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Marcus Scheiblecker

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Between Cointegration and Multicointegration: Modelling Time Series Dynamics by Cumulative Error Correction Models*

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Abstract

This study proposes a cumulative error correction model where the summing weights follow a geometrically decreasing function of prior deviations from equilibrium and are estimated from the data. It is shown that this approach is located in between the traditional error correction model – where no weight is given to deviations from steady state prior to the most recent period – and the error correction model based on the idea of multicointegration.

The presented form of accumulation does not change the order of integration of the series, like in the multicointegration approach of Granger and Lee (1989). Based on this model type, the relationship between US private consumption and real disposable income of private households is estimated. The short-run forces setting-off last period's deviations are much smaller than a VEC and a conventional single equation ECM suggests. Furthermore, the proposed model outperforms both others in respect of its forecasting power.

Keywords: Cumulative error correction model, cointegration, consumption, income.

JEL classification: C5, E21, E41.

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1. Introduction

Since the seminal work of Granger and Newbold (1974) the idea of cointegration based modelling has become very popular. The reason for this is not only because the authors have shown that regressing integrated time series on each other can cause spurious results, but also that cointegrated variables possess a so called Error-Correction-Models (ECM) representation[1]. Furthermore, such models have in many cases a reasonable economic interpretation. In ECMs, deviations from its long-run relationship (steady state) trigger some gravity forces which push back time series towards its steady state. The typical form of an ECM is

$$\Delta y_t = \alpha\xi_{t-1} + \gamma\Delta x_t + c + u_t \qquad (1)$$

where Δy_t and Δx_t are first order differences of time series integrated of order 1. ξ_t represents the so-called error correction term, measuring the distance between the steady state y^*_t and the time series value y_t

$$\xi_t = y_t - y_t^* \qquad (2)$$

with the cointegration relation representing the steady state, y^*_t being derived as a linear function of x_t and - if appropriately - of a constant and a deterministic time trend. The EC parameter $0>\alpha>-1$ partly settles deviations occurring at time $t-1$ in t.

However, the assumption that only the deviation from steady state of the most recent period influences short-term movements is not adequate in several economic applications. Past disequilibria between income and consumption can lead to an accumulation of a wealth stock which determines the consumption behavior of the future. Similarly, the mismatch between production and sales leads to inventories and between money demand and production to a money stock. As the usual ECMs in this case would result in a misspecification error[2], Granger and Lee (1989) developed the idea of multicointegration. Apart from the cointegrating relationship between the flows (first-level cointegration) there may be another coming from stocks. This they called second-level cointegration. Whereas usually the number of cointegrating relationships among n variables is at most $n-1$, with multicointegration it can be n as well.

Suppose that y_t and x_t are both I(1) and are cointegrated CI(1,1) so that

$$z_t = y_t - \phi x_t \qquad (3)$$

is I(0). (3) is the so-called first-level cointegration relationship. The authors further propose that past deviations from steady state accumulate to a stock

[1] This is called the Granger Representation Theorem (Engle and Granger, 1987).
[2] See for this Engsted and Johansen (1997) or Lee (1992).

variable $s_t = \sum_{i=1}^{t} z_t$. If now s_t cointegrates with either x_t or y_t we get another cointegration relationship (called second-level cointegration) so that $s_t - \kappa y_t$ forms again a stationary relationship

$$s_t - \kappa y_t = \sum_{j=1}^{t} y_t - \phi \sum_{j=1}^{t} x_t - \kappa y_t \qquad (4)$$

with s_t being I(1) as it stems from summing I(0) stock changes, and $\Sigma\ y_t$ and $\Sigma\ x_t$ being I(2) as both are summed I(1) variables.[3] Granger and Lee (1989) solved the estimation problem by a two step method as typically used for CI(1,1) variables. In a first step they estimated the first-level cointegration relation. The residuals as deviations from steady state were summed up and in a second stage they were regressed on the cumulated variables (summed y_t) for estimating the second order integrating relationship.

However, Engsted et al. (1997) have shown that in case of a two step method the first cointegrating relationship (of flows) must not be estimated. Otherwise the test statistics of the second one will have a different limiting distribution compared to normal settings. Furthermore, for I(2) based models usual asymptotic χ^2 inference is invalid and Johansen (2006) pointed out that it can be used only if a multicointegration relation is assumed with properties hardly met in reality. Engsted et al. (2007) proposed a single equation method were both forms of integration are teseted together and supplied tables with critical test levels.

The model presented here does not – like in the multicointegration approach – sum up past deviations s_t with equal weights. Instead another weighting scheme is chosen leaving the variable I(0). Therefore, there is no second cointegrating relationship necessary (κ=0). Compared to the traditional ECM it does not give to deviations prior to (t-1) a weight of zero. It is assumed instead that the weights are decreasing the further they are located in the past, according to a geometric process. It will be shown that this is a realistic assumption (at least as realistic as assuming a weights of zero or one) in several cases and has the advantage that the compilation of I(2) variables can be avoided. Furthermore, the estimation is very parsimony and outperforms traditional ECM approaches in terms of their forecasting properties.

2. The model

Like in the multicointegration approach it is assumed here that not only the last deviation from steady state (t-1) plays a role in the future adjustment process towards the steady state but also those further back in the past. However, in

[3] In the case of multicointegration the corresponding ECM considers adjustment mechanisms for the stock as well as the flow variables with $\Delta x_t = c + \alpha_1 (s_{t-1} - \kappa y_{t-1}) + \alpha_2 \xi_{t-1} + lagged(\Delta x_t, \Delta y_t) + u_t$

contrast to it deviations from steady state which are located in the further past have less influence on the adjustment process. Therefore, they are less important for explaning short-run dynamics than the ones which happened in the recent past. A typical application in economics is past investments forming today's capital stock. Depreciation reduces past investments so their contribution to capital stock is decreasing from period to period. Other forces reducing the impact of past deviations can be surprise inflation or changes in asset prices (house prices and securities), etc.

Instead of giving each deviation of the past the constant summation weight of one like Granger and Lee (1989) did, our weighting parameter is estimated from the data and is supposed to be smaller than one[4]. Based upon these considerations, a reformulation of (1) in

$$\Delta y_t = \beta \sum_{i=0}^{\infty} \lambda^i \xi_{t-1-i} + \gamma \Delta x_t + c + u_t \tag{5}$$

with the weight λ <1, can be made.

Koyck (1954) was first putting forward the transformation of an ADL model of the type (without considering here $\gamma\Delta x_t$)

$$F_t = c + \delta \sum_{j=0}^{\infty} \lambda^j S_{t-j} + \varepsilon_t \tag{6}$$

into an ARMAX(1,0,1) model

$$F_t = (1-\lambda)c + \delta\lambda^0 S_{t-0} + \lambda F_{t-1} + \varepsilon_t - \lambda\varepsilon_{t-1} \tag{7}$$

where λF_{t-1} represents the autoregressive [AR] part, $-\lambda\varepsilon_{t-1}$ the moving average [MA] part and S_t the regressor, which is therefore called the Koyck model.

In (6) the variable F_t is explained by a sum of a variable observed at consecutive time points in the past. The summing weights λ (called the retention rate) are defined over $0 \leq \lambda \leq 1$ so as their size is decreasing geometrically giving less weight to more distant observations. One advantage of this approach is immediately obvious. Only one parameter more than in the conventional ECM has to be estimated what makes a more parsimony estimation possible as this costs just one degree of freedom more. It is further noticeable that if the parameter δ is zero (i. e. no cointegration exists between the two series) then the retention parameter λ can not be retrieved from the model. If λ is zero the

[4] The assumption of a retention rate λ<1 is in so far important as otherwise the summation would result in an I(1) variable like in the case of multicointegration. In this case our model would be misspecified as a possibly existing second cointegrating relationship is not considered explicitly in our ECM.

conventional ECM results and if it is 1 the multicointegration method with equal weights as proposed by Granger and Lee (1989) emerges.[5]

If we replace S_{t-i} in equation (6) and (7) for ξ_{t-i-1} (so that $S_t = \xi_{t-1}$) F_t corresponds to the right hand side of equation (5)[6]. This can be transformed into

$$\Delta y_t = (1-\lambda)c + \beta\xi_{t-1} + \lambda\Delta y_{t-1} + \gamma\Delta x_t - \lambda\varepsilon_{t-1} + \varepsilon_t \qquad (8)$$

which is the ARMAX representation of our cumulative ECM given in (5). It is imediately clear that as long as λ <1 then all terms on either side of the equation are I(0) and hence the usual test statitics can be applied in order to determine the cointegrating relationship.

This time series representation reconciles somewhat the blame of ARIMA models – as introduced by Box and Jenkins (1970) – for their lack of theoretic content. At least this form seems to be based on economic theory as good as the conventional ECMs as well as the multicointegration method, as past deviations from steady state are considered. Equation (8) is an ARMAX model type which includes I(1) variables in levels – or at least their I(0) deviations from steady state – as well as an AR and an MA term. The only difference to pure time series model is that (8) demands the AR parameter to equal the MA one with a changed sign and that both terms are restricted to be of order one.

This model has interesting features as compared to the traditional one. Let us define "stability" as the situation where a time series is only driven by its exogenous short term variables (Δx_t) and the error term (ε_t), and steady state is the long run relation between flows as given by the cointegrating relationship. For the conventional ECM, stability is reached if $\Delta y_t = \alpha\xi_{t-1} + \gamma\Delta x_t + u_t$ collapses to $\Delta y_t = \gamma\Delta x_t + u_t$ i.e. $\xi_{t-1} = 0$ so if the time series has reached its steady state in the previous period.[7] For cumulated ECM the condition for stability is more complicated and it can be achieved even out of steady state. Disregarding the constant term, stability in (8) at time t is given if

$$\beta\xi_{t-1} + \lambda\Delta y_{t-1} - \lambda\varepsilon_{t-1} = 0 \qquad (9)$$

which can be transformed (the proof is given in the appendix) into

$$\xi_{t-1} = -\sum_{j=1}^{\infty}\lambda^j\xi_{t-1-j} \qquad (10)$$

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[5] As noted earlier this would not be exactly correct because the second-level cointegrating relationship is not considered explicitly.
[6] Apart from the regressor Δx_t.
[7] We rule out the possibility of α=0 as this would mean there is no cointegrating relationship among the series.

This means the condition for stability requires that the deviation from steady state in t-1 is off-set by the weighted sum of previously accumulated disequilibria i.e. the disequilibrium stock.

Stability in steady state ($\xi_{t-1} = 0$) at time t requires furthermore that either the retention rate is zero (i.e. the conventional ECM) or that the stock disequilibrium at time t-1 is zero. At this point in time the stock disequilibrium is represented by accumulation of deviations backward starting at t-2.

3. Modelling US consumption with cumECM

For testing the empirical relevance of this kind of cumulative ECM (cumECM) and comparing it to the conventional ECM method, we first use the classical textbook example of the cointegration between consumption and income. The data of US private consumption expenditure and private disposable income (both in nominal terms) were downloaded from the Bureau of Economic Analysis[8] on a quarterly basis beginning in the third quarter of 1954 and ending at the second quarter 2010. Figure 1 shows the development of both series in logs. Towards the end of the series the income series show a positive non-permanent shock due to the US Economics Stimulus Act's tax refunds starting from the second quarter 2008 as a policy reaction to the economic crises. Nearly at the same time consumption plunged and seems not to have recovered since. We estimate the models up to the end of 2007 (e.g. more than 200 observations) and retain the rest of the observations (e.g. 10 observations) for an evaluation of the forecasts.

According to the usual procedure both series were tested for unit roots. Table 1 gives the statistics for the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests[9] together with their critical values. Results strongly suggest that both variables are I(1).

As a next step, it has to be tested whether both series are cointegrated. In the past there has been developed a steadily rising number of cointegration tests. The most traditional is the 2-step approach proposed by Engle and Granger (1987) where the residuals of a linear combination of the variables supposed to be cointegrated (estimated by OLS regression) are tested for unit roots. Another very popular method is the Vector Error Correction (VEC) model as proposed by Johansen (1988). This allows for more than one cointegration relationship among the variables. Furthermore, they are capable of accounting for possible endogenity problems.

Here we use for the sake of transparency the traditional OLS approach by Engle and Granger (1987), but the existence of cointegration, the test for weak exogenity, and the size of the cointegrating parameters are cross-checked by the

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[8] www.bea.gov/national/nipaweb/SelectTable.asp?Selected=Y#S2 on the 12th Jan. 2011.

[9] Whether to include in the tested model a trend and/or an intercept was decided on the basis of their t-values.

VEC as well as the DOLS approach, in order to avoid the so-called second-order bias stemming from disregarded endogenity relations.

Table 1: Stationarity tests

	ADF (lag by SiC)			PP (BW:Newey-West using Bartlett k.)		
	H_0: series has a unit root			H_0: series has a unit root		
	T&C	C	none	T&C	C	none
log(PCE)	0.698	-1.543	3.255	0.814	-1.655	11.055
log(PDI)	0.943	-2.874	3.060	0.713	-1.902	11.169
D1log(PCE)	-5.757	-5.505	-1.044	-10.248	-10.079	-2.630
D1log(PDI)	-7.427	-5.328	-0.882	-14.232	-13.918	-4.862
1% level	-4.000	-3.460	-2.576	-4.000	-3.460	-2.575
5% level	-3.430	-2.874	-1.942	-3.430	-2.874	-1.942
10% level	-3.139	-2.574	-1.616	-3.139	-2.574	-1.616

Source: Own calculations

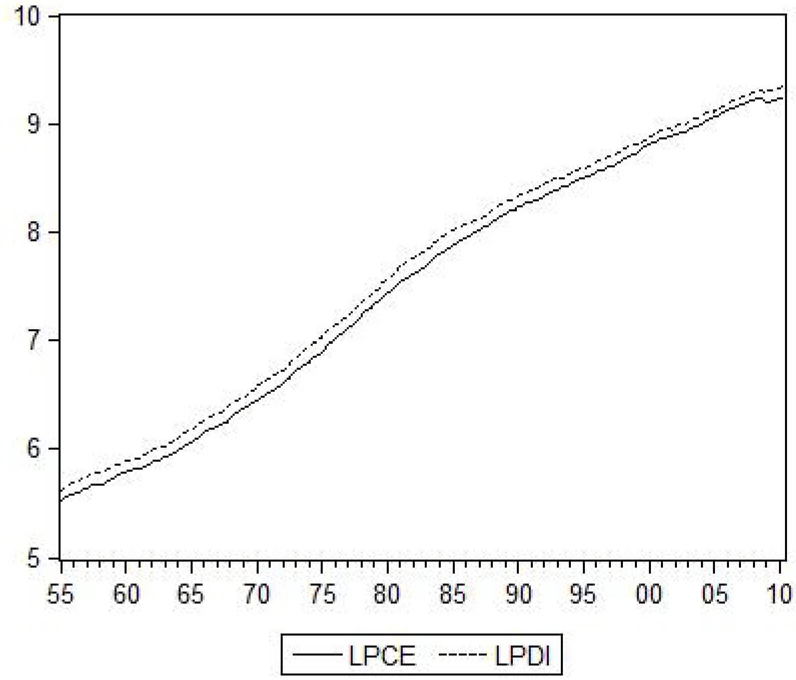
The cointegrating relationship has been tested by three approaches. The VEC based method was tested for several specifications: with an intercept included in the long term part of the model (Type 2), with an intercept including in the long term as well as one in the VAR part (Type 3), and an intercept in VAR part and a linear deterministic time trend as well as an intercept in the long term part (Type 4).[10]

For estimating the cointegrating relationship of consumption and income a VEC specification of Type 3, i.e. with two constants turned out as most appropriate. The number of lags was determined by the Wald lag exclusion test. The income elasticity of the cointegrating relationship was found to be around one in the long term. This result was robust to different lag length and is in line with theory. Siliverstovos (2003) found the same empirical evidence for the US between 1953 and 1984 using the multicointegration approach of Granger and Lee (1989). Table A1 in the appendix gives one of the estimation results. In the equation normalized for income, the EC parameter turned out to be insignificant which hints to weak exogenity of income.

This justifies the estimation of the cointegrating relationship by the traditional Engle and Granger OLS method without incurring a possible endogenity bias problem. The OLS gave a long run income elasticity of again slightly above 1. The results are given in Table 3. The estimated relation can only be regarded as a cointegrated one if residuals do not contain a unit roots. The ADF as well as the PP test have been carried out using the critical levels as reported by Davidson and MacKinnon (1993). According to both tests the null of a unit root was rejected at a critical value below 5%.

[10] This means that we exclude structures as proposed by Johansen (1991) without a constant in the long term part (Type 1) (unless the constant turns out to be insignificant) - which is thought to be necessary for scaling the different variables as can be easily observed by looking at Figure 1 - or a quadratic deterministic time trend (Type 5).

Figure 1: Logs of nominal Consumption and disposable Income



Our last method to estimate the cointegration relationship is the DOLS approach as proposed by Phillips and Loretan (1991), Saikkonen (1992) or Stock and Watson (1993). As the VEC model already hinted to a weak exogenity of income, the DOLS results should not be far away from both others. In order to determine the optimal size of leads and lags[11] of differences in regressors (as required in the DOLS approach in order to get rid of a possible endogenity bias) we followed the suggestion of Hayakawa and Kurozumi (2008) and cut insignificant leads. Table 3 shows that income elasticity lays again around 1 between the results of the VEC and the OLS. Estimation results as well as test statistics are given in Table 2A in the appendix together with Newy-West HAC standard errors. As with the traditional OLS, residuals were checked for unit roots and again the null of I(1) was rejected significantly.

Table 3: Long term income elasticises of consumption[12]

	VEC	Dynamic OLS	OLS
Parameter	1.0084***	1.0081***	1.0097***
se	(0.0033)	(0.0016)	(0.0023)
t-value	[307.243]	[615.656]	[430.959]

Source: Own calculations

[11] Here, the appropriate lead/lag length was determined by the Schwarz Information Criterion.
[12] For OLS and Dynamic OLS Newy-West HAC standard errors are given.

The next step was to set up conventional EC models in order to compare them with the proposed cumulated method. VEC approaches not only estimate the cointegrating relationship but also include an error correcting mechanism at the same time. This is regarded by many researchers as an advantage but here it is not, as we argue that specifications which consider only the deviation of the last period can lead to an omitted variable bias. While VECM care for auto-correlation including several lags of the differenced series they do not for deviations from steady-state. A bias arises as auto-correlation of deviations from steady state is a phenomenon typically observed in ECM applications.

As the income elasticities of the long term component derived from the DOLS and the traditional OLS method are virtually the same, and the method for setting up EC models based on them, too, we will concentrate in the following just on results derived by parameters estimated by OLS. In Table 4 the first two columns show the results for the conventional error correction models based on the VEC approach and the OLS approach for estimating the cointegrating relationship.

Both EC parameters β are significant and have the expected sign. The OLS output suggests that the deviation from steady state closes somewhat quicker with 13 percent to be corrected within a quarter. The short run influence of the current income growth (as represented by the $\gamma_{t(0)}$ parameter) on consumption growth is 0.37 in the OLS case. Using 6 parameters to be estimated, the R^2 shows that VEC is capable to explain 30% of the variation in consumption growth. The OLS ECM explains 40% with just 4 parameters.

Table 4: Error correction models explaining consumption growth

	Conv. ECM Based onVEC	Conv. ECM Based on OLS	CumECM unrestricted Based on OLS	CumECM restricted Based on OLS
β	-0.1089 (-4.4987)	-0.1316 (-6.9211)	-0.0539 (-2.4896)	-0.0534 (-2.7000)
$\gamma_{t(0)}$		0.3719 (8.3422)	0.3322 (6.8940)	0.3318 (7.7798)
$\gamma_{t(-1)}$	0.1049 (1.6933)			
$\gamma_{t(-2)}$	0.1900 (2.6187)			
$\lambda_{AR(1)}$	0.1417 (1.9534)		0.4413 (4.0638)	0.4420 (4.6084)
$\lambda_{AR(2)}$	-0.0025 (-0.0422))			
c	0.0099 (6.6111)	0.0110 (12.7190)	0.0040 (2.5093)	0.0040 (2.8046)
$\lambda_{MA(1)}$			-0.4508 (-3.1413)	-0.4420 (4.6084)
R^2	0.3056	0.4113	0.4542	0.4339
Skewness	-0.2241	-0.0925	-0.2161	-0.2067
Kurtosis	3.3519	3.0491	3.3626	3.3839
Jarque-Bera	2.8810	0.3249	2.8120	2.8385
Q-Stat(1)	0.406	0.728	0.467	0.330
Q-Stat(2)	0.048	0.002	0.051	0.050
Q-Stat(3)	0.107	0.005	0.113	0.110
Q-Stat(4)	0.107	0.012	0.113	0.104

Source: Own calculations[13]

Residuals seem neither to be significantly skewed nor to have tails deviating from those of a normal distribution, but in the OLS-case they suffer severely from auto-correlation at the second order. The null of no auto-correlation up to lag 4 is clearly rejected according to the Q-statistics.

To compare these results to the proposed cumulative ECM, the following model was estimated in accordance to equation (8)

$$\Delta lpce_t = \beta\xi_{t-1} + \gamma\Delta lpdi_t + c + \lambda_{AR(1)}\Delta lpce_{t-1} - \lambda_{MA(1)}\varepsilon_{t-1} + \varepsilon_t \quad (11)$$

with $\Delta lpce_t$ as the first order differences of logged US personal consumption expenditures in nominal terms, ξ_{t-1} the difference between the steady state (as given by the cointegrating relationship) and the logged personal consumption expenditures, a constant c, $\Delta lgdp_t$ the first order differences of logged nominal GDP and an error term ε_t. The results for the unrestricted version where $\lambda_{AR(1)}$ and $\lambda_{MA(1)}$ are estimated independently are shown in the third column of Table 4.

The weight given to the deviation from steady state in the previous period differs significantly from the VEC and the conventional EC OLS approach. Their ignorance of deviations located in the past beyond (t-1), biased their βs for more than the double. The short run influence of income growth on consumption growth is a little bit lower than for the others.

According to (7) the parameter of the AR(1) term λ (the retention rate) should be the same as of the MA(1) term but with an opposite sign. Table 4 shows that $\lambda_{(1)}$ = 0.5522 and $\lambda_{MA(1)}$ = -0.4854 if both are estimated independently. A Wald test, testing the null that $\lambda_{AR(1)} = -\lambda_{MA(1)}$ can not reject this.

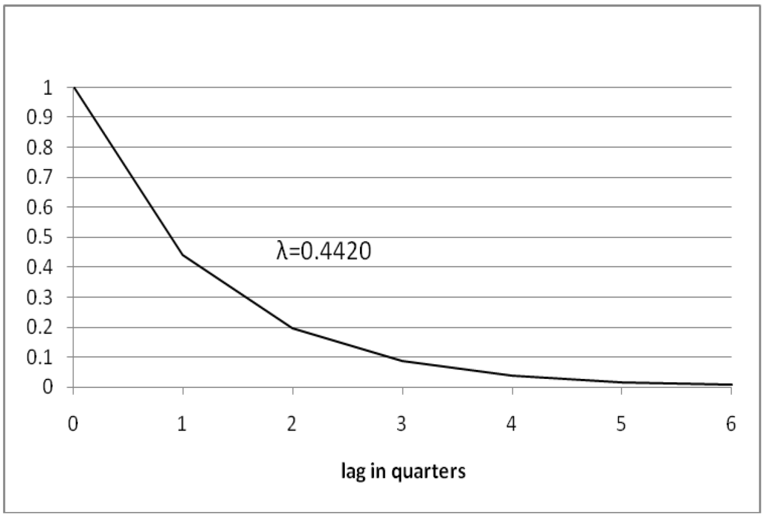
The last column in Table 4 shows the results of the estimation of a model where $\lambda_{AR(1)}$ and $\lambda_{MA(1)}$ were restricted to be the same with a different sign, as the theoretical Koyck model demands. Fransens and van Oest (2007) have shown that such a restriction requires maximum likelihood estimation and errors are non-normally distributed because of the so-called Davies (1987) problem. They propose several alternative test statistics of which here the average absolute t-statistics has been chosen and reported below the parameter values. [14] As the independently estimated lambdas are quite close to each other, it is no surprise that parameters and test statistics are too.

Figure 2 shows the weights of past deviations given by the cumECM method, where the retention parameter λ stems from the restricted model. The weight given to the last quarter's deviation from steady state is λ^0=1, like for conventional ECMs. The one for the deviation two periods ago is λ^1=0.442 and the

[13] VEC residuals for skewness, kurtosis and Jarque Bera were factorized according to the Dornik-Hansen inverse square root of residual correlation matrix.
[14] Critical values for testing β=0 are taken from Fransens and van Oest (2007) Table 2 page 294. For a sample size of 1000 the critical value of a 95% confidence level is 1.80.

one three periods ago has with λ^2= 0.195 already halved. The AR(1) parameter included in the VEC is considerably lower than the one found in the cumECM.

Figure 2: Retention rate



The Q-test statistics for auto-correlation are quite similar between both cumECMs and the VEC. Up to the fourth quarter in the past there exists no auto-correlation in residuals, with a borderline value for the second lag.

The significance of the retention rate clearly rejects the null that λ=0 so the traditional ECM seems to be invalid. A Wald test testing for λ=1 – as it is the case for the multicointegration approach of Granger and Lee (1989) – is rejected, too[15]. This is quite in line with a study of Lee (1996) where no evidence for multicointegeration between US consumption and income was found. Siliverstovos (2003), however, found empirical support of multicointegration between US consumption and income for the period 1953 – 1984, what we could not confirm.

4. Forecasting performance

In order to show the advantages of cumulative ECMs, their forecasting performance was evaluated. Here we focus on the simple mean squared error (MSE) criterion for the forecasting period from the first quarter of 2008 to the second quarter of 2010, e. g. ten observations. As explained above, the massive fiscal policy reaction of the US government in 2008, as a response to the international financial crises, led to a drift between income and consumption. In order to support private household consumption large tax credits were granted to private households. This innovation can reasonably be regarded as non-permanent and should therefore not enter in the cointegration relationship between income and consumption. As all our estimated models are estimated till the end of 2007, none of them captures this innovation. So all forecasts are

[15] In case of approaching the value of λ=1 the applied test statistics get more and more biased because the correct specification in this case would contain the second level cointegration term as required in the case of multicointegration.

lacking this innovation and the MSE statistics are influenced for all models the same. The exact specification of the different tested models is as given in Table 4 above.

Table 5: Forecasting performance

	Conv. ECM Based onVEC	Conv. ECM Based on OLS	CumECM Based on OLS unrestricted	CumECM Based on OLS restricted
Root Mean Squared Error	0.0124	0.0133	0.0111	0.0111
Theil Inequality Coefficient	0.5525	0.5372	0.5007	0.4901

As a further measures we use the Theil's inequality index, where our model is compared to the naive forecast dlpce(t+1) = dlpce(t). Values above 1 mean that the estimated model performs worse than the naive forecast and vice versa. According to Table 5 our proposed cumulated error correction model – whether estimated as a restricted or unrestricted form – again outperforms the conventional ECMs, whether based on a VEC estimation or not.

5. Conclusions

Traditional error correction models (ECM) based on cointegrating flow variables are specified just to balance the most recent deviation from steady state. Sometimes, the accumulation of past deviations forms a stock variable which are to be considered in the modelling process. Granger and Lee (1989) were first to propose a multicointegration model where not only the flows represent a cointegrating relationship (first-level cointegration) but also the sum of past deviations from steady state (second-level cointegration). The authors accumulated all past deviations with an equal unit weight to stock variables. As such an accumulation of I(1) variables leads to I(2) variables, usual statistics for testing the exisitence of cointegrating relationships are no longer valid.

A further, more convenient method is proposed here. The presented cumulative ECM is based on the assumption that the importance of past disequilibria for the current stock is decaying according to a geometric function. Koyck (1954) has shown that such a model possesses an ARMAX representation with AR and MA being of order one with parameters of the same size but with different signs. This parameter is called the retention rate. It can be shown that the traditional ECM is nested. The one based on multicointegration is not completely nested but can be tested with the proposed model, at least. While the first assumes a retention rate of zero the latter assumes it to be one. Furthermore it can be shown that the proposed kind of accumulation does not lead to I(2) variables, making the usual statistics for cointegration tests still valid.

Here our cumulative ECM was applied to estimate US consumption dynamics. It shows that it outperforms a VEC and a single equation EC model in respect of its forecasting power and parameters are estimated more parsimony. It explains at least the same percentage of variation by using less degrees of freedom.

For US consumption expenditures, last quarter's deviation from steady state is set-off by just 5% according to the cumulative ECM while the VECM gives 11% and the conventional single equation ECM 13%. The retention rate of the cumulative ECM suggests that deviations up to 6 quarters in the past are set-off at the same time. The estimated size of the retention rate with 0.44 rejects the traditional ECM (with a retention rate 0) as well as the one based on multicointegration (with a retention rate of 1) as being the right one.

References

Baba, Y., Hendry, D. F., Starr, R. M., 1992. The Demand for M1 in the U.S.A.,1960-1988, Review of Economic Studies, 59(1), 25-61.

Ball, L., 1992. Short-Run Money Demand, *NBER Working Paper*, 9235.

Box, G. E. P, Jenkins, G. M., 1970. Time Series Analysis - Forecasting and Control, San Francisco: Holden Day

Davidson, R., MacKinnon, J. G., 1993. Estimation and inference in Econometrics, New York, Oxford University Press.

Davies, R. B., 1997. Hypothesis testing when a nuisance parameter is present only under the alternative, Biometrika, 64, 247-254.

Engle, R. F., Granger, C. W. J., 1987. Cointegration and Error Correction: Representation, Estimation, and Testing, *Econometrica*, 55(2), 251-277.

Engsted, T., Johansen, S., 1997. Granger's Representation Theorem and Multicointegration, European University Institute, *Economics Working Paper*, 15.

Engsted, T., Gonzalo, J., Haldrup, N., 1997. Testing for multicointegration, *Economics Letters,* 56, 259–266.

Fransens, Ph. H., van Oest, R., 2007. On the econometrics of the geometric lag model, Economics Letters, 95, 291-296.

Goldfeld, St. M., 1973. The Demand for Money Revisited, *Brookings Papers on Economic Activity*, 3, 577-638.

Granger, C. W. J., Lee, T. H., 1989. Investigation of production, sales and inventory relations using multicointegration and non-symetric error correction model, *Journal of Applied Econometrics*, 4, 145-159.

Hayakawa, K., Kurozumi, E., 2008. The role of "leads" in the dynamic OLS estimation of cointegrating regression models, *Mathematics and Computers in Simulation*, 79, 3, 555-560.

Hess, G. D., Jones, Ch. S, Porter, R. D., 1994. The predictive failure of the Baba, Hendry and Starr model of the demand for M1 in the United States, Federal Reserve Bank of Kansas City, *Research Working Paper*, 06.

Hoffman, D. L., Rasche, R. H., 1991. Long-run income and interest elasticities of money demand in the united states, *Review of Economics and Statistics*, 73 (4), 665-674.

Johansen, S., 1991. Cointegration and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica*, 59, 6, 1551-1580.

Johansen, S., 1992. Determination of co-integration rank in the presence of a linear trend, *Oxford Bulletin of Economics and Statistics,* 54, 383–397.

Johansen, S., 2002. The statistical analysis of hypothesis on the cointegrating relations linear trend, *Oxford Bulletin of Economics and Statistics,* 54, 383–397.

Johansen, S., 2006. Statistical analysis of hypotheses on the cointegrating relations in the I(2) model, *Journal of Econometrics,* 132, 81-115.

Johansen, S., Juselius, K., 1990. Maximum likelihood estimation and inference on cointegration-with applications to the demand for money, *Oxford Bulletin of Economics and Statistics,* 52 (2), 169-210.

Kongsted, H. C., Nielsen, H. B., 2004. Analysing I(2) Systems by Transformed Vector Autoregressions, Oxford Bulletin of Economics and Statistics, 66, 3, 379-397.

Koyck, L. M., 1954. Distributed Lags and Investment Analysis, North-Holland, Amsterdam.

Lee, T. H., 1992. Stock-flow relationships in housing construction, Oxford Bulletin of Economics and Statistics, 54, 419-430.

Lee, T. H., 1996. Stock adjustment for multicointegrated series, *Empirical Economics*, 21, 563–639.

Lucas, R. E. Jr., 1988. Money Demand in the United States: A Quantitative Review, Carnegie-Rochester Conference Series on Public Policy, 29, 137-168.

Mehra, Y. P., 1993. The Stability of the M2 Demand Function: Evidence from an Error-Correction Model, *Journal of Money, Credit & Banking*, Vol. 25.

Phillips, P. C. B., Loretan, M., 1991. Estimating long-run economic equilibria, *Review of Economic Studies*, 58, 407-436.

Rao, B. B., 2007. Estimating short and long-run relationships: a guide for the applied economist, *Applied Economics*, 39: 13, 1613 — 1625.

Saikkonen, P., 1991. Asymptotically efficient estimation of cointegration regressions, *Econometric Theory*, 7, 1-21.

Siliverstovos, B., 2003. Multicointegration in US consumption data, German Institute for Economic Research, DIW, Discussion papers, 382.
Sims, C. ,(1980), Macroeconomics and reality, *Econometrica*, 48, 1–48.

Stock, J. H., Watson, M. W, 1993. A simple estimator of cointegrating vectors in higher order integrated systems, *Econometrica*, 61, 783-820.

Appendix

Table 1A: VEC estimates for consumption

Vector Error Correction Estimates
Date: 05/23/11 Time: 15:00
Sample (adjusted): 1955Q2 2010Q2
Included observations: 221 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LPCE(-1)	1.000000	
LPDI(-1)	-1.004924 (0.00370) [-271.715]	
C	0.142526	
Error Correction:	**D(LPCE)**	**D(LPDI)**
CointEq1	-0.099001 (0.02372) [-4.17391]	-0.032630 (0.03143) [-1.03807]
D(LPCE(-1))	0.239445 (0.07184) [3.33298]	0.498968 (0.09521) [5.24084]
D(LPCE(-2))	0.216219 (0.07020) [3.08014]	0.114452 (0.09303) [1.23027]
D(LPDI(-1))	0.113308 (0.06160) [1.83950]	-0.129635 (0.08163) [-1.58804]
D(LPDI(-2))	-0.022626 (0.06163) [-0.36714]	0.067478 (0.08167) [0.82622]
C	0.007557 (0.00134) [5.64712]	0.007550 (0.00177) [4.25750]
D(DUM200804)	-0.011571 (0.00470) [-2.46120]	-0.010912 (0.00623) [-1.75142]

Table 2A：DOLS estimates for consumtption

Dependent Variable: LPCE
Method: Least Squares
Date: 02/07/11 Time: 21:34
Sample (adjusted): 1956Q4 2010Q1
Included observations: 214 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LPDI	1.008108	0.001637	615.6556	0.0000
C	-0.120831	0.012675	-9.532662	0.0000
DLPDI(-8)	-0.360876	0.114392	-3.154734	0.0019
DLPDI(-7)	-0.404123	0.137684	-2.935157	0.0037
DLPDI(-6)	-0.346383	0.115494	-2.999141	0.0030
DLPDI(-5)	-0.327819	0.111026	-2.952633	0.0035
DLPDI(-4)	-0.297510	0.117867	-2.524108	0.0124
DLPDI(-3)	-0.318048	0.115406	-2.755901	0.0064
DLPDI(-2)	-0.262671	0.128215	-2.048675	0.0418
DLPDI(-1)	-0.233427	0.126648	-1.843120	0.0668
DLPDI	-0.431806	0.114451	-3.772846	0.0002
DLPDI(1)	0.257808	0.107259	2.403604	0.0171

R-squared	0.999849	Mean dependent var	7.559163
Adjusted R-squared	0.999840	S.D. dependent var	1.164032
S.E. of regression	0.014710	Akaike info criterion	-5.546129
Sum squared resid	0.043710	Schwarz criterion	-5.357382
Log likelihood	605.4358	Hannan-Quinn criter.	-5.469858
F-statistic	121233.2	Durbin-Watson stat	0.213586
Prob(F-statistic)	0.000000		

Table 3A: VEC estimates for real M1

Vector Error Correction Estimates
Date: 02/09/11 Time: 07:35
Sample (adjusted): 1971Q4 2010Q4
Included observations: 157 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1		
LM1R(-1)	1.000000		
LGDP(-1)	-0.387085 (0.12657) [-3.05827]		
R(-1)	0.026055 (0.01258) [2.07073]		
C	-2.139074 (0.60670) [-3.52578]		
Error Correction:	D(LM1R)	D(LGDP)	D(R)
CointEq1	-0.023246 (0.00853) [-2.72611]	-1.75E-18 (6.4E-18) [-0.27395]	-8.50E-17 (3.2E-16) [-0.26388]
D(LM1R(-1))	0.603126 (0.07365) [8.18947]	-6.00E-17 (6.7E-17) [-0.88927]	4.16E-15 (2.6E-15) [1.61412]
D(LM1R(-2))	0.167913 (0.06773) [2.47916]	7.72E-17 (6.2E-17) [1.24475]	-2.47E-15 (2.4E-15) [-1.04278]
D(LGDP(-1))	-0.140968 (0.09572) [-1.47272]	-8.06E-16 (8.8E-17) [-9.19127]	2.58E-14 (3.4E-15) [7.69990]
D(LGDP(-2))	0.001092 (0.09566) [0.01142]	-3.37E-16 (8.8E-17) [-3.84697]	-4.32E-15 (3.3E-15) [-1.28910]
D(R(-1))	-0.002940 (0.00073) [-4.03439]	2.75E-18 (6.7E-19) [4.11287]	3.70E-17 (2.6E-17) [1.45074]
D(R(-2))	0.000873 (0.00083) [1.04689]	3.01E-18 (7.6E-19) [3.93420]	-7.69E-17 (2.9E-17) [-2.63667]
D(LGDP)	0.035860 (0.09682) [0.37039]	1.000000 (8.9E-17) [1.1e+16]	-1.96E-14 (3.4E-15) [-5.78899]
D(R)	-0.001585 (0.00070) [-2.26742]	-7.93E-19 (6.4E-19) [-1.23723]	1.000000 (2.4E-17) [4.1e+16]
DUM200804	0.042059 (0.00652) [6.45193]	5.01E-18 (6.0E-18) [0.83850]	2.56E-16 (2.3E-16) [1.12392]

Table 4A: DOLS estimates for real M1

Dependent Variable: LM1R
Method: Least Squares
Date: 02/14/11 Time: 18:43
Sample (adjusted): 1971Q3 2010Q4
Included observations: 158 after adjustments
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LGDP	0.413640	0.046096	8.973490	0.0000
R	-0.019582	0.003782	-5.177254	0.0000
DR(-1)	0.012648	0.004864	2.600179	0.0102
DR	0.014207	0.004677	3.038018	0.0028
C	1.915245	0.215418	8.890831	0.0000

R-squared	0.880688	Mean dependent var	3.506615
Adjusted R-squared	0.877569	S.D. dependent var	0.208528
S.E. of regression	0.072964	Akaike info criterion	-2.366557
Sum squared resid	0.814541	Schwarz criterion	-2.269639
Log likelihood	191.9580	Hannan-Quinn criter.	-2.327198
F-statistic	282.3377	Durbin-Watson stat	0.056717
Prob(F-statistic)	0.000000		

Proof for page 4

Starting from equation (4), with disregarding the exogenous regressor and the constant

$$\Delta y_t = \beta \sum_{i=0}^{\infty} \lambda^i \xi_{t-1-i} + \varepsilon_t \tag{1A}$$

we get

$$\Delta y_t = \beta \lambda^0 \xi_{t-1} + \beta \sum_{i=1}^{\infty} \lambda^i \xi_{t-1-i} + \varepsilon_t \tag{2A}$$

Equation (7), again without constant and exogenous regressors, states that

$$\Delta y_t = \beta \xi_{t-1} + \lambda \Delta y_{t-1} - \lambda \varepsilon_{t-1} + \varepsilon_t \tag{3A}$$

If we equate now (2A) and (3A) and subtract on both sides $\beta\xi_{t-1}$ and ε_t we get

$$\beta \sum_{i=1}^{\infty} \lambda^i \xi_{t-1-i} = \lambda \Delta y_{t-1} - \lambda \varepsilon_{t-1} \tag{4A}$$

A substitution of this into (8) gives

$$\beta \xi_{t-1} + \beta \sum_{i=1}^{\infty} \lambda^i \xi_{t-1-i} = 0 \tag{5A}$$

which represents our required stability condition in steady state

$$\xi_{t-1} = -\sum_{j=1}^{\infty} \lambda^j \xi_{t-1-j}$$

as given in (9).