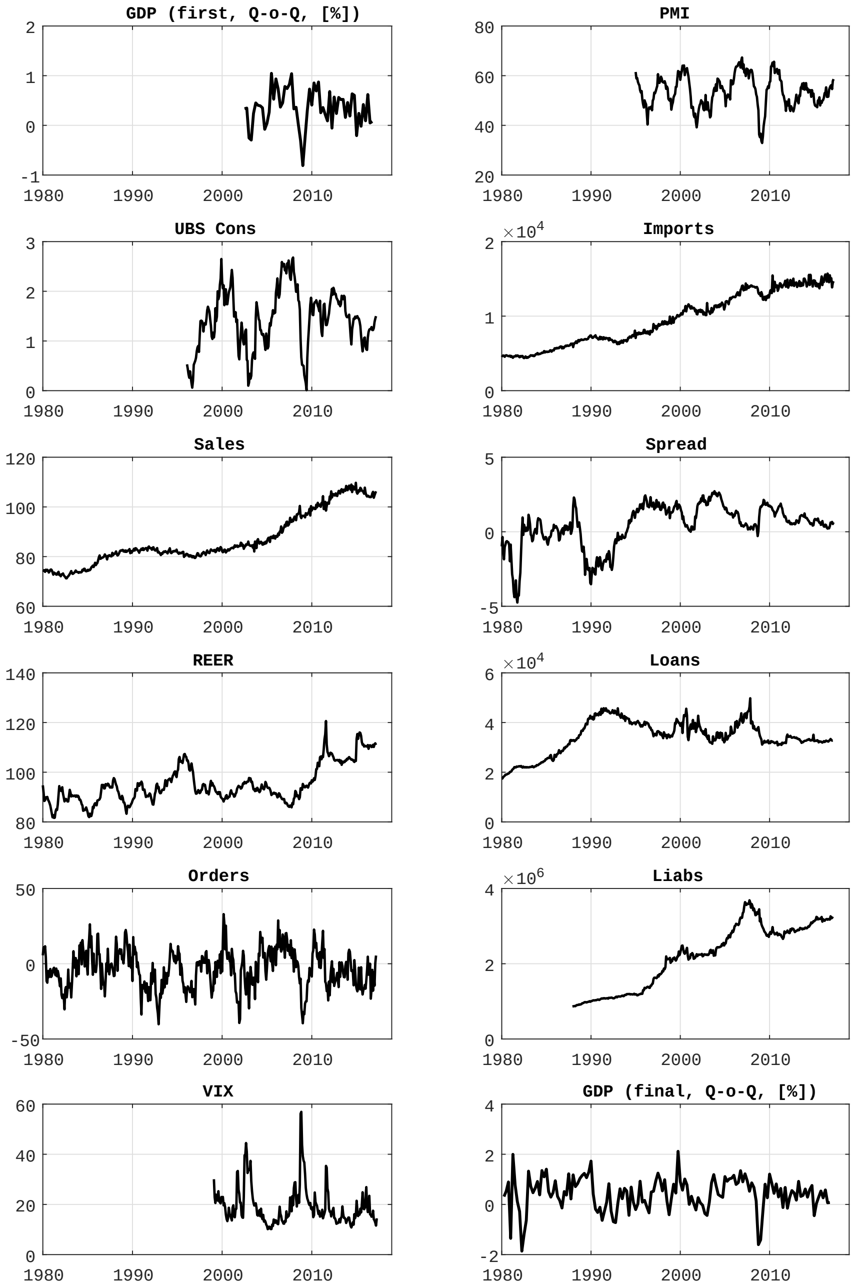
$$g_{X,annual} = \frac{\sum_{j=5}^{8} \prod_{i=2}^{j} g_i}{1 + \sum_{j=1}^{4} \prod_{i=1}^{j} g_i} - 1 \tag{84}$$

in which the quarterly growth rates of year 1 are referred to with $g_1, ..., g_4$, while the four quarters of year 2 are labelled with $g_5, ..., g_8$.

A.6 Additional figures and tables

The models presented and discussed in the main text feature a series of estimated coefficients of which only a few have been reported in the main text. Table 7 below lists the whole set of estimated parameters for both models including the standard deviation and the ratio of the point estimate and its standard deviation ($\sim$ t-values) for each parameter. Figure 5 shows the variable graphically.

Figure 5: Monthly indicator series used in the model



Note: See Table 1 for acronyms. Charts refer to data available on Friady 26/01/17.

Table 7: Estimated Parameters

		DSFM: $f_t \sim AR(2)$			MS-DFM: $f_t \sim MS$		
		Estimate	Standard Deviation	ratio	Estimate	Standard Deviation	ratio
Factor loadings							
GDP	(γ_q)	0.06	0.02	2.6	0.10	0.03	3.8
PMI	$(\in \boldsymbol{\gamma}_s)$	0.27	0.11	2.4	0.25	0.05	4.7
UBSc	$(\in \boldsymbol{\gamma}_s)$	0.23	0.10	2.4	0.22	0.05	4.6
Imports	$(\in \boldsymbol{\gamma}_h)$	0.02	0.01	2.3	0.07	0.01	4.9
Sales	$(\in \boldsymbol{\gamma}_h)$	0.02	0.01	1.8	0.06	0.00	15.2
Spread	$(\in \boldsymbol{\gamma}_h)$	-0.06	0.02	-2.3	-0.15	0.05	-3.2
REER	$(\in \boldsymbol{\gamma}_h)$	-0.02	0.01	-1.7	-0.13	0.08	-1.6
Loans	$(\in \boldsymbol{\gamma}_h)$	0.03	0.02	2.1	0.07	0.04	1.9
Orders	$(\in \boldsymbol{\gamma}_h)$	0.14	0.05	3.0	0.08	0.03	2.5
Assets	$(\in \boldsymbol{\gamma}_h)$	0.02	0.01	1.6	0.05	0.02	3.1
VSMI	$(\in \boldsymbol{\gamma}_h)$	-0.15	0.07	-2.2	-0.15	0.02	-7.6
Autoregressive coefficients							
factor (f_t)	$\phi_{f,1}$	1.49	0.21	7.3	-	-	-
	$\phi_{f,2}$	-0.55	0.19	-3.0	-	-	-
GDP	$\phi_{u_i,1}$	0.54	0.31	1.7	0.60	0.30	2.0
	$\phi_{u_i,2}$	-0.55	0.12	-4.5	-0.41	0.13	-3.2
PMI	$\phi_{u_i,1}$	0.73	0.07	11.0	0.77	0.06	13.0
	$\phi_{u_i,2}$	0.14	0.06	2.2	0.10	0.02	6.2
UBSc	$\phi_{u_i,1}$	1.06	0.06	16.4	1.14	0.10	11.0
	$\phi_{u_i,2}$	-0.21	0.06	-3.3	-0.22	0.05	-5.0
Imports	$\phi_{u_i,1}$	-0.61	0.05	-13.3	-0.61	0.06	-10.6
	$\phi_{u_i,2}$	-0.29	0.05	-6.4	-0.30	0.04	-7.6
Sales	$\phi_{u_i,1}$	-0.49	0.05	-10.5	-0.49	0.11	-4.7
	$\phi_{u_i,2}$	-0.23	0.05	-4.9	-0.22	0.09	-2.3
Spread	$\phi_{u_i,1}$	0.04	0.05	0.7	0.08	0.02	5.0
	$\phi_{u_i,2}$	-0.12	0.05	-2.4	-0.13	0.06	-2.2
REER	$\phi_{u_i,1}$	0.19	0.05	3.9	0.24	0.02	15.4
	$\phi_{u_i,2}$	-0.05	0.05	-1.1	-0.08	0.07	-1.1
Loans	$\phi_{u_i,1}$	-0.24	0.05	-5.1	-0.24	0.03	-9.4
	$\phi_{u_i,2}$	-0.15	0.05	-3.1	-0.13	0.03	-4.0
Orders	$\phi_{u_i,1}$	0.55	0.06	9.4	0.64	0.05	13.0
	$\phi_{u_i,2}$	0.16	0.05	3.2	0.21	0.05	4.3
Assets	$\phi_{u_i,1}$	-0.11	0.05	-2.0	-0.13	0.05	-2.5
	$\phi_{u_i,2}$	0.02	0.05	0.3	0.00	0.00	0.5
VSMI	$\phi_{u_i,1}$	0.97	0.08	12.7	1.04	0.07	14.4
	$\phi_{u_i,2}$	-0.27	0.10	-2.7	-0.30	0.08	-3.9
Variances							
Revision error	σ_ξ^2	0.34	0.12	2.9	0.35	0.11	3.3
GDP	(σ_q^2)	0.71	0.05	13.8	1.37	0.21	6.4
PMI	$(\in \Sigma_\nu)$	0.30	0.01	21.5	0.59	0.09	6.8
UBSc	$(\in \Sigma_\nu)$	0.29	0.01	21.7	0.64	0.10	6.3
Imports	$(\in \Sigma_\nu)$	0.84	0.03	29.7	1.23	0.21	5.9
Sales	$(\in \Sigma_\nu)$	0.89	0.03	29.8	1.54	0.22	7.0
Spread	$(\in \Sigma_\nu)$	0.97	0.03	29.2	1.75	0.29	6.1
REER	$(\in \Sigma_\nu)$	0.98	0.03	29.8	1.75	0.76	2.3
Loans	$(\in \Sigma_\nu)$	0.96	0.03	29.6	1.61	0.45	3.6
Orders	$(\in \Sigma_\nu)$	0.59	0.02	26.4	1.49	0.13	11.5
Assets	$(\in \Sigma_\nu)$	0.99	0.04	26.4	1.67	0.29	5.7
VSMI	$(\in \Sigma_\nu)$	0.46	0.03	15.4	0.80	0.07	12.2
Markov-switching coefficients							
	$\mu(\zeta_t = 0)$	-	-	-	0.27	0.06	4.6
	$\mu(\zeta_t = 1)$	-	-	-	2.94	0.46	6.3
	p_{00}	-	-	-	0.98	0.18	5.4
	p_{11}	-	-	-	0.86	0.19	4.6

Note: See Table 1 for acronyms, data transformation and a description of these indicators.