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An Evaluation of the Forecasting Performance of Three Econometric Models for the Eurozone and the USA

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This paper compares the forecasting performance of three different econometric models for the Eurozone and the USA: A vector auto regression (VAR), a Bayesian vector auto regression (BVAR), and a structural vector error correction model (SVEC). The forecast evaluation is based on 19 vintages of real time data for output, inflation rates, interest rates, the exchange rate and the money stock from the 4th quarter of 2004 until the the 1st quarter of 2010. The oil price is used as the only exogenous variable in the model. Imposing a stringent set of long-run assumptions on the econometric model results in less accurate forecasts. The difference is significant for several variables and forecast horizons. Reducing the comparison to data from the pre-financial crisis period reduces the size of forecast errors but does not change the overall picture.

JEL Classification: C32, C53, E37.

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1 Introduction

In this paper the forecasting performance of three different econometric models for the Eurozone and the USA is assessed: A vector auto regression (VAR), a Bayesian vector auto regression (BVAR), and a structural vector error correction model (SVEC) which is based on Gaggl, Kaniovski, Prettner, & Url (2009).

VARs can be regarded as an economic-theory-free way to capture dynamics in multiple time series. Due to the large number of parameters to be estimated VAR models, however, are often inefficient and suffer from over-parameterization and a low number of degrees of freedom. BVAR models can partly overcome these disadvantages by including *a priori* value assumptions into the estimation. Since Granger (1981) and Engle & Granger (1987) error correction models are known to be superior to VARs in the presence of co-integration in the estimated variables. Clements & Hendry (2008), however, show that this this need not be necessarily the case when equilibrium shifts occur within the forecasting period. Then SVEC models, in fact, do not 'error correct' but 'equilibrium correct'. This built-in equilibrium, however, is outdated by the time the shift has occurred creating a series of forecast errors.

Section 2 provides a detailed overview of the data set, its manipulation and transformation, as well as an explanation for the use of real-time data. Section 3 explains the three models used for forecasting and highlights the most important differences between them. Section 4 provides the reader with an illustration of the statistical tools used to assess forecast accuracy. In section 5 the forecast errors for the period 2004:Q4 to 2010:Q1 are presented. Since the results appear to vary widely among the three models, section 6 includes a test examining whether the differences are statistically significant or not. Following the reasoning of Clements & Hendry (2008), forecast errors are recalculated in section 7 covering quarterly forecast errors for 2004:Q4 to 2008:Q2 only, i.e. before the recent financial crisis (a possible equilibrium shift) has occurred. The last section offers concluding remarks.

2 Data

Data were obtained from the Austrian Institute of Economic Research, the Statistical Warehouse of Eurostat and the Federal Reserve Bank of Philadelphia. The time series for the Eurozone- and US- price levels, GDP levels, and money supply M1 consist of real-time data (for a detailed explanation of real-time data see section 2.1). Oil prices and interest rates are regular time series, since they are not subject to revision and their historical values do not change when time progresses.

The dataset consists of 23 vintages or time series. The first vintage includes data as observed in the 4th quarter of 2004 and ranges from the first quarter of 1970 to the third quarter of 2004; vintage #23 consists of data as observed in the second quarter of 2010 and ranges from the first quarter of 1970 to the first quarter of 2010. The following summary explains the meaning of abbreviations for the variables and which transformations have been applied for the use in the models.

- exch the natural logarithm of the normalized nominal Euro per US-Dollar exchange rate (base: first quarter 2000 = 1).
- hez natural logarithm of the normalized Eurozone M1 real per capita money stock in relation to real per capita GDP (base: first quarter 2000 = 1).
- hus natural logarithm of the normalized US M1 real per capita money stock in relation to real per capita GDP (base: first quarter 2000 = 1).
- pez natural logarithm of the Eurozone consumer price index (base: first quarter of 2000 = 1).
- pus natural logarithm of the US consumer price index (base: first quarter 2000 = 1).
- rez natural logarithm of $(1+r_{ez}/100)$, where r_{ez} is the annualized average 3 month interest rate in the Eurozone.

rus natural logarithm of $(1+r_{US}/100)$, where r_{US} is the annualized average 3 month interest rate in the USA.

yez natural logarithm of the normalized real per capital GDP in the Eurozone (base: first quarter of 2000 = 1).

yus natural logarithm of the normalized real per capital GDP in the USA (base: first quarter of 2000 = 1).

pd *pez* minus *pus;* the price differential

And as exogenous variable:

poil natural logarithm of the import price for crude oil in US-Dollars.[1]

For estimation purposes all variables were seasonally adjusted using the program Tramoseats in Eviews.

2.1 Real Time Data

The meaning of real-time data in economics is a different one than that of real-time data in finance. In finance, e.g. on stock markets real-time data are obtained to get the latest information on prices or values to be able to react immediately on their changes. In economics, real-time data are used from a backward looking perspective, i.e. we look how the latest available information at a given point in time looked like back in history. When economic forecasts are performed one has to rely on the actual data. Economic indicators, however, are usually preliminary estimates themselves and subject to several revisions. Therefore, the use of such real-time data for forecasts creates a different conclusion than what would have been obtained by reapplying the same forecasting technique with revised data at a future point in time. Real-time time series are also called vintages. Like a good vintage wine is made from grapes of the same vintage year, a real-time vintage consists of data that were obtained at the same point in time.

[1] A "d" in front of a variable's name indicates that first differences have been calculated, e.g. *dpoil* comprises of first differences of *poil*, hence, contains one observation less.

To sum up, real-time datasets allow estimating models and simulating forecasts as if they had been computed at the date of the forecast, e.g. when looking at vintage set #1 (1970:Q1 – 2004:Q3) in this paper, we are in the position of a researcher performing forecasts in the end of year 2004 for the following quarters. Thanks to real time data we get 19 forecasts from each of the three models estimated.

Table 1: Data Set Structure

Date of forecast	Vintage	Estimation sample	Forecast period
2004:Q4	1	1970:Q1 – 2004:Q3	2004:Q4 – 2005:Q3
2005:Q1	2	1970:Q1 – 2004:Q4	2005:Q1 – 2005:Q4
⋮	⋮	⋮	⋮
2009:Q2	19	1970:Q1 – 2009:Q1	2009:Q2 – 2010:Q1

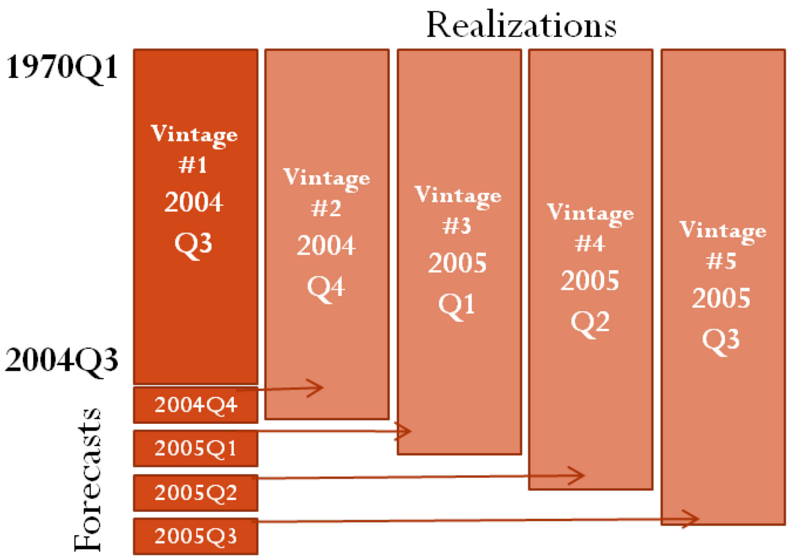

Figure 1: Forecasts and corresponding realizations

2.2 Data Manipulation

I define the Eurozone as the aggregate of twelve countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. Annual population data were obtained from the Austrian Institute of Economic Research and were interpolated to achieve quarterly data in the same way as in Gaggl, Kaniovski, Prettner, & Url (2009), i.e. by using the Boot et al. (1967) method in Ecotrim.

US Data

US time series for *CPI, GDP, Money Stock M1* are real-time data and were obtained from the Federal Reserve Bank of Philadelphia`s Real-Time Data Research Center. Thanks to the excellent correctness of the dataset and its convenient structure no further adjustments had to be made except changing the base year and computing 3-month averages to get quarterly data for CPI, GDP and Money Stock M1.

Eurozone Data

Real-time data for the Eurozone (*CPI, GDP, Money Stock M1*) were obtained from Eurostat`s Statistical Warehouse and/or the Euro-Area Business Cycle Network (EABCN). Unfortunately Eurozone vintages do not range back to 1970. The first observation for Eurozone GDP and CPI is 1991Q1 while the first observation for Eurozone Money Stock M1 is 1980Q1. The missing years are filled up by calculating chaining factors over the incomplete vintages' earliest (i.e. 1980 – 1984 and 1991 – 1995) four years available. Therefore these last four years of data in each incomplete vintage are set in relation to data from 2010. Then, these factors are chain linked with actual available data (year 2010) back for the period of 1970 to 1980 or 1970 to 1991 where necessary.

3 Models

I estimate an unrestricted vector auto regression model (VAR), a structural vector error correction model (SVEC) based on Gaggl et al. (2009) and a Bayesian vector auto regression model (BVAR) based on Sims & Zha (1998). For all three models the initial estimation window is 1970:Q4 – 2004:Q3 (i.e. real-time vintage set #1) and the initial forecast window is 2004:Q4 – 2005:Q3. The last estimation window is 1970:Q4 – 2009:Q1 (i.e. vintage set #19), while the last forecast window is 2009:Q2 – 2010:Q1. Since realizations are needed to calculate forecast errors, the first 19 vintages are used for estimation, while vintages 20 to 23 are used for comparison only. As equations 1 to 4 show, forecast errors, e^h_{N+v}, are calculated by subtracting one-, two-, three- and four-step ahead forecast values, $\hat{y}^h_{N+v-1}$, based on the information set available at time *N*+v-1 from the corresponding one-, two-, three- and four-following quarters' realizations, y_{N+v}, corresponding to the first publication of the realization of *y* in *N*+v :

$$e^{h=1}_{N+v} = y_{N+v} - \hat{y}^{h=1}_{N+v-1} \tag{1}$$

$$e^{h=2}_{N+v+1} = y_{N+v+1} - \hat{y}^{h=2}_{N+v-1} \tag{2}$$

$$e^{h=3}_{N+v+2} = y_{N+v+2} - \hat{y}^{h=3}_{N+v-1} \tag{3}$$

$$e^{h=4}_{N+v+3} = y_{N+v+3} - \hat{y}^{h=4}_{N+v-1} \tag{4}$$

The forecast step size is indicated by the letter *h=1,…,4*. *N* shows the last available observation in vintage #1, i. e. the third quarter of 2004. The number of vintages is given by v = 1, …, 19.

3.1 Vector Auto Regression *(VAR)*

A VAR is a *n*-equation, *n*-variable linear model in which each variable is explained by its own lagged values plus current and past values of the remaining *n* - 1 variables;

see Stock & Watson (2001). A mathematical representation of a *pth-order* vector auto regression, denoted *VAR*(*p*), is:

$$y_t = c + \Phi_1 y_{t-1} + \Phi_1 y_{t-2} + \cdots + \Phi_p y_{t-p} + \psi x_t + \varepsilon_t, \quad (5)$$

where y_t is the (*n* x 1) vector of endogenous variables, x_t is the vector of exogenous variables, $\Phi_1 \ldots \Phi_p$ and ψ are matrices of coefficients to be estimated, and ε_t is a vector of innovations which are i.i.d. N~(0; ∑).

VAR models can be used for data description, forecasting, structural inference and policy analysis. They are a neutral way of observing interdependencies between variables since no structural assumptions – except the choice of the variables themselves and the lag length – are necessary. Therefore it can be regarded as a (economic-) theory-free way to capture dynamics in multiple time series. The same logic applies for forecasts made from VAR estimations.

Due to the large number of parameters to be estimated, however, VAR models are often inefficient and suffer from over-parameterization and a low number of degrees of freedom.

3.1.1 Stationary and Nonstationary Time Series – Unit Roots

According to Wooldridge (2006, p. 381) a strictly stationary process is one whose probability distributions are stable over time in the following sense: "if we take any collection of random variables in the sequence and then shift that sequence ahead *h* time periods, the joint probability distribution must remain unchanged"; i.e. for every collection of time indices 1≤ $t_1 < t_2 < \cdots < t_m$, the joint distribution of a stochastic process $(x_{t_1}, x_{t_2}, \ldots, x_{t_m})$ is the same as the joint distribution of $(x_{t_{1+h}}, x_{t_{2+h}}, \ldots, x_{t_{m+h}})$ for all *h* ≥ 1. When times series exhibit unit roots they are *non-stationary*, however.

The Augmented Dickey-Fuller (ADF) unit root tests in tables 12 - 24 in the appendix show strong evidence for unit roots in the levels of our data. First differences appear as stationary, however, with mixed results for inflation rates. Second differences do not show any indication for unit roots at all (see section 3.2 on cointegration for further theoretical considerations).

To determine the optimal lag length of the VAR, I use the Akaike information criterion and the sequential modified likelihood ratio test statistic (LR) as suggested by Sims (1980); see tables 28 and 29. As in Gaggl et al. (2009) an optimal lag length of 2 is the result.

3.1.2 VAR Estimation

Given these results, I estimate an unrestricted VAR(2) in differences

$$\Delta y_t = c + \Phi_1 \Delta y_{t-1} + \Phi_2 \Delta y_{t-2} + \psi(\mathrm{L})\Delta x_t + \varepsilon_t, \tag{6}$$

with the following vector of eight endogenous variables,

$$\Delta y = (ddpez, dexch, dhez, dpd, drez, drus, dyez, dyus), \tag{7}$$

with the oil price building the vector of exogenous variables,

$$\Delta x = (dpoil, dpoil(-1), dpoil(-2)). \tag{8}$$

It is common and often recommended to routinely take first differences of non-stationary time series before estimation (as shown above). This, however, can result in a misspecified regression if important dynamic relations between the variables in levels are lost due to taking differences. The next section addresses this issue.

3.2 Structural Vector Error Correction Model (*SVEC*)

The vector error correction model adds error correction terms and imposes long-run restrictions on the unrestricted VAR in differences. The SVEC model used in this paper is based on Gaggl, Kaniovski, Prettner, & Url (2009).

3.2.1 Co-integration

"Co-integration means that although many developments can cause permanent changes in the individual elements of y_t, there is some long-run equilibrium relation tying the individual components together, represented by a linear combination $ß' y_t$." Hamilton (1994, p.573)

Or in the words of Granger (1987): "If each element of a vector of time series x_t first achieves stationarity after differencing, but a linear combination $ß' x_t$ is already stationary, the time series x_t are said to be co-integrated with co-integrating vector ß."

Co-integrated models are based on the assumption that the endogenous variables are integrated of order one, I(1), meaning that they are non-stationary. In other words, a lot of economic time series behave like I(1) processes, i.e. they show an upward trend or drift, however, when compared with other variables they do not drift away from each other, i.e. are cointegrated. Several co-integrating vectors may exist within a multiple time series model.

3.2.2 SVEC Model Estimation

I apply the Johansen cointegration test to vintage set #19 which covers almost the full sample available and find six co-integrating relations – like Gaggle at al. (2009) do – according to the trace statistic (see Tables 30 and 31 in the appendix). The results from the trace test indicate six cointegrating relations, whereas the results from the maximum eigenvalue test indicate two relations. These results are only provided as side-information, however, as I estimate the SVEC model in exactly the same way as Gaggl et al. (2009) did.

As can be seen in Gaggl et al. (2009, p.214), "steady-state conditions derived from economic theory as identifying restrictions for estimation of the cointegrating vectors" are used while short-term dynamics are entirely data-driven.

The following equation shows a general vector error correction model:

$$\Delta y_t = c - \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \sum_{i=0}^{p-1} \psi_i \Delta x_{t-i} + \varepsilon_t, \quad (9)$$

where Δy_t is the *m*x1 vector of endogenous variables in first differences and *c* is a *m*x1 vector of constants. Γ_i are the *mxm* coefficient matrices describing the short-term response to past variations in lagged endogenous variables, *p* is the order of the vector autoregressive process in levels, ε_t is an *m*x1 vector of innovations, i.i.d. N~(0; Σ) and ψ_i are the coefficient matrices of the lagged exogenous variable Δx_t. The matrix Π ($\Pi=\alpha\beta'$) is the error correction mechanism if the elements of y_t are integrated of order one and relates Δy_t to past values of y_t.

3.2.3 Steady-State Conditions

Additionally, Gaggl et al. (2009) derive steady-state relations from a dynamic open economy model which can also be used to get restrictions for the estimation of the co-integrating vectors of the SVEC model.[2]

The following part is identical to the representation in Gaggl et al. (2009, p. 213). The vector $y_t = (m_t, y_t, i_t, \Delta p_t, i_t^*, (p_t - p_t^*), e_t, y_t^*,)$ includes the real money stock, real output levels of home and foreign, nominal interest rates, the inflation rate, the price differential between home and foreign, and the exchange rate. Given these

[2] See the process of deriving steady-state conditions from a macroeconomic model in Gaggl et al. (2009, Chapter 2) or more detailed in Romer (2006).

variables, the steady-state equilibrium conditions suggest the following set of restrictions on the coefficients of matrix **β** containing the co-integrating vectors:

$$m_t - \beta_{22} y_t + \beta_{23} i_t = b_{10} \xi_{1t+1} \tag{10}$$

$$i_t - \Delta p_t = b_{20} + \xi_{2t+1} \tag{11}$$

$$i_t - i_t^* = b_{30} + \xi_{3t+1} \tag{12}$$

$$p_t - p_t^* - e_t = b_{40} + \xi_{4t+1} \tag{13}$$

$$y_t - y_t^* = b_{50} + \xi_{5t+1} \tag{14}$$

where $\boldsymbol{\beta}'\boldsymbol{y_{t-1}} = \xi_t$ with long-run equilibrium errors, ξ_{it} $i = 1,2, \ldots ,5$, having mean zero.

Equations 10 - 14 feature 0 or 1 restrictions, except equation 10 that contains two free coefficients. All in all, the macroeconomic model proposes five long-run steady-state conditions. Gaggl et al. (2009) finally assume the following model structure:

$$\Delta y_t = \alpha \beta' \mathrm{c} - \alpha \beta' y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \sum_{i=0}^{p-1} \psi_i \Delta poil_{t-i} + \varepsilon_t \tag{15}$$

including all possible combinations of the potential long run restrictions and get the following matrix of cointegrating vectors

$$\boldsymbol{\beta}' = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix}. \tag{16}$$

This means 24 restrictions are imposed on the matrix **β**. The 8 x 1 coefficient vectors ψ_i show the dynamic response of the system to current and previous changes in the oil price.

3.3 Bayesian Vector Auto Regression (BVAR)

"Let Γ be an (mx1) vector of parameters to be estimated from a sample of observations y_t. Classical statistics assumes that there exists a true value for Γ_i. This true value is regarded as an unknown but fixed number. An estimator $\hat{\gamma}_i$ is constructed from the data, and $\hat{\gamma}_i$ is therefore a random variable. In classical statistics, the mean and probability limit of the random variable $\hat{\gamma}_i$ are compared with the true value γ_i." Hamilton (1994, p. 35)

In Bayesian statistics, however, the γ_i itself is regarded as a random variable. The goal of Bayesian statistical analysis is to describe the uncertainty about γ_i in terms of a probability distribution.

Bayes' law is the foundation of Bayesian statistics,

$$p(\mathrm{B}|\mathrm{A}) = \frac{p(\mathrm{A}|\mathrm{B})\mathrm{p}(\mathrm{B})}{p(\mathrm{A})} \tag{17}$$

where *A* and *B* are some random variables.

For our purposes, let us replace *A* by *y* and *B* by Γ to get:

$$p(\Gamma|y) = \frac{p(y|\Gamma)\mathrm{p}(\Gamma)}{p(y)}. \tag{18}$$

The term $p(\Gamma|y)$ is called posterior density; the probability distribution function for the data given the parameters of the model, $p(y|\Gamma)$, is referred to as the likelihood function and $p(\Gamma)$ as the prior; see Koop (2003, pp. 1-5).

The prior does not depend on data, i.e. contains only non-data information available, it allows to be adjusted to our needs. Contrarily, empirical Bayesian methods often use data-based information to choose the prior.

3.3.1 BVAR Estimation

According to Koop (2003) there is an ongoing debate about the importance of unit roots and cointegration in Bayesian analysis; see Koop (2003, p. 299) for discussion. Since no predominant Bayesian theory exists about that issue, I estimate a BVAR with the following vector of endogenous variables:

$$y = (hez, yez, rez, pd, rus, dpez, exch,\ yus) \tag{19}$$

and

$$x = (poil) \tag{20}$$

as exogenous variable. I estimate the Bayesian VAR (BVAR) model with a random walk prior as described in Sims and Zha (1998) using the MSBVAR (Markov-switching Bayesian reduced form vector auto regression model setup and posterior mode estimation) package for R-software. The model for y is:

$$y\Phi_0 = C + \Phi(L)y_{t-1} + \psi(L)x_t + \varepsilon_t, \tag{21}$$

where y_{t-1} is the *m*x1 vector of observations, $\Phi(L)$ is a *mxm* matrix polynomial of lag operator *L* with lag length *p* and nonnegative powers and $\mathrm{x_t}$ is the *m*x1 vector of the exogenous variable. *C* is a constant vector. The model can be rewritten in matrix form:

$$\mathbf{Y\Phi_0} - \mathbf{X\Phi_+} = \mathbf{E}, \tag{22}$$

where $\mathbf{Y}$ is a *Txm matrix*, $\mathbf{\Phi_0}$ is *mxm,* $\mathbf{X}$ *is* Tx*k,* $\mathbf{\Phi_+}$ is *kxm*, and $\mathbf{E}$ is *Txm*. $\mathbf{X}$ contains the lagged $\mathbf{Y}$'s and a column of 1's corresponding to the constant, *T* is the number of observations, *m* is the number of equations, and *k* = *mp*+1 is the number of coefficients corresponding to $\mathbf{X}$. The conditional likelihood function in compact form can be written in the following way:

$$L(\boldsymbol{Y}|\mathbf{\Phi})\boldsymbol{\alpha}|\mathbf{\Phi_0}|^{\boldsymbol{T}}\exp[-0.5trace(\mathbf{Z\Phi})'(\mathbf{Z\Phi})] \tag{23}$$

$$\boldsymbol{\alpha}|\mathbf{\Phi}_0|^{T}\exp[-0.5\boldsymbol{\phi}'(\boldsymbol{I}\otimes\boldsymbol{Z}'\boldsymbol{Z})\boldsymbol{\phi}]$$

Thereby $\boldsymbol{\phi}$ has prior p.d.f.

$$\pi(\boldsymbol{\phi}) = \pi_0(\boldsymbol{\phi}_0)\varphi\big(\boldsymbol{\phi}_+ - \mu(\boldsymbol{\phi}_0); H(\boldsymbol{\phi}_0)\big), \tag{24}$$

where $\pi_0(\,)$ is a marginal distribution of $\boldsymbol{\phi}$ and $\varphi(\cdot\,;\,\Sigma)$ is the standard normal p.d.f. with covariance matrixΣ. Combining (23) and (24) we get the posterior density function of$\boldsymbol{\phi}$:

$$q(\boldsymbol{\phi})\ \boldsymbol{\alpha}\ \pi_0(\boldsymbol{\phi_0})|\Phi(0)|^T|H(\boldsymbol{\phi}_0|^{-\frac{1}{2}} \tag{25}$$

$$\times \exp[-0.5(\boldsymbol{\phi}'_0(\boldsymbol{I}\otimes\boldsymbol{Y}'\boldsymbol{Y})\boldsymbol{\phi}_0 - 2\boldsymbol{\phi}'_+(\boldsymbol{I}\otimes\boldsymbol{X}'\boldsymbol{Y})\boldsymbol{\phi}_0 + \boldsymbol{\phi}'_+(\boldsymbol{I}\otimes\boldsymbol{X}'\boldsymbol{X})\boldsymbol{\phi}_+$$

$$+\big(\boldsymbol{\phi}_+ - \mu(\boldsymbol{\phi}_0)\big)' H(\boldsymbol{\phi}_0)^{-1}(\boldsymbol{\phi}_+ - \mu(\boldsymbol{\phi}_0))].$$

For a more detailed discussion see Sims and Zha (1998, pp. 950 – 952).

3.3.2 The Random Walk Prior

The Litterman (1986) prior for a reduced form model suggests that a random walk model for each variable in the system is a reasonable assumption. As Sims and Zha (1998, p. 954) show "it suggests that beliefs about the reduced form coefficient matrix

$$\mathbf{B} = \boldsymbol{\Phi}_+\boldsymbol{\Phi}_{\mathbf{0}}^{-\mathbf{1}} \tag{26}$$

should be centered on an identity matrix for the top *m* rows and zeros for the remaining rows". They make the conditional distribution for $\boldsymbol{\Phi}_+$ Gaussian with mean of $\boldsymbol{\Phi}_0$ in the first *m* rows and 0 in the remaining rows, or

$$E[\boldsymbol{\Phi}_+|\boldsymbol{\Phi}_0] = \begin{bmatrix} \boldsymbol{\Phi}_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{27}$$

Sims and Zha (1998, p. 954) assume "the prior conditional covariance matrix of the coefficients in $\boldsymbol{\Phi}_+$ follows the same pattern that Litterman gave to the prior covariance matrix on reduced form coefficients." This means, they make the conditional prior independent across elements of $\boldsymbol{\Phi}_+$ and with the conditional standard deviation of the coefficient on lag l of variable *j* in equation *i* given by

$$\frac{\lambda_0\lambda_1}{\sigma_j l^{\lambda_3}} \; . \tag{28}$$

The hyperparameter λ_0 stands for the overall tightness of the prior and controls the beliefs on $\mathbf{\Phi}_0$, λ_1 controls the standard deviation or tightness of the beliefs around the random walk prior, λ_3 stands for the lag decay, i.e. the rate at which the prior variance shrinks with increasing lag length. The vector of parameters, $\sigma_1, \dots, \sigma_m$, contains scale factors. The last row of $\boldsymbol{\Phi}_+$ corresponds to the constant term. Sims and Zha give it a conditional prior mean of zero and a standard deviation controlled by $\lambda_0\lambda_4$, where λ_4 is a separate hyperparameter that controls the standard deviation or tightness around the intercept.

I use the MSBVAR program in R-software for the estimation of the model. Thereby, λ_5 controls the standard deviation or tightness around the exogenous variable, i.e. the oil price, μ_5 stands for the sum of coefficients, where larger values imply difference stationarity, and μ_6 is a drift parameter where larger values allow for common trends. I use the following values for lambdas $\lambda_0 = 1, \lambda_1 = 0.2$, $\lambda_3 = \lambda_4 = \lambda_5 = 1$, and $\mu_5 = 0$, $\mu_6 = 0.1$ as suggested in Sims & Zha (1998) and lag length *p*=2.

4 Measures of forecast accuracy

The most straightforward way to evaluate the accuracy of a forecast is to compare the forecast, $\hat{y}_{N+v-1}^{h}$, with the actual economic outcome, y_{N+v} (the realization). Ignoring the fact that a forecast can have an influence on the realized outcome (this question does not arise within the context of this paper), a forecast can be regarded as good if the degree of congruence is high.

The following analysis covers the quarterly forecasts of the VAR, SVEC and BVAR models for the Eurozone and the US-American economy.

The forecast accuracy is examined on the basis of different criteria which will be divided into three groups; see Baumgartner (2002, p. 194):

1. Measures of statistical accuracy
2. Theil coefficients, i.e. comparison with "naïve" forecasts
3. Tests for the correct sign of the forecast and its significance

For all measures of accuracy discussed the following relation holds: The smaller the value of the measure the better this model predicts real-time realizations. The mean forecast error (*ME*) shows the average deviation of the projected values ($\hat{y}_{N+v-1}^{h}$) from the actual (realized) values (y_{N+v}). Its value is close to zero if the forecast accuracy is high or if under- and overestimations cancel each other. A value close to zero indicates unbiased forecast values but information going beyond the direction of the forecasts is impossible to gather from the *ME*. Equation (29) shows the mean error of the one-step forecast level, where $e_{N+v}^{h=1}$ is derived from equation (1). Equation (30) shows the mean error on the four-step forecast level.

$$ME^{h=1} = \frac{1}{19}\sum_{v=1}^{19}(e_{N+v}^{h=1}) \qquad v=1,\dots,19 \qquad (29)$$

$$\vdots$$

$$ME^{h=4} = \frac{1}{19}\sum_{v=1}^{19}(e_{N+v+3}^{h=4}) \qquad v=1,\ldots,19 \qquad (30)$$

The mean absolute error (*MAE*) is calculated by taking the average of the absolute values of the forecast errors and allows statements about the overall forecast accuracy:

$$MAE^{h=1} = \frac{1}{19}\sum_{v=1}^{19}\left|e_{N+v}^{h=1}\right| \qquad v=1,\ldots,19 \qquad (31)$$

$$\vdots$$

$$MAE^{h=4} = \frac{1}{19}\sum_{v=1}^{19}\left|e_{N+v+3}^{h=4}\right| \qquad v=1,\ldots,19 \qquad (32)$$

The mean squared error (*MSE*) also measures the accuracy of the forecast. Hereby the squares of the forecast errors are added and the average is taken. In contrast to the *MAE* the forecast errors do not enter the equation linearly but by the power of two. The *MSE*, therefore, puts more emphasis on large forecast errors:

$$MSE^{h=1} = \frac{1}{19}\sum_{v=1}^{19}(e_{N+v}^{h=1})^{2} \qquad v=1,\ldots,19 \qquad (33)$$

$$\vdots$$

$$MSE^{h=4} = \frac{1}{19}\sum_{v=1}^{19}(e_{N+v+3}^{h=4})^{2} \qquad v=1,\ldots,19 \qquad (34)$$

The root mean squared error (*RMSE*) gives back a statistic that has the same dimension as the underlying variable:

$$RMSE^{h} = \sqrt{MSE^{h}}. \qquad (35)$$

It can be split up in two ways that allow further insight into the accuracy of the forecast errors.

$$UM^{h} + US^{h} + UC^{h} = 1 \qquad (36)$$

$$UM^h + UR^h + UD^h = 1 \tag{37}$$

The bias (*UM*), variance (*US*), and regression (*UR*) proportions should be small, while the co-variance (*UC*) and distribution (*UD*) proportions, respectively, should be close to 1 for a good forecast[3].

Variables with less variation are obviously easier to predict than those who are subject to wide variation. To take that into account I divided the $RMSE$ by the corresponding variable's realization's standard deviation, SD_y, i.e. $RMSE/SD_y$. This measure allows easier comparison between different variables.

The Theil inequality statistics allow an easy interpretation as they are standardized to one. This statistic relates our forecast errors to "naïve" forecasts. In the case of the Theil *W* statistic, the "naïve" forecast is the hypothesis of "no change in the rate of change", whereas the Theil *U*'s "naïve" forecast relates to the hypothesis of "no change in the level". A Theil *W* or *U* value smaller than 1 indicates superiority of the model's forecast over the "naïve" one.

$$Theil\,W = \sqrt{\frac{\sum_{v=1}^{19}\left(e_{N+v+i}^{h}\right)^2}{\sum_{v=1}^{19}(y_{N+v} - y_{N+v-h})^2}} \qquad i=0,\ldots,3 \qquad v=1,\ldots,19 \tag{38}$$

$$Theil\,U = \sqrt{\frac{\sum_{v=1}^{19}\left(e_{N+v+i}^{h}\right)^2}{\sum_{v=1}^{19} y_{N+v+h-1}^2}} \qquad i=0,\ldots,3 \qquad v=1,\ldots,19 \tag{39}$$

[3] See the methodological annex for a detailed explanation of these proportions.

The ratio of congruence (*ER*) is calculated as the ratio of correctly projected sign changes to all changes of direction based on a congruency table; see Baumgartner (2005, p. 205) and Bleymüller, Gehlert, & Herbert (2008).

$$ER = \frac{a+b}{a+b+c+d} \tag{40}$$

ER is tested using a χ^2-distributed test of independence with one degree of freedom. The null hypothesis states that the sign of the change of the forecast and the sign of the change of the realization are statistically independent. The χ^2 test statistic is defined as follows:

$$\chi^2 = \frac{(a+b+c+d)(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}. \tag{41}$$

Every *ER* is checked for statistical significance and marked by a star "*" when the corresponding p-value is below 0.10. In case of perfect sign prediction (c = 0 and d = 0), the test statistic shows an undefined value (division by zero) and cannot be inserted into its probability distribution. For that case the corresponding entry in the tables will be "1.00", meaning perfect sign congruence, without available statistical significance though.

5 Forecasting performance between 2004:Q4 – 2010:Q1

The following section provides the results of the forecasting competition between the three models. As explained in section three, 19 one-, two-, three- and four-step ahead forecast errors were calculated for each model (see equations 1-4 as well as figure 1). These errors are now analyzed by applying the measures of forecast accuracy from section 4.

The analysis of the forecasting performance will focus mainly on GDP, Eurozone inflation and the exchange rate, as these variables are of greater importance for economic researchers than the other variables (M1, price differential, and interest rates).

5.1 One-step ahead forecasts

As can be seen in table 2, the Bayesian VAR model clearly outperforms the unrestricted VAR model and the error correction model in predicting interest rates (*rez, rus*) and GDP levels (*yez, yus*). The same holds for the EUR/USD exchange rate with the BVAR showing superior performance in terms of Theil coefficients and other measures.

The unrestricted VAR performs best at predicting Eurozone inflation rate and the price differential. Only for the Eurozone inflation rate and the price differential the structural error correction model performs better than the BVAR. Furthermore, the SVEC model predicts worse than the unrestricted VAR model on every field.

Concerning the sign test, the SVEC model performs better than the VAR for all statistical significant results, while BVAR performs better than the two other models for the Eurozone inflation rate.

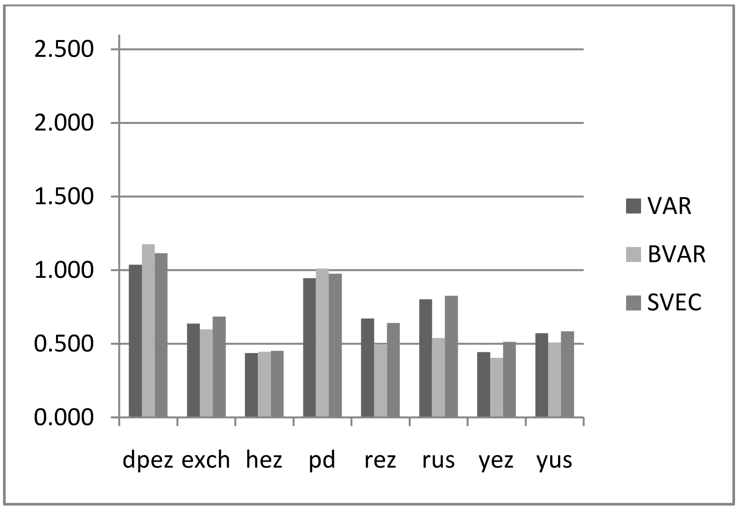

Figure 2: One-step RMSE/SD_y

5.2 Two-step ahead forecasts

Root mean squared errors for two-step forecasts partly increased by more than 50% compared to one-step results.

Table 3 shows similar results as table 2. The Bayesian VAR exhibits lower RMSE values and lower Theil coefficients than the unrestricted VAR for three variables (*rez, rus, yus*). For the Eurozone GDP, however, the VAR model shows a slightly lower Theil coefficient of 0.641 versus 0.664 of the BVAR.

The VAR model is the best model at predicting all four left hand side variables (*dpez, exch, hez, pd*).

In table 3 the SVEC model again shows the weakest performance of all three models. It shows higher errors than the VAR for every forecasted variable. Concerning the correct sign of the forecasts, the BVAR outperforms both models with statistically significant results for inflation rate and Eurozone interest rates. The SVEC is the best sign predictor among all models for Eurozone GDP.

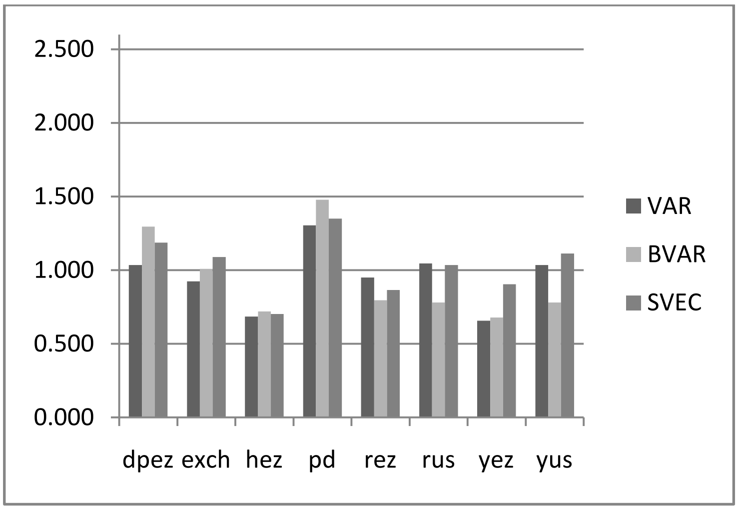

Figure 3: Two-step RMSE/SD_y

5.3 Three-step ahead forecasts

Figure 4 shows RMSE/SD_y results for three-step ahead forecasts. Again the values increased significantly compared to the previous step size. Root mean squared errors for GDP more than doubled compared to one-step results.

The BVAR turns out to be the best model to predict both GDP levels and interest rates with a forecast step size of three. The BVAR is the only model that exhibits lower than one Theil coefficients for US-GDP forecasts, meaning that the other models add less information than the naïve forecasts. Concerning the correct sign, both VAR and SVEC show better results.

For Eurozone inflation rates all three models exhibit Theil W coefficients higher than one, with VAR- and SVEC- models performing better than the BVAR, however.

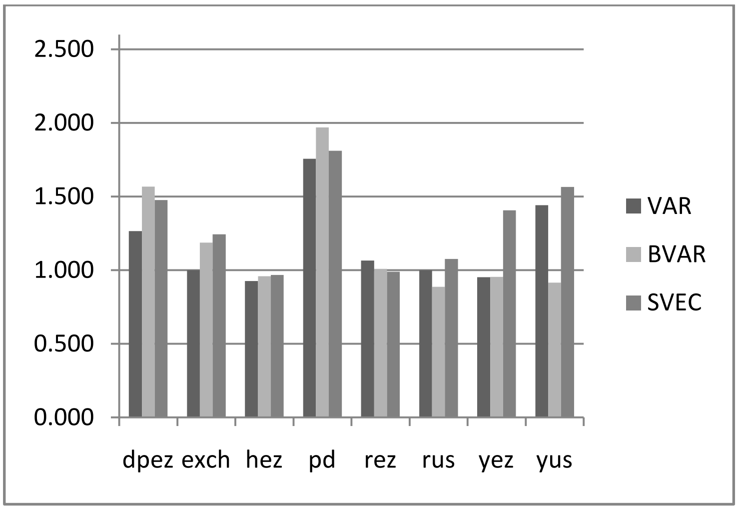

Figure 4: Three-step RMSE/SD_Y

5.4 Four-step ahead forecasts

Also on the four-step level the structure among the variables stays the same, existing gaps widened however. Table 5 shows that the BVAR is superior to VAR and SVEC in forecasting GDP levels, while VAR and SVEC are superior at Eurozone inflation rate. Interestingly, four-step errors for the inflation rate are smaller than for the three-step level for all models. VAR and SVEC even exhibit Theil W coefficients lower than unity again.

SVEC is the worst predictor for GDP levels in terms of accuracy, while it is the only model with 100% correct sign predictions. However, as the mean error (ME) shows SVEC under predicts GDP levels more than the two others.

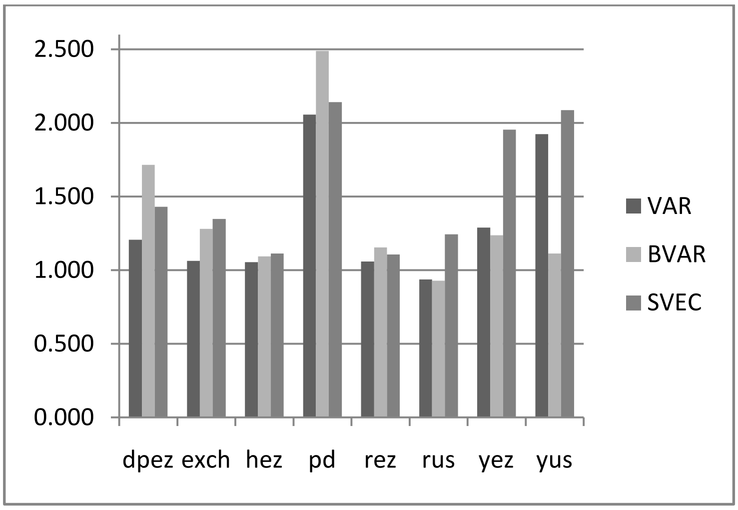

Figure 5: Four-step RMSE/SD_y

5.5 Chapter summary

Four specific patterns can be found after having observed and compared one-, two-, three-, and four-step forecast errors.

- The Bayesian VAR is the best predictor for both GDP levels in terms of statistical accuracy. This holds in particular for US-GDP, while the unrestricted VAR can compete with BVAR for Eurozone GDP forecasts at eye level. Mean errors show that VAR and SVEC models under-predict GDP levels on average for the observed period.
- VAR and SVEC are clearly superior to BVAR at predicting Eurozone inflation rate and the price differential. BVAR systematically under predicts *dpez*. VAR, however, outperforms SVEC on all step sizes.
- While SVEC cannot compete with the two other models in terms of statistical accuracy, it shows the best results among all models at predicting the correct sign of GDP development.
- SVEC Theil W coefficients for three- and four-step ahead GDP forecasts are above unity.

Table 2: One-step ahead forecast errors

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.007	0.006	-0.001	0.003	0.002	-0.002	-0.004
MAE	0.003	0.035	0.017	0.006	0.005	0.008	0.006	0.006
MSE	0.000	0.002	0.001	0.000	0.000	0.000	0.000	0.000
RMSE	**0.004**	**0.047**	**0.024**	**0.010**	**0.007**	**0.011**	**0.010**	**0.008**
UM	0.03	0.02	0.05	0.00	0.13	0.03	0.04	0.20
US	0.17	0.10	0.02	0.03	0.18	0.23	0.01	0.03
UC	0.80	0.88	0.92	0.96	0.69	0.74	0.95	0.77
UR	0.10	0.30	0.12	0.32	0.37	0.48	0.08	0.00
UD	0.88	0.68	0.83	0.67	0.50	0.49	0.88	0.80
$RMSE/SD_R$	1.036	0.636	0.437	0.945	0.673	0.801	0.443	0.572
Theil W	*0.890*	*0.621*	*0.522*	*0.992*	*0.727*	*1.015*	*0.618*	*0.660*
Theil U	*0.615*	*0.154*	*0.074*	*0.295*	*0.218*	*0.287*	*0.133*	*0.080*
ER	0.58*	0.53*	0.84	0.21	0.37	0.42	0.84*	0.84*

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.002	0.001	0.001	0.003	-0.001	-0.002	-0.001	-0.001
MAE	0.003	0.033	0.018	0.007	0.003	0.005	0.006	0.005
MSE	0.000	0.002	0.001	0.000	0.000	0.000	0.000	0.000
RMSE	**0.004**	**0.045**	**0.025**	**0.011**	**0.005**	**0.008**	**0.009**	**0.007**
UM	0.17	0.00	0.00	0.09	0.03	0.05	0.02	0.02
US	0.09	0.03	0.01	0.01	0.04	0.01	0.01	0.08
UC	0.74	0.97	0.99	0.90	0.93	0.94	0.97	0.91
UR	0.14	0.19	0.09	0.26	0.17	0.03	0.08	0.00
UD	0.69	0.81	0.91	0.65	0.80	0.92	0.90	0.98
$RMSE/SD_R$	1.176	0.598	0.445	1.010	0.491	0.539	0.404	0.508
Theil W	*1.012*	*0.602*	*0.531*	*1.057*	*0.532*	*0.683*	*0.569*	*0.586*
Theil U	*0.698*	*0.145*	*0.075*	*0.315*	*0.159*	*0.193*	*0.121*	*0.071*
ER	0.84*	0.26	1.00	0.11	0.74*	0.58	0.89*	0.68

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	0.000	-0.003	0.003	0.001	0.002	-0.004	-0.006	-0.004
MAE	0.003	0.038	0.018	0.006	0.005	0.008	0.008	0.006
MSE	0.000	0.003	0.001	0.000	0.000	0.000	0.000	0.000
RMSE	**0.004**	**0.051**	**0.025**	**0.010**	**0.007**	**0.012**	**0.012**	**0.008**
UM	0.01	0.00	0.01	0.01	0.06	0.09	0.29	0.23
US	0.07	0.11	0.06	0.04	0.22	0.16	0.01	0.02
UC	0.92	0.88	0.92	0.95	0.72	0.75	0.70	0.75
UR	0.21	0.34	0.19	0.35	0.42	0.40	0.06	0.01
UD	0.78	0.66	0.80	0.65	0.52	0.51	0.64	0.76
$RMSE/SD_R$	1.115	0.684	0.452	0.975	0.640	0.827	0.513	0.584
Theil W	*0.960*	*0.687*	*0.539*	*1.024*	*0.691*	*1.040*	*0.704*	*0.674*
Theil U	*0.662*	*0.166*	*0.076*	*0.304*	*0.207*	*0.297*	*0.154*	*0.081*
ER	0.58*	0.47*	0.89*	0.21	0.37	0.79	0.89*	0.84*

Table 3: Two-step ahead forecast errors

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.013	0.013	-0.002	0.005	0.004	-0.004	-0.008
MAE	0.003	0.055	0.030	0.009	0.008	0.013	0.009	0.010
MSE	0.000	0.005	0.002	0.000	0.000	0.000	0.000	0.000
RMSE	**0.004**	**0.070**	**0.040**	**0.013**	**0.011**	**0.017**	**0.014**	**0.015**
UM	0.03	0.03	0.11	0.03	0.22	0.06	0.09	0.26
US	0.04	0.02	0.01	0.11	0.24	0.24	0.02	0.01
UC	0.94	0.95	0.88	0.87	0.54	0.69	0.90	0.72
UR	0.17	0.27	0.15	0.52	0.44	0.54	0.16	0.10
UD	0.80	0.69	0.74	0.45	0.34	0.40	0.75	0.64
RMSE/SD_R	1.035	0.924	0.685	1.303	0.949	1.047	0.656	1.036
Theil W	*0.694*	*0.761*	*0.607*	*1.068*	*0.817*	*1.117*	*0.641*	*0.966*
Theil U	*0.656*	*0.225*	*0.116*	*0.357*	*0.337*	*0.421*	*0.184*	*0.149*
ER	0.53	0.47	0.89	0.26	0.32	0.37	0.84*	0.89

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.003	0.007	0.004	0.007	-0.003	-0.003	-0.002	0.000
MAE	0.004	0.061	0.033	0.012	0.006	0.009	0.010	0.009
MSE	0.000	0.006	0.002	0.000	0.000	0.000	0.000	0.000
RMSE	**0.005**	**0.076**	**0.042**	**0.014**	**0.010**	**0.012**	**0.015**	**0.011**
UM	0.42	0.01	0.01	0.23	0.09	0.07	0.01	0.00
US	0.11	0.00	0.00	0.02	0.01	0.04	0.01	0.10
UC	0.47	0.99	0.99	0.75	0.90	0.89	0.97	0.90
UR	0.04	0.28	0.15	0.36	0.19	0.04	0.18	0.02
UD	0.54	0.71	0.84	0.41	0.72	0.89	0.81	0.98
RMSE/SD_R	1.295	1.008	0.719	1.478	0.796	0.781	0.679	0.781
Theil W	*0.864*	*0.873*	*0.643*	*1.207*	*0.683*	*0.840*	*0.664*	*0.727*
Theil U	*0.821*	*0.245*	*0.122*	*0.405*	*0.283*	*0.314*	*0.190*	*0.113*
ER	0.84*	0.16	1.00	0.11	0.79*	0.53	0.89*	0.58

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.008	0.009	0.002	0.002	-0.006	-0.013	-0.009
MAE	0.003	0.065	0.032	0.009	0.007	0.012	0.014	0.011
MSE	0.000	0.007	0.002	0.000	0.000	0.000	0.000	0.000
RMSE	**0.005**	**0.083**	**0.041**	**0.013**	**0.010**	**0.016**	**0.020**	**0.016**
UM	0.07	0.01	0.05	0.03	0.03	0.15	0.46	0.34
US	0.01	0.03	0.05	0.11	0.32	0.14	0.01	0.00
UC	0.93	0.95	0.90	0.86	0.65	0.70	0.53	0.66
UR	0.28	0.38	0.25	0.54	0.57	0.41	0.10	0.11
UD	0.65	0.61	0.70	0.43	0.40	0.44	0.44	0.55
RMSE/SD_R	1.187	1.089	0.703	1.351	0.866	1.035	0.905	1.113
Theil W	*0.782*	*0.929*	*0.624*	*1.109*	*0.745*	*1.114*	*0.871*	*1.039*
Theil U	*0.752*	*0.265*	*0.119*	*0.370*	*0.308*	*0.417*	*0.253*	*0.160*
ER	0.58	0.37	1.00	0.16	0.47	0.74	0.95*	1.00

Table 4: Three-step ahead forecast errors

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	0.000	-0.018	0.021	-0.004	0.007	0.003	-0.008	-0.013
MAE	0.004	0.067	0.045	0.010	0.010	0.014	0.014	0.015
MSE	0.000	0.006	0.003	0.000	0.000	0.000	0.000	0.000
RMSE	**0.005**	**0.079**	**0.057**	**0.016**	**0.014**	**0.017**	**0.020**	**0.021**
UM	0.00	0.05	0.14	0.05	0.24	0.04	0.16	0.42
US	0.01	0.01	0.01	0.16	0.18	0.12	0.01	0.01
UC	0.99	0.94	0.86	0.78	0.58	0.84	0.83	0.57
UR	0.38	0.18	0.20	0.65	0.41	0.44	0.21	0.14
UD	0.62	0.76	0.66	0.30	0.35	0.53	0.63	0.44
RMSE/SD_R	1.265	0.999	0.926	1.757	1.065	1.000	0.953	1.442
Theil W	*1.004*	*0.826*	*0.696*	*1.296*	*0.809*	*0.951*	*0.720*	*1.136*
Theil U	*0.760*	*0.247*	*0.159*	*0.438*	*0.414*	*0.450*	*0.254*	*0.210*
ER	0.53*	0.47*	0.95	0.32*	0.21	0.26	0.84*	1.00

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.004	0.013	0.008	0.011	-0.005	-0.005	-0.002	0.001
MAE	0.005	0.075	0.050	0.016	0.009	0.012	0.015	0.011
MSE	0.000	0.009	0.004	0.000	0.000	0.000	0.000	0.000
RMSE	**0.006**	**0.094**	**0.059**	**0.018**	**0.013**	**0.015**	**0.020**	**0.013**
UM	0.45	0.02	0.02	0.35	0.14	0.10	0.01	0.01
US	0.08	0.00	0.00	0.02	0.00	0.09	0.01	0.09
UC	0.47	0.98	0.98	0.63	0.86	0.81	0.98	0.90
UR	0.14	0.33	0.23	0.39	0.20	0.03	0.27	0.07
UD	0.41	0.65	0.75	0.26	0.66	0.87	0.72	0.92
RMSE/SD_R	1.567	1.188	0.958	1.969	1.009	0.886	0.954	0.916
Theil W	*1.249*	*1.000*	*0.729*	*1.436*	*0.762*	*0.859*	*0.726*	*0.721*
Theil U	*0.941*	*0.294*	*0.165*	*0.491*	*0.392*	*0.399*	*0.254*	*0.133*
ER	0.89*	1.00	1.00	0.11	0.74*	0.47	0.84*	0.37

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.010	0.015	0.004	0.000	-0.010	-0.022	-0.016
MAE	0.004	0.082	0.049	0.012	0.009	0.014	0.022	0.017
MSE	0.000	0.010	0.004	0.000	0.000	0.000	0.001	0.001
RMSE	**0.006**	**0.098**	**0.060**	**0.017**	**0.013**	**0.019**	**0.029**	**0.023**
UM	0.06	0.01	0.07	0.06	0.00	0.31	0.55	0.50
US	0.00	0.00	0.04	0.16	0.25	0.03	0.00	0.00
UC	0.94	0.99	0.90	0.78	0.75	0.66	0.45	0.50
UR	0.48	0.38	0.31	0.66	0.56	0.22	0.12	0.13
UD	0.46	0.60	0.62	0.28	0.44	0.47	0.33	0.37
RMSE/SD_R	1.476	1.244	0.968	1.811	0.988	1.075	1.406	1.564
Theil W	*1.176*	*1.045*	*0.729*	*1.339*	*0.750*	*1.042*	*1.045*	*1.230*
Theil U	*0.886*	*0.308*	*0.166*	*0.452*	*0.384*	*0.484*	*0.375*	*0.228*
ER	0.68	0.37	1.00	0.11	0.53*	0.84*	0.95*	1.00

Table 5: Four-step ahead forecast errors

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	0.000	-0.024	0.029	-0.005	0.007	0.001	-0.012	-0.019
MAE	0.004	0.073	0.055	0.010	0.011	0.014	0.017	0.019
MSE	0.000	0.007	0.005	0.000	0.000	0.000	0.001	0.001
RMSE	**0.005**	**0.083**	**0.070**	**0.017**	**0.015**	**0.018**	**0.025**	**0.028**
UM	0.01	0.09	0.17	0.08	0.22	0.00	0.22	0.49
US	0.02	0.02	0.00	0.21	0.08	0.03	0.01	0.00
UC	0.97	0.90	0.82	0.71	0.70	0.97	0.77	0.51
UR	0.32	0.18	0.21	0.69	0.33	0.31	0.29	0.24
UD	0.67	0.74	0.61	0.22	0.45	0.68	0.49	0.27
RMSE/SD_R	1.205	1.063	1.054	2.057	1.060	0.937	1.290	1.924
Theil W	*0.718*	*0.903*	*0.714*	*1.310*	*0.738*	*0.805*	*0.817*	*1.311*
Theil U	*0.750*	*0.255*	*0.187*	*0.460*	*0.446*	*0.465*	*0.324*	*0.276*
ER	0.53	0.47*	1.00	0.32*	0.16	0.32	0.84*	1.00

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.005	0.019	0.012	0.014	-0.007	-0.007	-0.002	0.004
MAE	0.005	0.084	0.063	0.019	0.012	0.014	0.019	0.014
MSE	0.000	0.010	0.005	0.000	0.000	0.000	0.001	0.000
RMSE	**0.007**	**0.099**	**0.073**	**0.021**	**0.016**	**0.018**	**0.024**	**0.016**
UM	0.61	0.04	0.03	0.46	0.18	0.14	0.00	0.06
US	0.06	0.00	0.00	0.03	0.00	0.16	0.01	0.04
UC	0.32	0.96	0.97	0.51	0.82	0.70	0.99	0.90
UR	0.06	0.37	0.28	0.38	0.20	0.01	0.41	0.20
UD	0.33	0.59	0.69	0.16	0.62	0.84	0.59	0.75
RMSE/SD_R	1.714	1.281	1.094	2.488	1.154	0.929	1.236	1.114
Theil W	*1.013*	*1.088*	*0.765*	*1.574*	*0.798*	*0.814*	*0.782*	*0.752*
Theil U	*1.067*	*0.308*	*0.194*	*0.557*	*0.486*	*0.461*	*0.311*	*0.160*
ER	0.95	0.05	1.00	0.05	0.79*	0.53	0.84*	0.26

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.002	-0.010	0.024	0.006	-0.004	-0.016	-0.030	-0.022
MAE	0.004	0.091	0.061	0.014	0.012	0.017	0.030	0.022
MSE	0.000	0.011	0.006	0.000	0.000	0.001	0.001	0.001
RMSE	**0.005**	**0.105**	**0.074**	**0.018**	**0.015**	**0.023**	**0.038**	**0.030**
UM	0.16	0.01	0.10	0.11	0.06	0.49	0.62	0.56
US	0.00	0.00	0.03	0.20	0.12	0.00	0.00	0.00
UC	0.84	0.99	0.87	0.69	0.81	0.51	0.38	0.44
UR	0.36	0.45	0.33	0.68	0.46	0.10	0.15	0.21
UD	0.48	0.54	0.57	0.21	0.48	0.41	0.23	0.23
RMSE/SD_R	1.430	1.349	1.113	2.141	1.106	1.243	1.955	2.087
Theil W	*0.842*	*1.150*	*0.758*	*1.373*	*0.768*	*1.089*	*1.225*	*1.421*
Theil U	*0.890*	*0.324*	*0.198*	*0.479*	*0.465*	*0.617*	*0.491*	*0.299*
ER	0.79	0.26	1.00	0.05	0.74*	0.89	1.00	1.00

6 Forecast evaluation

Within this section the differences between the forecast errors of all three models are analyzed. So far we have used measures of statistical accuracy to assess the forecasting performance. We saw that the Bayesian VAR and the unrestricted VAR produced better results than the structural vector error correction model for GDP prediction. On the other hand, the SVEC model and the unrestricted VAR performed better than BVAR at forecasting inflation rates, with mixed results for the EUR/USD exchange rate. Section 6.1 shows whether the notion of superiority and inferiority of one model over the other used in section 5 is interestingly large or not. Hence, the differences between the outcomes of the three models are assessed for statistical significance, e.g. are SVEC forecasts statistically significant worse predictors for GDP values than those of the other two models?

6.1 Diebold Mariano test

The Diebold Mariano test examines the difference between two forecasts. The null hypothesis of the test assumes equal accuracy of the two forecasts. Let e_t^A and e_t^B be the forecast errors of the same step size of models A and B. The loss function shall be $g(e_t^i)$, with *i*=A, B, where g is the absolute value of the forecast errors. The Diebold Mariano test is based on the loss differential of

$$d_v = g(e_v^A) - g(e_v^B). \qquad (42)$$

The Diebold Mariano test statistic is

$$DM = \frac{\bar{d}}{\sqrt{\frac{1}{19} V(\bar{d})}} \qquad (43)$$

where

$$\bar{d} = \frac{1}{19} \sum_{v=1}^{19} d_v \qquad (44)$$

and

$$V(\bar{d}) = \frac{1}{19}\left(\gamma_0 + 2\sum_{\tau=1}^{h-1}\gamma_\tau\right) \tag{45}$$

where $\gamma_\tau = cov(d_\nu, d_{\nu-\tau})$. For a detailed discussion see Diebold & Mariano (1994). The present analysis uses the absolute errors as loss functions to examine whether statistically significant differences exist between the forecasts of the three models.

The results in table 6 show that there is – according to the Diebold Mariano test – a statistically significant difference in the predicting accuracy for Eurozone GDP (yez) between the SVEC model and the VAR. The same holds for US-GDP, except for the one-step ahead forecast horizon. In other words, the null hypothesis of equal expected absolute errors can be rejected on the 5% level and we can conclude that VAR is superior to SVEC when it comes to GDP forecasting. The same holds for three- and four-step forecasts of the EUR/USD exchange rate and the price differential.

The second part of table 6 shows the p-values from the Diebold Mariano test for the structural vector error correction model and the Bayesian VAR. With p-values above 0.30 the null hypothesis of equal expected absolute errors cannot be rejected for US-GDP forecast errors. This indicates that there is no statistically significant difference between SVEC and BVAR predictions for *yus*. Concerning Eurozone GDP, for one-, two- and three-step ahead forecasts the null of equal errors can be rejected on the 5% level. No evidence for statistically different errors can be found for Eurozone inflation rates.

Table 6: Diebold Mariano Test

SVECM vs. VAR

p-value	dpez	exch	hez	pd	rez	rus	yez	yus
1-step	0.407	0.390	0.107	0.845	0.891	0.946	**0.001**	0.612
2-step	0.159	0.124	**0.010**	**0.000**	0.419	0.826	**0.000**	**0.000**
3-step	**0.039**	**0.039**	**0.000**	**0.001**	0.647	0.991	**0.000**	**0.000**
4-step	0.057	**0.015**	**0.000**	**0.012**	0.937	0.689	**0.000**	**0.000**

SVECM vs. BVAR

p-value	dpez	exch	hez	pd	rez	rus	yez	yus
1-step	0.404	0.069	0.931	0.402	0.063	**0.006**	**0.000**	0.352
2-step	0.798	0.473	0.823	0.128	0.691	0.090	**0.013**	0.374
3-step	0.593	**0.045**	0.936	0.162	0.799	0.572	**0.039**	0.246
4-step	0.244	0.173	0.809	0.105	0.298	0.484	0.066	0.371

The results from the Diebold Mariano test confirm the findings of section 5. When it comes to Eurozone GDP forecasts one can indeed speak of a statistically significant superiority of the VAR and the BVAR model over the structural error correction model. Concerning US GDP (yus) there is, however, no clear evidence that BVAR forecasts are more accurate than SVEC forecasts. The overall picture shows that VAR and SVEC deliver statistically significant different results regarding our variables of attention, whereas BVAR and SVEC results do not differ to the same extent but less.

6.2 Forecast error evolution – Eurozone GDP

Figure 6 shows the Eurozone GDP forecast error evolution over all 19 quarters for each step size separately (from left to right, from top to bottom). It can be clearly seen how the forecast errors started to increase in quarter 16 (2008:Q2) due to the disturbances of the financial crisis. While BVAR had the smallest mean errors of all three models in tables 2 – 5, figure 6 shows that this result is somehow misleading as VAR presents by far the smallest errors before the beginning of the financial crisis. To further analyze this issue section 7 provides an analysis for the pre-crisis data set only.

No evidence can be found for an improvement of the forecasting performance with increasing estimation sample sizes over the time period. This holds for all graphs in subsection 6.2.

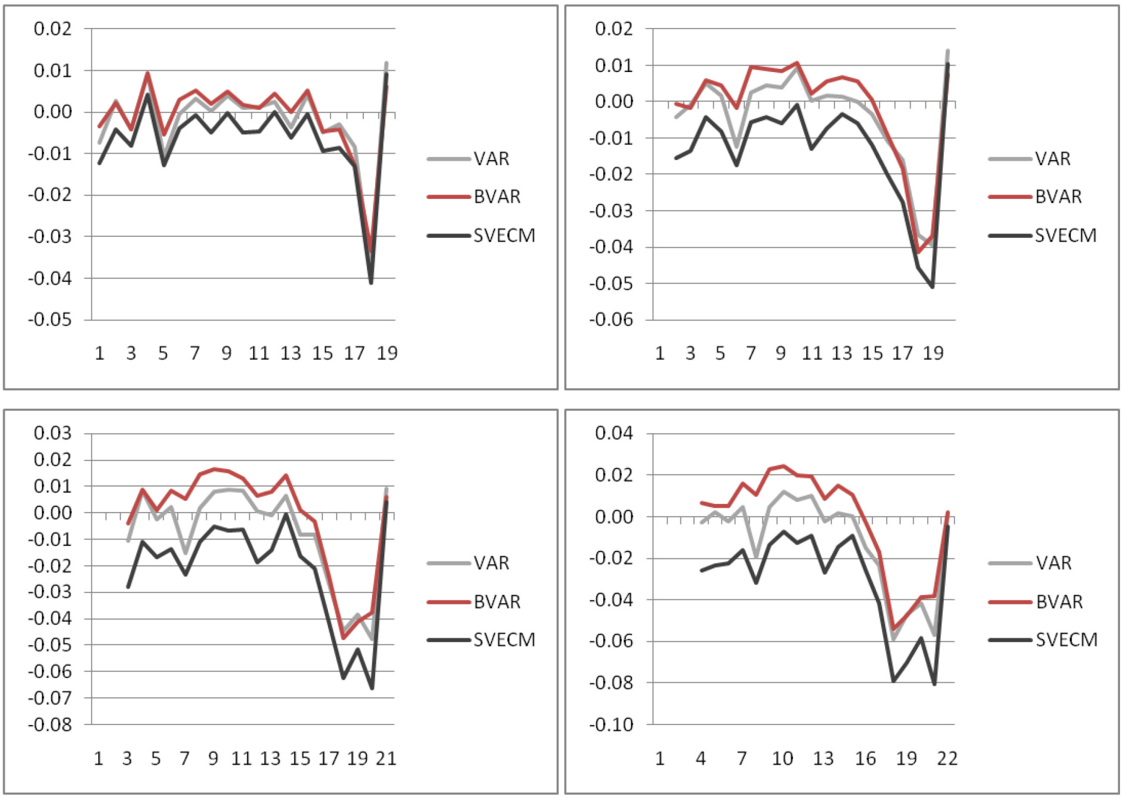

Figure 6: 1-, 2-, 3- and 4-step forecast errors - Eurozone GDP

Figure 7 shows the Eurozone GDP forecast error evolution of all step sizes for each model separately in one graph. One can see clearly how forecasts become more imprecise with higher forecast step-size.

Figure 8 shows histograms of one-step forecast errors for the Eurozone GPD. Most errors are concentrated around unity. Again one can see that SVEC typically over-estimates the GDP values.

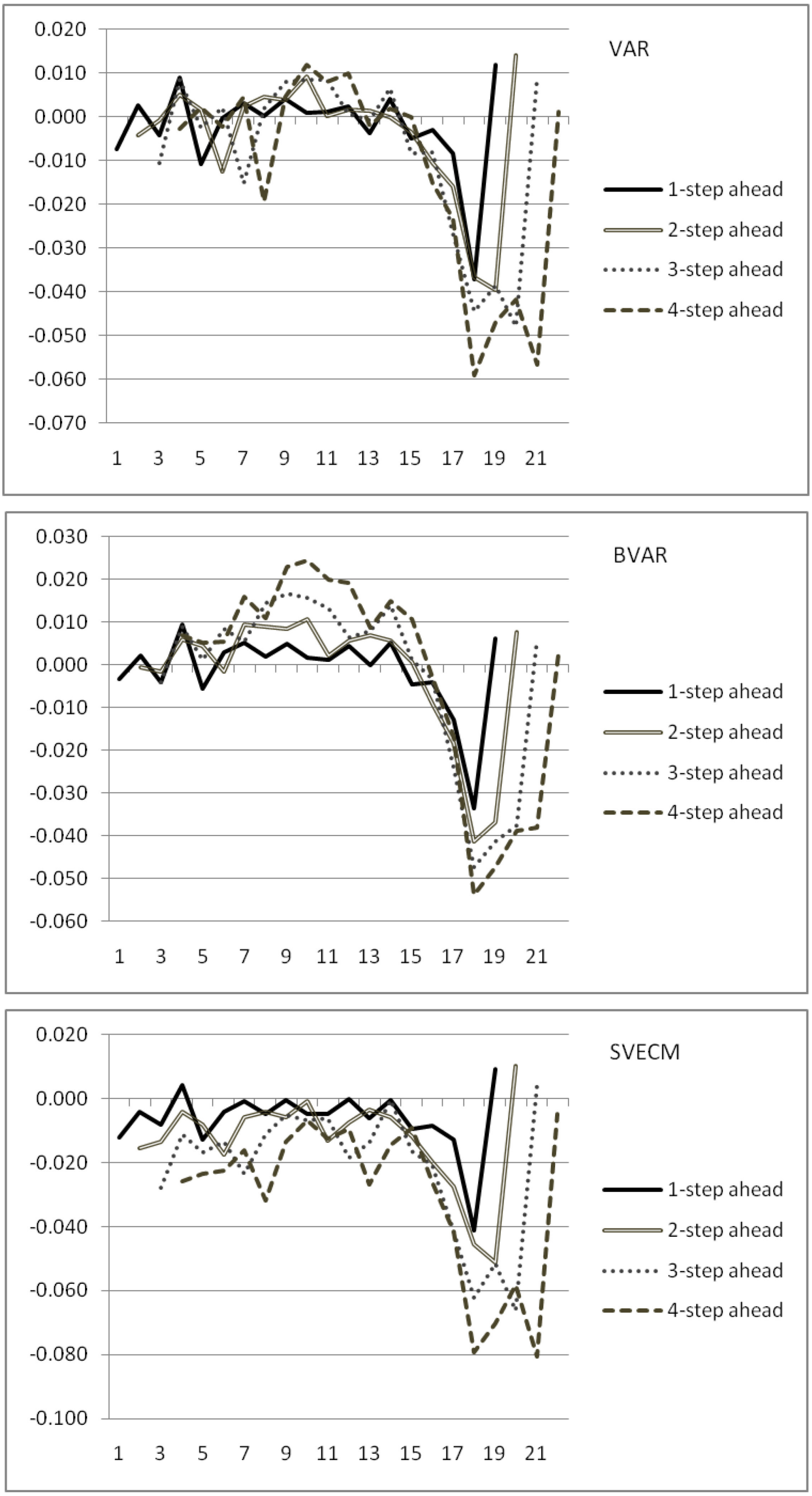

Figure 7 : 1 – 4 step forecast errors from VAR, BVAR and SVEC – Eurozone GDP

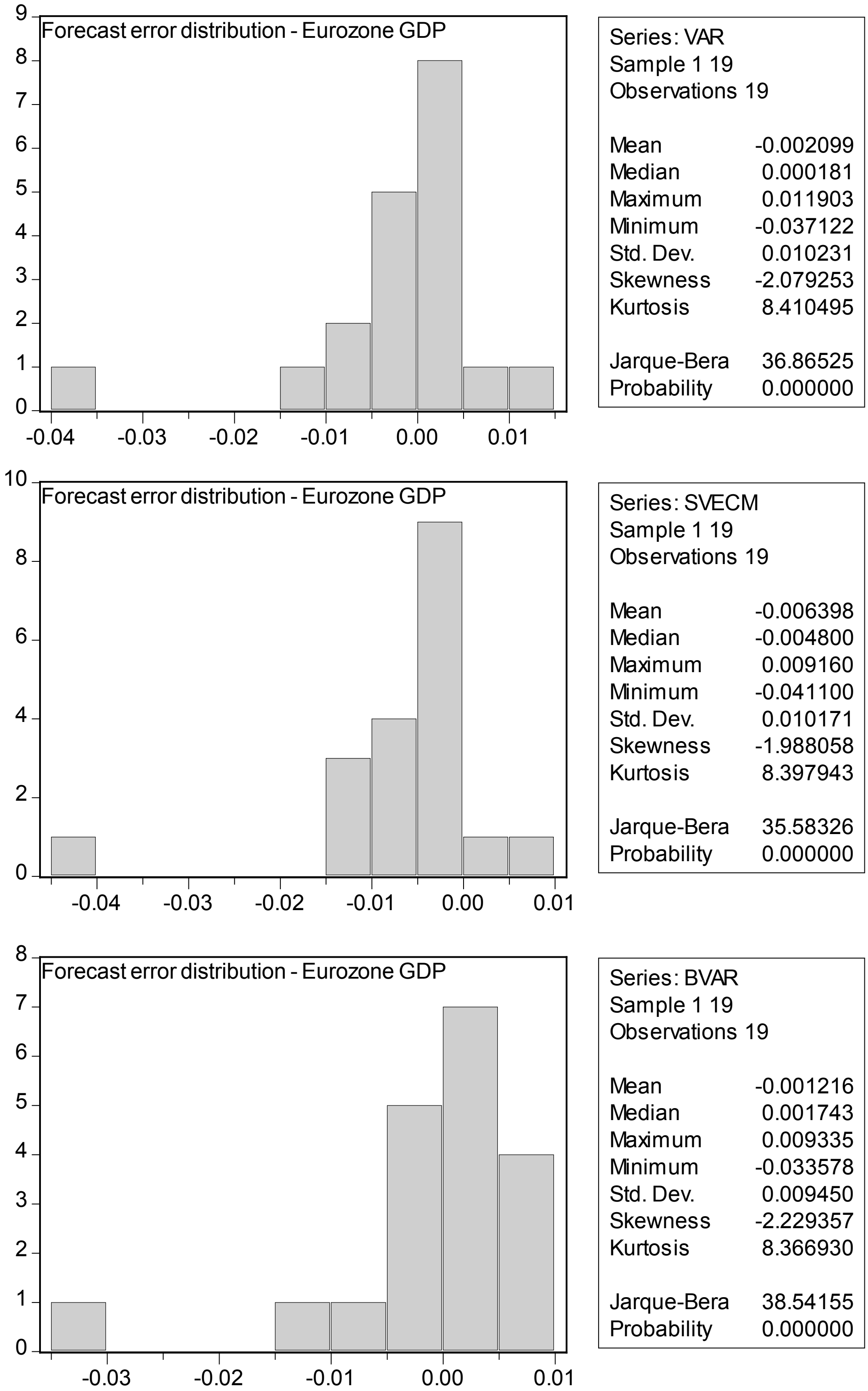

Figure 8: Eurozone GDP one-step forecast error distribution for all three models

7 Forecasting performance between 2004:Q4 – 2008:Q2

The following analysis is structured the same way as section five, ignoring however, the economic development after the recent financial and economic crisis had started with all its consequences on the data. The outbreak of the recent crisis can be regarded as a break point in the data, as well as an "abnormal" event, from the actual perspective at least.

Only vintage sets #1 to #12 are within this new data horizon. The first one-step ahead forecasts predict 2004:Q4, the last ones 2007:Q3. The first four-step ahead forecasts predict values for 2005:Q3, while the last ones predict values for 2008:Q2 i.e. right before the outbreak (Lehman bankruptcy) of the (sub-prime) financial crisis that evolved into an economic crisis by the end of 2008. We will see that forecast errors become much smaller with the new pre-crisis data horizon. Furthermore, as figures 6 and 7 already showed the ranking of the models changes.

7.1 One-step ahead forecasts

Figure 9 shows one-step root mean squared forecast errors divided by the standard deviation of the corresponding realizations. The SVEC model performs better than in figure 2. SVEC is the best predictor for the EUR/USD exchange rate and for the price differential before the financial crisis had started.

Except for the inflation rate and exchange rate, variables show up to 50% smaller root mean errors and Theil coefficients in the 'before the crisis' statistics as compared to statistics from section five.

Except that, the basic structure of the results stays the same. Still the BVAR model is superior in predicting GDP levels and interest rates at the one-step level. On the other hand, VAR and SVEC deliver better predictions for Eurozone inflation rate and the price differential.

Mean errors in table 7 show how SVEC still over-predicts GDP development, while the sign of the forecasts is right 100% of time.

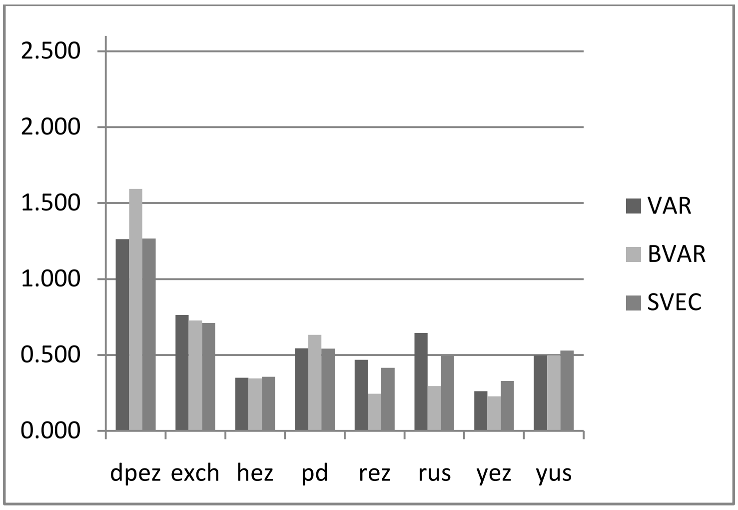

Figure 9: One-Step RMSE/SD$_y$ - before the crisis

7.2 Two-step ahead forecasts

Table 8 indicates that BVAR is not the best predictor for GDP development anymore. The VAR model is superior to the other models in all terms of statistical accuracy. Furthermore, SVEC shows lower Theil coefficients than BVAR for US-GDP. Interestingly, Eurozone inflation rates are better predicted over two-steps than over one-step. In figure 10, the SVEC model loses all its number one positions from figure 9 to the VAR model. It performs better than BVAR for *dpez, exch, pd and yus*, however.

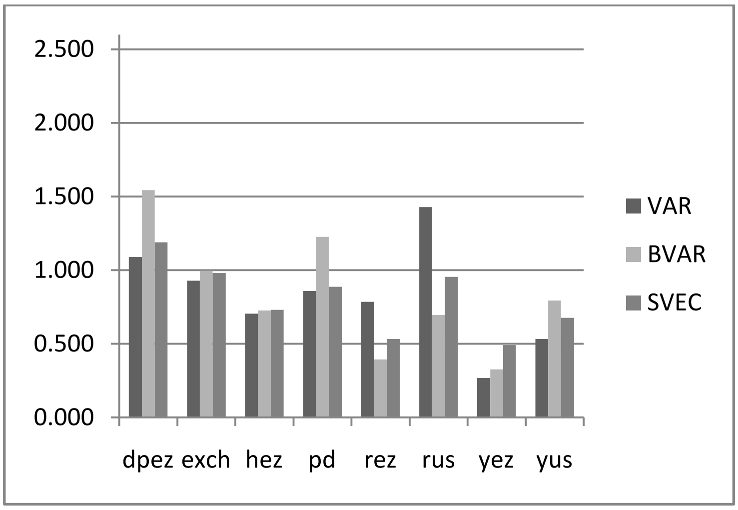

Figure 10: Two-step RMSE/SD$_Y$ - before crisis

7.3 Three-step ahead forecasts

The VAR model delivers the best GDP forecasts at the three-step level. The same applies for Eurozone inflation and EUR/USD exchange rate. As table 9 shows, it is clearly the best performing model.

The SVEC model performs better than BVAR for inflation rate, exchange rate, price differential and US GDP. Sign tests for the three most important variables (*dpez, yez, yus*) show good results for all models.

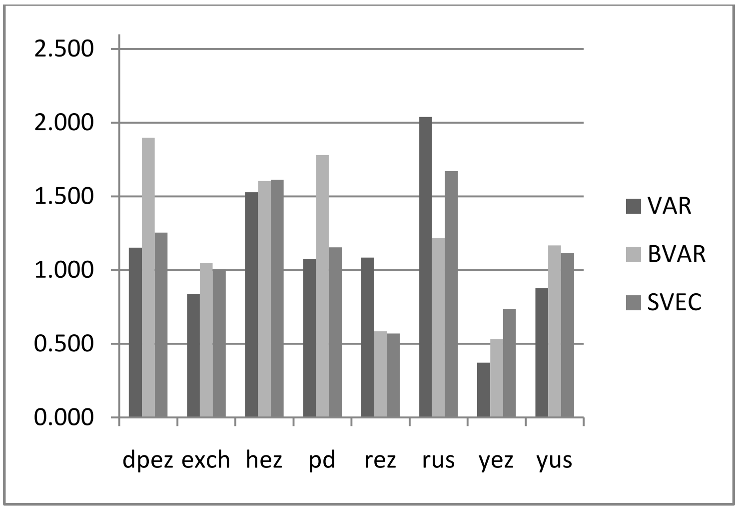

Figure 11: Three-step RMSE/SD$_Y$ - before crisis

7.4 Four-step ahead forecasts

The VAR is by far the best predictor for both GDP development as well as for the exchange rate, Eurozone inflation and price differential in terms of all featured statistics. Theil coefficients, especially for Eurozone GDP are clearly below unity. Furthermore, VAR is the only model that exhibits mean errors below 0.01 for GDP values, indicating that it delivers unbiased forecasts, while BVAR systematically under-, and SVEC systematically over-predicts GDP values.

SVEC shows the best sign prediction of all models except for the exchange rate. Its forecasts suffer from too high mean errors, however.

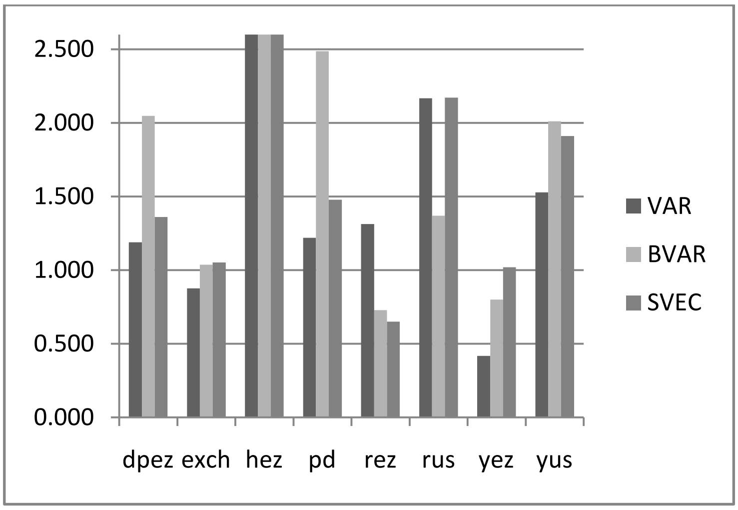

Figure 12: Four-step RMSE/SD_Y - before crisis

7.5 Chapter Summary

The following specific patterns can be found for one-, two-, three-, and four-step ahead forecasts within the pre-crisis analysis.

- The unrestricted VAR model is the best predictor for GDP levels on every step size, except for Eurozone GDP on the one-step level where BVAR exhibits lower errors. Behind the VAR no clear second position can be assigned since SVEC and BVAR show different rankings for every step size. A comparison of mean errors shows, however, that SVEC over-estimates and BVAR under-estimates GDP development, while VAR exhibits mostly unbiased forecasts.

- VAR and SVEC are the better models at forecasting Eurozone inflation rate on every step size. However, Theil W coefficients are close to unity on the one-step forecast horizon and above unity at the three-step horizon. BVAR exhibits higher Theil coefficients and higher mean errors than the two other models.
- The unrestricted VAR is a better predictor in terms of statistical accuracy than SVEC for every variable on every step size, except for EUR/USD exchange rate on the one-step horizon.
- Root mean squared errors increase by up to 50% from each step to the next for all variables except the inflation rate.

Table 7: One-step ahead forecast errors - before crisis

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.007	0.006	0.000	0.003	0.004	0.000	-0.001
MAE	0.003	0.024	0.011	0.004	0.003	0.006	0.004	0.004
MSE	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
RMSE	**0.003**	**0.032**	**0.017**	**0.005**	**0.004**	**0.007**	**0.005**	**0.006**
UM	0.11	0.05	0.12	0.01	0.60	0.38	0.00	0.03
US	0.12	0.08	0.15	0.02	0.00	0.09	0.08	0.01
UC	0.77	0.87	0.73	0.98	0.40	0.53	0.92	0.96
UR	0.27	0.30	0.25	0.15	0.01	0.20	0.03	0.11
UD	0.62	0.64	0.63	0.85	0.39	0.42	0.97	0.86
RMSE/SD_R	1.262	0.763	0.349	0.545	0.467	0.645	0.261	0.496
Theil W	*0.972*	*0.509*	*0.476*	*0.707*	*0.834*	*0.954*	*0.432*	*0.763*
Theil U	*0.580*	*0.123*	*0.054*	*0.164*	*0.122*	*0.153*	*0.076*	*0.057*
ER	0.67*	0.58*	0.92	0.08	0.33	0.42	1.00	1.00

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.002	-0.004	-0.001	0.003	0.000	0.002	0.002	0.002
MAE	0.003	0.025	0.012	0.005	0.002	0.002	0.004	0.005
MSE	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
RMSE	**0.004**	**0.031**	**0.017**	**0.006**	**0.002**	**0.003**	**0.004**	**0.006**
UM	0.33	0.01	0.00	0.36	0.01	0.31	0.15	0.11
US	0.06	0.00	0.17	0.00	0.09	0.14	0.09	0.00
UC	0.62	0.99	0.83	0.64	0.90	0.55	0.76	0.89
UR	0.30	0.14	0.29	0.03	0.03	0.08	0.04	0.03
UD	0.37	0.84	0.71	0.62	0.96	0.61	0.81	0.85
RMSE/SD_R	1.593	0.726	0.345	0.631	0.245	0.294	0.227	0.497
Theil W	*1.230*	*0.561*	*0.471*	*0.804*	*0.440*	*0.432*	*0.405*	*0.762*
Theil U	*0.732*	*0.117*	*0.053*	*0.190*	*0.064*	*0.070*	*0.066*	*0.057*
ER	1.00	0.25	1.00	1.00	0.92*	0.42	1.00	0.83

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.003	0.004	0.001	0.002	-0.001	-0.004	-0.002
MAE	0.002	0.024	0.012	0.004	0.003	0.004	0.005	0.005
MSE	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
RMSE	**0.003**	**0.030**	**0.017**	**0.005**	**0.003**	**0.005**	**0.006**	**0.006**
UM	0.05	0.01	0.05	0.05	0.41	0.03	0.45	0.07
US	0.08	0.05	0.21	0.02	0.04	0.12	0.06	0.01
UC	0.87	0.94	0.74	0.93	0.55	0.85	0.48	0.92
UR	0.33	0.26	0.33	0.13	0.09	0.28	0.03	0.11
UD	0.62	0.73	0.62	0.82	0.50	0.69	0.52	0.82
RMSE/SD_R	1.266	0.710	0.357	0.541	0.416	0.492	0.328	0.530
Theil W	*0.978*	*0.508*	*0.487*	*0.701*	*0.727*	*0.677*	*0.501*	*0.814*
Theil U	*0.582*	*0.115*	*0.055*	*0.163*	*0.109*	*0.116*	*0.095*	*0.061*
ER	0.58*	0.50*	1.00	0.08	0.33	0.83	1.00	1.00

Table 8: Two-step ahead forecast errors - before crisis

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	0.000	-0.014	0.011	-0.001	0.006	0.008	0.001	-0.001
MAE	0.003	0.044	0.019	0.005	0.006	0.010	0.004	0.004
MSE	0.000	0.003	0.001	0.000	0.000	0.000	0.000	0.000
RMSE	**0.003**	**0.051**	**0.027**	**0.007**	**0.007**	**0.012**	**0.005**	**0.006**
UM	0.02	0.07	0.16	0.03	0.78	0.45	0.04	0.07
US	0.25	0.00	0.39	0.10	0.00	0.25	0.06	0.06
UC	0.73	0.93	0.45	0.87	0.22	0.30	0.90	0.88
UR	0.14	0.20	0.54	0.37	0.00	0.40	0.02	0.20
UD	0.84	0.73	0.30	0.60	0.22	0.16	0.95	0.73
RMSE/SD_R	1.088	0.927	0.705	0.858	0.785	1.429	0.267	0.532
Theil W	*0.677*	*0.657*	*0.523*	*0.800*	*1.025*	*1.185*	*0.323*	*0.604*
Theil U	*0.547*	*0.187*	*0.083*	*0.218*	*0.212*	*0.262*	*0.073*	*0.057*
ER	0.67	0.50	1.00	0.08	0.33	0.25	1.00	1.00

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.003	-0.001	-0.002	0.008	0.000	0.004	0.005	0.006
MAE	0.004	0.048	0.023	0.009	0.003	0.005	0.006	0.007
MSE	0.000	0.003	0.001	0.000	0.000	0.000	0.000	0.000
RMSE	**0.005**	**0.055**	**0.027**	**0.010**	**0.003**	**0.006**	**0.006**	**0.009**
UM	0.50	0.00	0.00	0.57	0.02	0.36	0.57	0.44
US	0.11	0.03	0.45	0.00	0.31	0.07	0.10	0.00
UC	0.39	0.97	0.55	0.43	0.67	0.57	0.34	0.55
UR	0.08	0.16	0.65	0.06	0.15	0.00	0.06	0.04
UD	0.42	0.84	0.35	0.37	0.83	0.64	0.37	0.52
RMSE/SD_R	1.544	0.996	0.725	1.226	0.394	0.695	0.326	0.794
Theil W	*0.981*	*0.849*	*0.554*	*1.134*	*0.500*	*0.590*	*0.406*	*0.896*
Theil U	*0.776*	*0.201*	*0.085*	*0.312*	*0.107*	*0.128*	*0.089*	*0.085*
ER	1.00	0.17	1.00	1.00	1.00	0.42	1.00	0.67

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.009	0.007	0.003	0.003	-0.002	-0.008	-0.004
MAE	0.003	0.047	0.021	0.005	0.003	0.007	0.008	0.006
MSE	0.000	0.003	0.001	0.000	0.000	0.000	0.000	0.000
RMSE	**0.004**	**0.054**	**0.028**	**0.007**	**0.005**	**0.008**	**0.010**	**0.007**
UM	0.06	0.03	0.07	0.16	0.42	0.04	0.72	0.23
US	0.17	0.00	0.50	0.07	0.04	0.42	0.04	0.05
UC	0.77	0.97	0.44	0.77	0.53	0.54	0.23	0.72
UR	0.24	0.21	0.66	0.30	0.12	0.66	0.02	0.19
UD	0.70	0.76	0.28	0.55	0.46	0.31	0.26	0.59
RMSE/SD_R	1.189	0.979	0.731	0.887	0.532	0.954	0.492	0.676
Theil W	*0.690*	*0.773*	*0.543*	*0.835*	*0.693*	*0.811*	*0.544*	*0.771*
Theil U	*0.597*	*0.198*	*0.086*	*0.226*	*0.144*	*0.175*	*0.134*	*0.072*
ER	0.67	0.33	1.00	1.00	0.50	0.83	1.00	1.00

Table 9: Three-step ahead forecast errors - before crisis

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	0.000	-0.020	0.014	-0.002	0.009	0.007	0.001	-0.005
MAE	0.002	0.054	0.032	0.006	0.009	0.014	0.006	0.007
MSE	0.000	0.003	0.002	0.000	0.000	0.000	0.000	0.000
RMSE	**0.003**	**0.058**	**0.040**	**0.008**	**0.010**	**0.016**	**0.008**	**0.009**
UM	0.00	0.12	0.12	0.06	0.80	0.20	0.03	0.27
US	0.06	0.07	0.50	0.22	0.00	0.17	0.07	0.14
UC	0.93	0.81	0.38	0.72	0.20	0.63	0.90	0.58
UR	0.26	0.03	0.72	0.53	0.01	0.58	0.01	0.33
UD	0.74	0.85	0.16	0.41	0.19	0.22	0.96	0.40
RMSE/SD_R	1.152	0.839	1.528	1.076	1.085	2.040	0.371	0.878
Theil W	*1.004*	*0.702*	*0.632*	*0.787*	*1.100*	*1.108*	*0.322*	*0.695*
Theil U	*0.486*	*0.203*	*0.122*	*0.244*	*0.280*	*0.341*	*0.097*	*0.083*
ER	0.75*	0.33	0.92	0.08	0.17	0.25*	1.00	1.00

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.004	0.000	-0.005	0.012	-0.001	0.003	0.009	0.010
MAE	0.004	0.058	0.038	0.013	0.004	0.008	0.010	0.010
MSE	0.000	0.005	0.002	0.000	0.000	0.000	0.000	0.000
RMSE	**0.005**	**0.072**	**0.042**	**0.014**	**0.005**	**0.009**	**0.011**	**0.012**
UM	0.56	0.00	0.02	0.73	0.01	0.13	0.70	0.70
US	0.06	0.11	0.57	0.00	0.24	0.07	0.08	0.00
UC	0.37	0.89	0.41	0.26	0.75	0.79	0.23	0.29
UR	0.18	0.15	0.82	0.04	0.03	0.20	0.04	0.05
UD	0.26	0.85	0.16	0.22	0.96	0.66	0.26	0.25
RMSE/SD_R	1.898	1.048	1.605	1.781	0.584	1.220	0.532	1.168
Theil W	*1.683*	*0.928*	*0.690*	*1.270*	*0.559*	*0.688*	*0.502*	*0.925*
Theil U	*0.802*	*0.254*	*0.128*	*0.403*	*0.151*	*0.204*	*0.138*	*0.110*
ER	1.00	1.00	1.00	1.00	0.92	0.33	1.00	1.00

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.001	-0.011	0.009	0.005	0.002	-0.005	-0.013	-0.008
MAE	0.003	0.061	0.035	0.007	0.004	0.009	0.013	0.008
MSE	0.000	0.005	0.002	0.000	0.000	0.000	0.000	0.000
RMSE	**0.003**	**0.069**	**0.042**	**0.009**	**0.005**	**0.013**	**0.015**	**0.011**
UM	0.11	0.02	0.04	0.30	0.19	0.16	0.74	0.48
US	0.04	0.06	0.59	0.13	0.10	0.14	0.05	0.10
UC	0.85	0.91	0.37	0.57	0.72	0.70	0.20	0.42
UR	0.26	0.13	0.81	0.35	0.24	0.54	0.02	0.24
UD	0.63	0.85	0.15	0.35	0.58	0.30	0.24	0.28
RMSE/SD_R	1.253	1.004	1.613	1.154	0.570	1.672	0.737	1.116
Theil W	*1.111*	*0.883*	*0.670*	*0.855*	*0.575*	*0.942*	*0.589*	*0.874*
Theil U	*0.529*	*0.243*	*0.129*	*0.261*	*0.147*	*0.280*	*0.192*	*0.106*
ER	0.83*	0.25	1.00	1.00	0.67	1.00	1.00	1.00

Table 10: Four-step ahead forecast errors - before crisis

Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	0.000	-0.037	0.014	-0.003	0.010	0.005	0.001	-0.007
MAE	0.003	0.065	0.039	0.006	0.010	0.016	0.006	0.007
MSE	0.000	0.005	0.003	0.000	0.000	0.000	0.000	0.000
RMSE	**0.003**	**0.074**	**0.051**	**0.009**	**0.012**	**0.018**	**0.008**	**0.012**
UM	0.01	0.25	0.08	0.09	0.81	0.07	0.03	0.37
US	0.09	0.18	0.66	0.44	0.00	0.08	0.00	0.23
UC	0.90	0.57	0.27	0.46	0.19	0.85	0.96	0.39
UR	0.29	0.00	0.87	0.68	0.02	0.73	0.02	0.43
UD	0.71	0.75	0.05	0.23	0.17	0.20	0.95	0.20
RMSE/SD_R	1.188	0.876	3.711	1.219	1.313	2.165	0.417	1.526
Theil W	*0.673*	*0.758*	*0.659*	*0.661*	*1.068*	*1.040*	*0.305*	*0.806*
Theil U	*0.515*	*0.241*	*0.155*	*0.237*	*0.319*	*0.399*	*0.096*	*0.114*
ER	0.75	0.33*	1.00	1.00	1.00	0.42*	1.00	1.00

Bayesian Vector Auto Regression

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.005	-0.007	-0.012	0.016	0.000	0.003	0.014	0.014
MAE	0.005	0.074	0.053	0.016	0.005	0.009	0.014	0.015
MSE	0.000	0.008	0.003	0.000	0.000	0.000	0.000	0.000
RMSE	**0.006**	**0.087**	**0.055**	**0.017**	**0.006**	**0.012**	**0.015**	**0.016**
UM	0.63	0.01	0.05	0.81	0.00	0.05	0.82	0.79
US	0.06	0.21	0.72	0.00	0.20	0.09	0.02	0.05
UC	0.31	0.79	0.23	0.19	0.80	0.86	0.17	0.17
UR	0.14	0.10	0.91	0.05	0.00	0.49	0.00	0.11
UD	0.22	0.89	0.04	0.14	1.00	0.46	0.18	0.10
RMSE/SD_R	2.048	1.036	3.970	2.486	0.728	1.369	0.801	2.010
Theil W	*1.140*	*0.895*	*0.787*	*1.371*	*0.546*	*0.678*	*0.582*	*1.045*
Theil U	*0.887*	*0.285*	*0.166*	*0.483*	*0.177*	*0.252*	*0.183*	*0.150*
ER	1.00	1.00	1.00	1.00	1.00	0.42	1.00	0.25

Structural Vector Error Correction Model

	dpez	exch	hez	pd	rez	rus	yez	yus
ME	-0.002	-0.020	0.009	0.007	0.000	-0.011	-0.018	-0.011
MAE	0.003	0.077	0.045	0.009	0.005	0.011	0.018	0.011
MSE	0.000	0.008	0.003	0.000	0.000	0.000	0.000	0.000
RMSE	**0.004**	**0.089**	**0.054**	**0.010**	**0.006**	**0.018**	**0.019**	**0.015**
UM	0.20	0.05	0.03	0.51	0.00	0.33	0.84	0.56
US	0.05	0.14	0.73	0.19	0.10	0.02	0.02	0.16
UC	0.75	0.81	0.24	0.30	0.90	0.64	0.14	0.28
UR	0.26	0.11	0.93	0.33	0.31	0.46	0.00	0.31
UD	0.54	0.84	0.04	0.16	0.69	0.21	0.16	0.13
RMSE/SD_R	1.360	1.052	3.917	1.477	0.649	2.172	1.019	1.911
Theil W	*0.741*	*0.916*	*0.708*	*0.840*	*0.506*	*1.075*	*0.690*	*1.004*
Theil U	*0.589*	*0.289*	*0.163*	*0.287*	*0.158*	*0.401*	*0.233*	*0.143*
ER	1.00	0.08	1.00	1.00	1.00	1.00	1.00	1.00

7.6 Comparison of the results

- While the BVAR model is best at predicting GDP values on almost all forecast step sizes in the wide time span 2004:Q4 – 2010:Q1 including the recent financial crisis the unrestricted VAR shows the best performance for two-, three-, and four-step ahead Eurozone GDP forecasts before the crisis as well as the best performance on every step for US-GDP.
- For both sample sizes holds, that VAR and SVEC predict Eurozone inflation rates and price differentials better than BVAR. Furthermore, the VAR predicts Eurozone inflation rates best for all step sizes in sections 5 and 7.
- In section 5 Theil W coefficients of all models for Eurozone inflation were above unity at the two-step level indicating weak forecasts. In section 7, however, the same problem occurs on the three-step level.
- Given the results of both sections, SVEC shows the best performance at delivering the correct sign of the forecasts.

8 Conclusion

Table 11 shows how the ranking of the three models changes with the different forecast horizons. While the Bayesian vector auto regression performs best when predicting GDP evolution over the long time span 2004:Q4 – 2010:Q1 including the financial crisis (section 5), the unrestricted VAR shows the best performance over the short time span 2004:Q4 - 2008:Q2, with mixed results for one-step ahead forecasts only. The structural vector error correction model shows the highest forecast errors

Table 11: Best performing model according to root mean squared errors

2004:Q4 - 2010:Q1	**dpez**	**exch**	**yez**	**yus**
1-step ahead	VAR/BVAR/SVEC	BVAR	BVAR	BVAR
2-step ahead	VAR	VAR	VAR	BVAR
3-step ahead	VAR	VAR	BVAR/VAR	BVAR
4-step ahead	VAR / SVEC	VAR	BVAR	BVAR

2004:Q4 - 2008:Q2	**dpez**	**exch**	**yez**	**yus**
1-step ahead	VAR/SVEC	SVEC	BVAR	VAR/BVAR/SVEC
2-step ahead	VAR	VAR	VAR	VAR
3-step ahead	VAR/SVEC	VAR	VAR	VAR
4-step ahead	VAR	VAR	VAR	VAR

of all three models regarding our variables of main interest, namely the Eurozone- and the U.S.-GDP, over both time spans. The Diebold Mariano test in section 6 mostly confirms these findings. According to it there is a statistically significant difference between BVAR and VAR forecast errors on the one hand and SVEC forecast errors on the other hand favoring the former. The SVEC model, however, turns out to be the best sign predictor for GDP movements in both sections. For both sample sizes holds that VAR and SVEC better predict Eurozone inflation rates than BVAR. Furthermore, sections 5 and 7 show how forecast errors (e.g. RMSE/SD_R) increase strongly with growing forecast step-size.

As Clements & Hendry (2008, p.3) showed, error correction models do "in fact not error correct when equilibria shift" but become equilibrium-correction models. These in-built equilibria, however, loose their validity after shifts have occurred. Therefore error correction models force "variables back to relationships that reflect the previous equilibria" –so they will 'correct' in an inappropriate way. Notwithstanding the above, we see that the structural error correction model in our analysis performs worse than the two other models not only over the long time span including the possible equilibrium shift, but also over the second, shorter one.

As a result, the main question to be asked is why two models without any underlying ecnomic theory can outperform the structural error correction model which incorporates long-run restrictions derived from a dynamic open economy model. One clearly has to rethink the process of obtaining the co-integration vectors and furthermore, has to question the underlying macroeconomic theory. This working paper shows that higher complexity and more effort devoted to a model need not necessarily make it a more successful tool when it comes to forecasting, in fact, in this case the opposite is true.

Methodological annex

P_t forecast for year t,

h forecast horizon (quarters)

R_t realisation of year t,

T number of observations.

$s^2{}_p = \frac{1}{T}\sum_{t=1}^{T}(P_t - \bar{P})^2$ variance of a forecast

$s^2{}_r = \frac{1}{T}\sum_{t=1}^{T}(R_t - \bar{R})^2$ variance of actual results

$r = \frac{\frac{1}{T}\sum_{t=1}^{T}(R_t - \bar{R})(P_t - \bar{P})}{s_r s_p}$ correlation coefficient

$UM = \frac{(\overline{R_t} - \overline{P_t}^2)}{MSE}$ bias proportion of the MSE

$US = \frac{(s_r s_p)^2}{MSE}$ variance proportion of the MSE

$UC = \frac{2(1-r) s_r s_p}{MSE}$ covariance proportion of the MSE

$UR = \frac{(s_p r s_r)^2}{MSE}$ regression proportion of the MSE

$UD = \frac{(1-r^2) s^2{}_r}{MSE}$ distribution proportion of the MSE

Appendix

Table 12: ADF Test

Null Hypothesis: EXCH_19 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.632919	0.0886
Test critical values:	1% level	-3.472813	
	5% level	-2.880088	
	10% level	-2.576739	

*MacKinnon (1996) one-sided p-values.

Table 13: ADF Test

Null Hypothesis: HEZ_19 has a unit root
Exogenous: Constant
Lag Length: 5 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.946464	1.0000
Test critical values:	1% level	-3.473967	
	5% level	-2.880591	
	10% level	-2.577008	

*MacKinnon (1996) one-sided p-values.

Table 14: ADF Test

Null Hypothesis: HUS_19 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.503773	0.5292
Test critical values:	1% level	-3.472813	
	5% level	-2.880088	
	10% level	-2.576739	

*MacKinnon (1996) one-sided p-values.

Table 15: ADF Test

Null Hypothesis: PD_19 has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.378496	0.0132
Test critical values:	1% level	-3.473382	
	5% level	-2.880336	
	10% level	-2.576871	

*MacKinnon (1996) one-sided p-values.

Table 16: ADF Test

Null Hypothesis: PEZ_19 has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.845362	0.0031
Test critical values:	1% level	-3.473096	
	5% level	-2.880211	
	10% level	-2.576805	

*MacKinnon (1996) one-sided p-values.

Table 17: ADF Test

Null Hypothesis: PUS_19 has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.165264	0.0241
Test critical values:	1% level	-3.473672	
	5% level	-2.880463	
	10% level	-2.576939	

*MacKinnon (1996) one-sided p-values.

Table 18: ADF Test

Null Hypothesis: REZ_19 has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.402792	0.5797
Test critical values:	1% level	-3.473096	
	5% level	-2.880211	
	10% level	-2.576805	

*MacKinnon (1996) one-sided p-values.

Table 19: ADF Test

Null Hypothesis: RUS_19 has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.024713	0.2761
Test critical values:	1% level	-3.473382	
	5% level	-2.880336	
	10% level	-2.576871	

*MacKinnon (1996) one-sided p-values.

Table 20: ADF Test

Null Hypothesis: YEZ_19 has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.946623	0.3103
Test critical values:	1% level	-3.473096	
	5% level	-2.880211	
	10% level	-2.576805	

*MacKinnon (1996) one-sided p-values.

Table 21: ADF Test

Null Hypothesis: YUS_19 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.306403	0.6259
Test critical values:	1% level	-3.472813	
	5% level	-2.880088	
	10% level	-2.576739	

*MacKinnon (1996) one-sided p-values.

Table 22: ADF Fisher Unit Root Test summary - levels

Null Hypothesis: Unit root (individual unit root process)
Series: EXCH_19, HEZ_19, HUS_19, PD_19, PEZ_19, PUS_19, REZ_19, RUS_19, YEZ_19, YUS_19
Automatic lag length selection based on SIC: 1 to 5
Total number of observations: 1536
Cross-sections included: 10

Method	Statistic	Prob.**
ADF - Fisher Chi-square	40.7201	0.0040
ADF - Choi Z-stat	-1.32578	0.0925

** Probabilities for Fisher tests are computed using an asymptotic Chi -square distribution. All other tests assume asymptotic normality.

Intermediate ADF test results UNTITLED

Series	Prob.	Lag	Max Lag	Obs
EXCH_19	0.0886	1	13	155
HEZ_19	1.0000	5	13	151
HUS_19	0.5292	1	13	155
PD_19	0.0132	3	13	153
PEZ_19	0.0031	2	13	154
PUS_19	0.0241	4	13	152
REZ_19	0.5797	2	13	154
RUS_19	0.2761	3	13	153
YEZ_19	0.3103	2	13	154
YUS_19	0.6259	1	13	155

Table 23: ADF Fisher Unit Root Test summary - first differences

Null Hypothesis: Unit root (individual unit root process)
Series: EXCH_19, HEZ_19, HUS_19, PD_19, PEZ_19, PUS_19, REZ_19, RUS_19, YEZ_19, YUS_19
Automatic lag length selection based on SIC: 0 to 4
Total number of observations: 1536
Cross-sections included: 10

Method	Statistic	Prob.**
ADF - Fisher Chi-square	259.812	0.0000
ADF - Choi Z-stat	-12.5344	0.0000

** Probabilities for Fisher tests are computed using an asymptotic Chi -square distribution. All other tests assume asymptotic normality.

Intermediate ADF test results D(UNTITLED)

Series	Prob.	Lag	Max Lag	Obs
D(EXCH_19)	0.0000	0	13	155
D(HEZ_19)	0.0012	4	13	151
D(HUS_19)	0.0000	0	13	155
D(PD_19)	0.0113	2	13	153
D(PEZ_19)	0.6297	1	13	154
D(PUS_19)	0.1727	3	13	152
D(REZ_19)	0.0000	1	13	154
D(RUS_19)	0.0000	2	13	153
D(YEZ_19)	0.0013	1	13	154
D(YUS_19)	0.0000	0	13	155

Table 24: ADF Fisher Unit Root Test summary - second differences

Null Hypothesis: Unit root (individual unit root process)
Series: EXCH_19, HEZ_19, HUS_19, PD_19, PEZ_19, PUS_19, REZ_19, RUS_19, YEZ_19, YUS_19
Automatic lag length selection based on SIC: 0 to 6
Total number of observations: 1522
Cross-sections included: 10

Method	Statistic	Prob.**
ADF - Fisher Chi-square	834.238	0.0000
ADF - Choi Z-stat	-27.4439	0.0000

** Probabilities for Fisher tests are computed using an asymptotic Chi -square distribution. All other tests assume asymptotic normality.

Intermediate ADF test results D(ADFDIFFSUMMARY,2)

Series	Prob.	Lag	Max Lag	Obs
D(EXCH_19,2)	0.0000	1	13	153
D(HEZ_19,2)	0.0000	6	13	148
D(HUS_19,2)	0.0000	1	13	153
D(PD_19,2)	0.0000	1	13	153
D(PEZ_19,2)	0.0000	0	13	154
D(PUS_19,2)	0.0000	1	13	153
D(REZ_19,2)	0.0000	4	13	150
D(RUS_19,2)	0.0000	3	13	151
D(YEZ_19,2)	0.0000	0	13	154
D(YUS_19,2)	0.0000	1	13	153

Table 25: Time Series (Vintage set #19) in levels

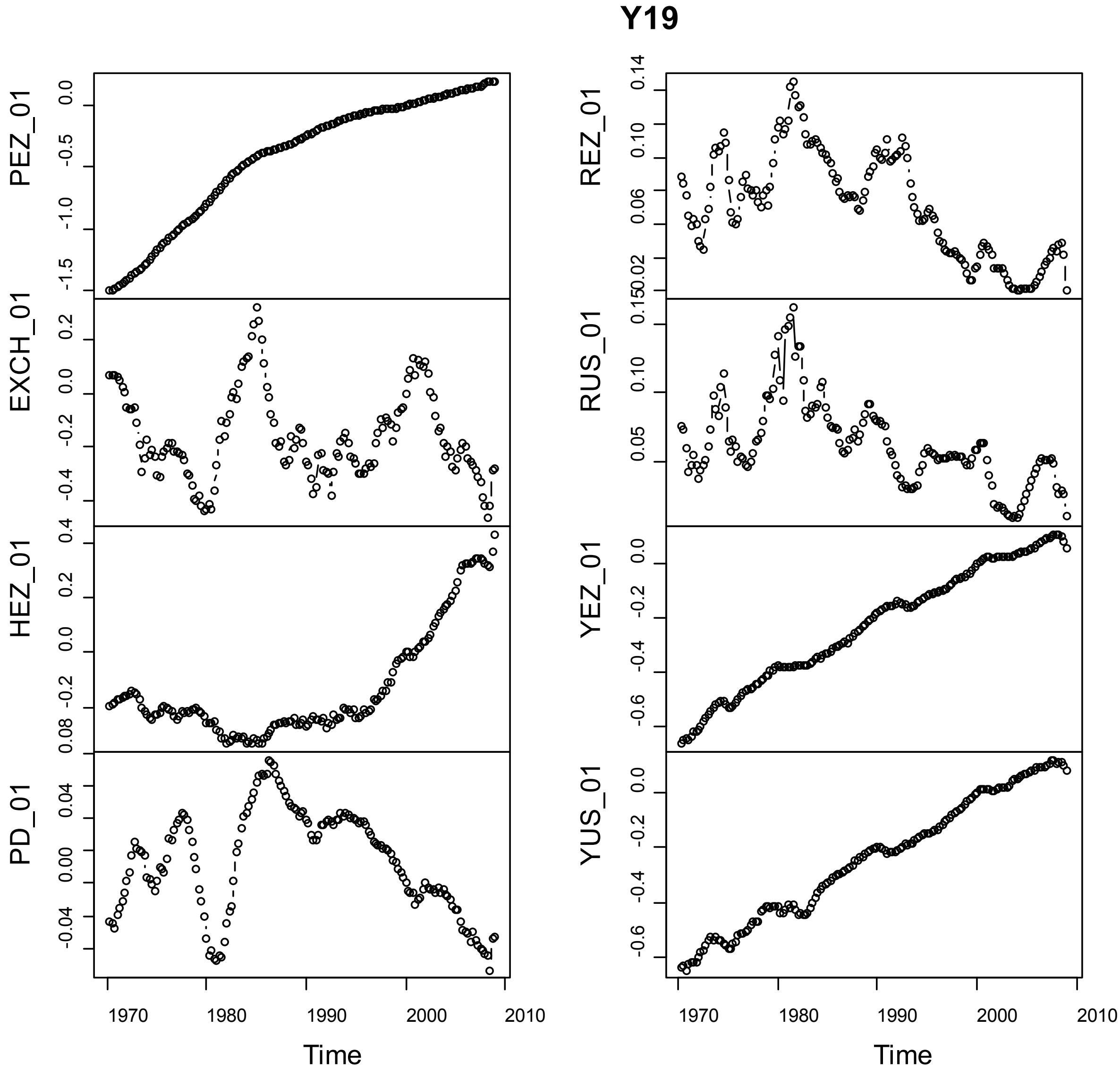

Table 26: Time Series (Vintage set #19) in first differences

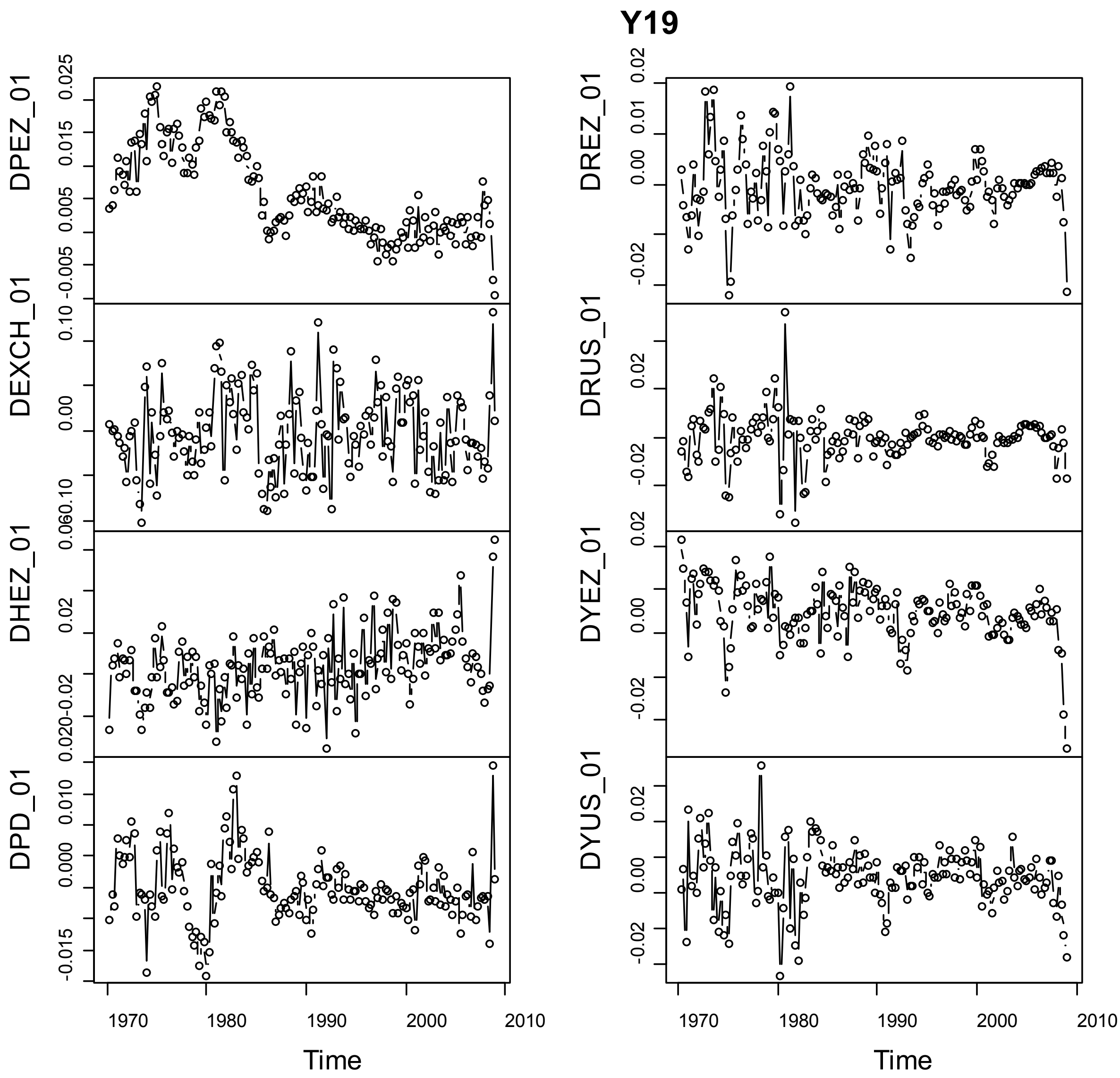

Table 27: Time series PEZ (vintage #19) in second differences

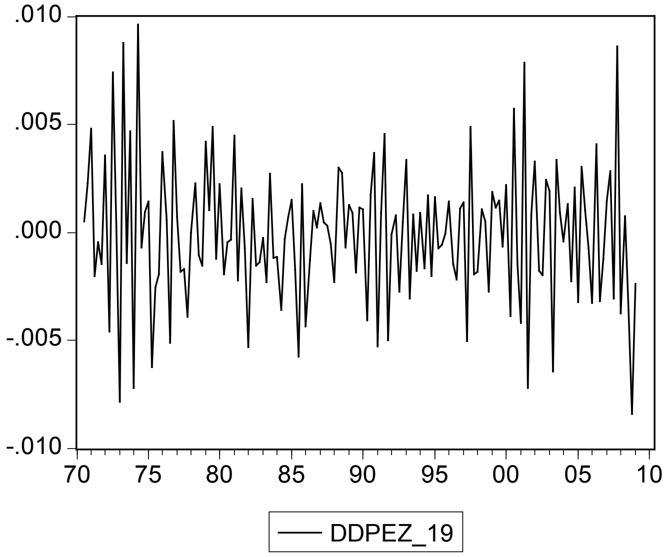

Table 28: Tests on lag length for VAR in differences

Table 29: Tests on lag length for VAR in levels

VAR Lag Order Selection Criteria
Endogenous variables: DDPEZ_19 DEXCH_19 DHEZ_19 DPD_19 DREZ_19 DRUS_19 DYEZ_19 DYUS_19
Exogenous variables: C DPOIL DPOIL(-1) DPOIL(-2)
Date: 11/24/10 Time: 17:39
Sample: 1970Q1 2009Q1
Included observations: 152

Lag	LogL	LR	FPE	AIC	SC	HQ
0	4149.312	NA	4.10e-34	-54.17515	-53.53855*	-53.91654
1	4308.074	292.4573	1.18e-34	-55.42203	-53.51221	-54.64619*
2	4398.744	157.4801*	8.39e-35*	-55.77295*	-52.58992	-54.47990
3	4438.118	64.24037	1.19e-34	-55.44892	-50.99267	-53.63863

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

VAR Lag Order Selection Criteria
Endogenous variables: PEZ_19 EXCH_19 HEZ_19 PD_19 REZ_19 RUS_19 YEZ_19 YUS_19
Exogenous variables: C
Included observations: 155

Lag	LogL	LR	FPE	AIC	SC	HQ
0	2019.078	NA	7.42E-22	-25.94940	-25.79232	-25.88559
1	4375.146	4438.527	1.06E-34	-55.52446	-54.11074*	-54.95024*
2	4460.244	151.5303*	8.15E-35*	-55.79670*	-53.12634	-54.71206

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Table 30: Johansen Cointegration Test for levels

Sample (adjusted): 1971Q2 2009Q1
Included observations: 152 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: EXCH_19 HEZ_19 HUS_19 PD_19 PEZ_19 REZ_19 RUS_19 YEZ_19 YUS_19
Lags interval (in first differences): 1 to 4
Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.346733	294.3401	208.4374	0.0000
At most 1 *	0.344860	229.6232	169.5991	0.0000
At most 2 *	0.252474	165.3414	134.6780	0.0002
At most 3 *	0.212312	121.1115	103.8473	0.0022
At most 4 *	0.176410	84.83628	76.97277	0.0111
At most 5 *	0.142894	55.33567	54.07904	0.0384
At most 6	0.096446	31.89820	35.19275	0.1087
At most 7	0.068005	16.48253	20.26184	0.1530

Trace test indicates 6 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.346733	64.71696	59.24000	0.0132
At most 1 *	0.344860	64.28176	53.18784	0.0026
At most 2	0.252474	44.22992	47.07897	0.0978
At most 3	0.212312	36.27521	40.95680	0.1532
At most 4	0.176410	29.50061	34.80587	0.1876
At most 5	0.142894	23.43747	28.58808	0.1982
At most 6	0.096446	15.41568	22.29962	0.3418
At most 7	0.068005	10.70504	15.89210	0.2744

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level

Table 31: Johansen Cointegration Test for first differences

Date: 11/22/10 Time: 18:08
Sample (adjusted): 1970Q4 2010Q1
Included observations: 158 after adjustments
Trend assumption: No deterministic trend
Series: DPEZ_23 DEXCH_23 DHEZ_23 DPD_23 DREZ_23 DRUS_23 DYEZ_23 DYUS_23 DHUS_23
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.546590	502.2206	179.5098	0.0000
At most 1 *	0.439083	377.2493	143.6691	0.0000
At most 2 *	0.405689	285.8966	111.7805	0.0000
At most 3 *	0.356473	203.6808	83.93712	0.0000
At most 4 *	0.347768	134.0357	60.06141	0.0000
At most 5 *	0.176108	66.51366	40.17493	0.0000
At most 6 *	0.113426	35.90651	24.27596	0.0011
At most 7 *	0.095018	16.88479	12.32090	0.0081
At most 8	0.007001	1.109972	4.129906	0.3400

Trace test indicates 8 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.546590	124.9713	54.96577	0.0000
At most 1 *	0.439083	91.35272	48.87720	0.0000
At most 2 *	0.405689	82.21581	42.77219	0.0000
At most 3 *	0.356473	69.64506	36.63019	0.0000
At most 4 *	0.347768	67.52204	30.43961	0.0000
At most 5 *	0.176108	30.60715	24.15921	0.0058
At most 6 *	0.113426	19.02172	17.79730	0.0326
At most 7 *	0.095018	15.77482	11.22480	0.0075
At most 8	0.007001	1.109972	4.129906	0.3400

Max-eigenvalue test indicates 8 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

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