THE TREATMENT OF SEASONALITY IN ERROR CORRECTION MODELS

A CASE STUDY FOR AN AUSTRIAN CONSUMPTION FUNCTION

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1. Introduction

In their seminal paper on a consumption model, Davidson, Hendry, Srba and Yeo (1978), hereafter denoted DHSY, developed an appropriate econometric methodology for modelling dynamic relationships. These authors used seasonally unadjusted data to estimate an error correction model (ECM) for UK consumption. Unadjusted data were chosen because Sims (1974) and Wallis (1974) have shown that seasonal adjustment can distort relations between variables. In order to deal with seasonality, DHSY chose a specification in which seasonal lags and the seasonal difference operator played a prominent role. This variant of a seasonal ECM has had a strong influence on subsequent research. A version of such a seasonal ECM for Austrian nondurable consumption can be found in Thury and Wueger (1994).

In a recent paper, Harvey and Scott (1994) critizised the approach of DHSY because it leads to dynamic misspecifcation, if the seasonal effects change gradually over time. They propose to treat seasonality as an unobserved component which could change slowly over time. In the present paper, we take up this suggestion and introduce an unobserved seasonal component into an ECM for Austrian consumption. Before doing this, we present an analysis of stylized facts for real consumer expenditure on nondurables and services and for real personal disposable income. We concentrate on these two variables, because they form the core of the consumption model in Thury and Wueger.

Unobserved components provide valuable information about stylized facts of the modelled series. When building an unobserved components model, one can proceed in two principal ways. One can specify directly a model for each unobserved component which, in some way, captures one's prior beliefs about the component. This is the socalled Structural Time Series (STS) approach, and some basic references are Engle (1978), Gersch and Kitagawa (1983), Harvey and Todd (1983) and Harvey (1989). Alternatively, since observations are only available on the overall series, one can proceed by identifying first a model for the observed series, and then derive appropriate models for the components that are compatible with the overall model. This is the so-called ARIMA Model Based (AMB) approach, and basic references are Box, Hillmer and Tiao (1978), Burman (1980), Hillmer and Tiao (1982), and Bell and Hillmer (1984). We use the STS approach in this paper.

For both approaches, the model has the form

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \qquad (2.1)$$

where, in our estimated models, y_t is the logarithm of the observed series, μ_t the trend, γ_t the seasonal, and ε_t the irregular. All three components are stochastic, but can be deterministic in limiting cases. In general, μ_t becomes stationary after differencing, while γ_t is stationary when multiplied by the seasonal summation operator

$$S(L) = 1 + L + L^2 + \cdots + L^{s-1}, \qquad (2.2)$$

where γ_{jt} is given by

$$\begin{bmatrix} \gamma_{jt} \\ \gamma^*_{jt} \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma^*_{j,t-1} \end{bmatrix} + \begin{bmatrix} \omega_{jt} \\ \omega^*_{jt} \end{bmatrix}$$

with $\lambda_j = 2\pi j/s,$ for $j = 1,2,\cdots,s/2$ and

$$\gamma_{jt} = -\gamma_{j,t-1} + \omega_{jt}, \qquad for \quad j = s/2.$$

ω_{jt} and ω^*_{jt} are uncorrelated zero mean white noise processes with a common variance σ^2_ω. The larger this variance, the more past observations are discounted in estimating the seasonal pattern. γ^*_{jt} appears by construction in order to form γ_{jt} [see Harrison and Akram (1993)].

A model consisting of *(2.1)* with *(2.3)* and *(2.4)* will be referred to as basic structural model (BSM). The variances $\sigma^2_\eta, \sigma^2_\zeta, \sigma^2_\omega,$ *and* σ^2_ε, usually referred to as hyperparameters in the literature, can be estimated by maximum likelihood as shown in Harvey (1989). Once this is done, the trend and seasonal components may be extracted by a smoothing algorithm as in Koopman (1993).

We now turn to an analysis of consumption and income. Unless otherwise stated, the sample period is 1961:1 to 1995:4, and estimation is by exact ML using the STAMP 5.0 package.

2.2 Consumer Expenditure on Nondurables and Services

frequency distribution of the residuals. As it is to be expected from the outcome of the Q-test, we observe a small number of residuals, which are significantly different from zero and, consequently, we have some residual autocorrelation. But from the CUSUM test and the shape of the frequency distribution, we would conclude that the residuals are a normally distributed random variable.

Fig. 3. Basic Structural Model for Consumption

Figure 4 depicts the extracted unobserved components for trend, seasonal and irregular. Since σ_{ζ}^{2} is extremely small, the trend is practically a random walk with drift. As we see from the q-ratios, fluctuations in the level of the trend are the most important source of the variations in consumption. But, since these fluctuations in the level exhibit no regular repetitive pattern, they cannot be captured by introducing a cyclical component. Although σ_{ω}^{2} is relatively small too, it is large enough to allow a quite marked decline in the amplitude of the seasonal component. The irregular is definitely white noise of a small order of magnitude.

Fig. 4. Unobserved Components for Consumption

Figures 5a and 5b provide some insight in the forecasting performance of our consumption model. The perhaps surprising finding of this forecast test is, besides the general accuracy of the resulting predictions, that there exists no observable difference in the quality of one-step ahead forecasts and extrapolations. A forecast horizon of three years is already relatively long and, nevertheless, extrapolations and realizations show no tendency whatsoever to drift apart.

Fig. 5a. One-Step Ahead Forecasts for Consumption
Fig. 5b. Extrapolations of Consumption

performance of the BSM for dispoable income is significantly inferior (already for one-step ahead forecasts and, above all, for extrapolations). The model has a tendency to systematically overestimate future income levels.

Fig. 8a. One-Step Ahead Forecasts of Disposable Income
Fig. 8b. Extrapolations for Disposable Income

3. Multivariate Analysis of Consumption and Income

Univariate models for consumption and income are often considered as inappropriate by economists. Economic theory presupposes a strong relationship between consumption and income and, as a consequence, these two variables should be modelled jointly.

3.1 Seemingly Unrelated Time Series Equations

Seemingly unrelated time series equation (SUTSE) models, as multivariate structural time series models are called in the literature [see Harvey and Koopman (1996)], have a similar form to univariate models, except that $\tilde{y}_t$ is now a $N \times 1$ vector of observations, that is

$$\tilde{y}_t = \tilde{\mu}_t + \tilde{\gamma}_t + \tilde{\varepsilon}_t, \qquad (3.1)$$

where we use the superscript '~' to denote vectors or matrices. In a SUTSE model, each series is modelled as in the univariate case, but the disturbancies may be correlated across series. Thus, if $\tilde{\varepsilon}_t = \{\varepsilon_{1t}, \cdots, \varepsilon_{Nt}\}'$ is the irregular disturbance,

$$Var(\tilde{\varepsilon}_t) = \tilde{\Sigma}_\varepsilon . \qquad (3.2)$$

the SUTSE model is exactly identical to the performance of the two univariate models.

Fig. 11a. One-Step Ahead Forecasts of Consumption and Income
Fig. 11b. Extraspolations for Consumption and Income

A closer inspection of the covariance matrices for the component disturbances reveals that the covariance matrix of the slope disturbance does not have full rank implying the presence of a common trend. Econometricians consider this as indication for cointegration between consumption and income. But, imposing the relevant restrictions on our SUTSE model and reestimating does not improve the resulting outcome.

4. Seasonal Error Correction Models

The basic formulation of an error correction model (ECM) is

$$\nabla y_t = \theta + \beta \, \nabla x_t + \lambda \left(y_{t-1} - \alpha \, x_{t-1} \right) + \varepsilon_t \, , \qquad (4.1)$$

where $\varepsilon_t \sim NID\left(0, \sigma^2_\varepsilon\right)$. For expositional purposes, we use a very simple model. Our estimated consumption functions are more complex, containing more explanatory variables and a more realistic error correction with the inflation rate and wealth as additional variables. This error correction term will be denoted by the acronym 'ECM_t' in the relevant tables.

As it stands, model *(4.1)* does not allow for seasonal effects. Unfortunately, the standard ECM approach does not offer any guidance how seasonal effects should be incorporated. In their original paper, DHSY tried to capture seasonality by taking seasonal differences obtaining the following model:

But, if *(4.3)* is a good representation of the data generating process (DGP), implying that the seasonality is stochastic, the DHSY specification can have several shortcomings. If the seasonal patterns of y_t and x_t are different, the term $(y_{t-s} - \alpha x_{t-s})$ will itself display seasonal fluctuations. In such a situation, seasonal differencing is often not sufficient to remove seasonal effects completely. This is explicitly noted by Hendry and Ungern-Sternberg (1981), who added deterministic seasonal dummies to the original DHSY model. This solution is also adopted by Thury and Wueger (1994). Moreover, serial correlation in the error term of DHSY model can induce misspecification effects.

4.2 The Austrian Consumption Function

Applying ECM modelling to Austrian data, Thury and Wueger (1994) obtain a stable dynamic relationship between the logarithms of real consumer expenditure on nondurables and services (c_t), real personal disposable income (y_t), real wealth (w_t), and the price deflator for nondurable consumption (p_t) over the period 1961:1 to 1992:4. Additionally, a set of dummy variables capturing an Easter effect and the effects of several fiscal policy measures together with the already mentioned deterministic seasonal dummies are also included in this model. Without indicating it explicitly in the relevant tables, the Easter and the fiscal policy dummies are included in all estimated consumption functions of this paper (also in the univariate and bivariate structural time series models for consumption). This model is reestimated here by maximizing a likelihood function, computed via the Kalman filter, using the STAMP 5.0 package.

fluctuations in consumption growth. In summary, we would say that an ECM with a stochastic seaonal seems to be a very attractive specification. Any signs of misspecification in terms of serial corrrelation are removed and, additionally, certain inconsistencies, which plagued the old specification of Thury and Wueger, also disappear. Above all, the somewhat strange mixture of variables in first and fourth differenced form (or both for the inflation rate) and the recourse to an Almon type lag structure for changes in wealth now become obsolete. In ECM/SEAS, all variables enter as first differences. Moreover, no seasonal dummies are required to capture remaining deterministic seasonality.

The results of Table 5 provide clear evidence that adding three years of new observations has no effect on the resulting estimates. As we shall see below, some structural changes obviously occured during the eighties, but not in the first years of the nineties.

Table 5. ECMs for Consumption Growth
1961:1 - 1995:4

In order to test for equation stability, we estimate our consumption models for three different sample periods, namely 1961:1 to 1980:4, 1961:1 to 1985:4, and 1961:1 to 1990:4, and make for each sample period post-sample predictions with a forecast horizon of 20 quarters. The resulting test statistics are given in Table 6. Neither the Chow test nor a parameter stability test provide evidence of structural change. The parameter stability test is given by

$$pst = \frac{1}{H} \sum_{t=T+1}^{T+H} \frac{e_t^2}{\sigma_\varepsilon^2},$$

model. However, the resulting improvement was far less pronounced than the one achieved by Harvey and Scott for the UK consumption function of DHSY.

6. References

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Table 1. Univariate Structural Time Series Model for Consumption

Parameter/Test	Estimate	q-ratio
σ^2_η	2.7471×10^{-5}	1.00
σ^2_ς	1.4169×10^{-7}	0.01
σ^2_ω	7.8000×10^{-6}	0.28
σ^2_ε	1.0372×10^{-5}	0.38
$\hat{\sigma}$	0.0128	
$\hat{\beta}$	0.0055 (3.75)	
R^2	0.9983	
R^2_s	0.7804	
Normality Test	1.1000 [0.58]	
$H(45)$	0.5254 [0.98]	
$Q(4)$	8.0733 [0.00]	
$Q(8)$	11.2374 [0.05]	
T	140	

The figure in parentheses is a *t*-value. Figures in square bracktes are *p*-values.

Table 2. Univariate Structural Time Series Model for Income

Parameter/Test	Estimate	q-ratio
σ^2_η	10.3084×10^{-5}	1.00
σ^2_ς	1.0697×10^{-7}	0.00
σ^2_ω	7.3757×10^{-6}	0.07
σ^2_ε	1.6734×10^{-5}	0.16
$\hat{\sigma}$	0.0174	
$\hat{\beta}$	0.0069 (3.75)	
R^2	0.9973	
R^2_s	0.6403	
Normality Test	3.5110 [0.17]	
$H(45)$	0.7864 [0.79]	
$Q(4)$	2.3888 [0.12]	
$Q(8)$	7.6686 [0.18]	
T	140	

The figure in parentheses is a *t*-value. Figures in square bracktes are *p*-values.

Table 3. Seemingly Unrelated Time Series Equation (SUTSE) Model for Consumption and Income

Parameter/Test	Consumption Estimate	Consumption q-ratio	Disposable Income Estimate	Disposable Income q-ratio
σ^2_η	2.8601×10^{-5}	1.00	10.0073×10^{-5}	1.00
σ^2_ς	1.3501×10^{-7}	0.00	1.4941×10^{-7}	0.00
σ^2_ω	7.8515×10^{-6}	0.27	7.3182×10^{-6}	0.07
σ^2_ε	9.1239×10^{-6}	0.32	1.8076×10^{-5}	0.18
$\hat{\sigma}$	0.0127		0.0170	
$\hat{\beta}$	0.0056 (3.84)		0.0063 (3.75)	
R^2	0.9983		0.9974	
R^2_s	0.7814		0.6560	
Normality Test	2.4370 [0.30]		2.8850 [.24]	
$H(45)$	0.5063 [0.99]		0.8487 [.71]	
$Q(4)$	8.3197 [0.00]		2.8925 [.09]	
$Q(8)$	11.8336 [0.04]		8.6909 [.12]	
T	140		140	

The figure in parentheses is a *t*-value. Figures in square bracktes are *p*-values.

Table 4. ECMs for Consumption Growth
1961:1 - 1992:4

ECM		ECM /AGG		ECM /SEAS	
$\nabla_4 c_t$		$\nabla_4 c_t$		∇c_t	
$\nabla_4 y_t$	.301 (7.31)	$\nabla_4 y_t$	.216 (4.56)	∇y_t	.252 (4.78)
$A1\nabla w_t$	-.249 (-5.73)	$A1\nabla w_t$	-.207 (-4.39)	∇w_t	-.115 (-5.76)
$\nabla\nabla_4 p_t$	-.321 (-3.35)	$\nabla\nabla_4 p_t$	-.379 (-3.45)	$\nabla\nabla p_t$	-.427 (-4.76)
ECM_{t-4}	-.358 (-10.76)	$S(L)ECM_{t-1}$	-0.95 (-8.08)	ECM_{t-1}	-.135 (-7.85)
p.e.v.	9.00×10^{-5}		11.73×10^{-5}		9.13×10^{-5}
σ_ε^2	-		-		4.80×10^{-6}
σ_ω^2	-		-		1.72×10^{-5}
R^2	.725		.642		.994
Normality	.449		.540		2.342
$H(42)$	1.114		.814		.714
$Q(1)$	6.513*		4.393*		-
$Q(2)$	7.750*		5.027		3.380
$Q(3)$	8.058*		5.030		3.686
$Q(4)$	9.343*		6.785		6.454
$Q(8)$	15.597*		14.745		9.897

Explanatory variables for each regression are listed in columns 1,3, and 5. The dependent variables are shown at the head of the table in line 2. t-statistics are shown in parrentheses. The normality test is the Bowman -Shenton test, distributed approximately as χ_2^2, and *H(m)* is a heteroscedasticity test distributed approximately as $F_{m,m}$. *Q(P)* denotes the Box-Ljung Q statistic based on the first P residual autocorrelations. Note that the coefficients of determination, R^2, are not directly comparable for dependent variables in first and fourth differences. A * denotes significance at the 5% level.

Table 5. ECMs for Consumption Growth
1961:1 - 1995:4

ECM $\nabla_4 c_t$		**ECM /AGG** $\nabla_4 c_t$		**ECM /SEAS** ∇c_t	
$\nabla_4 y_t$	.304 (7.94)	$\nabla_4 y_t$	.220 (5.03)	∇y_t	.247 (5.08)
$A1\nabla w_t$	-.256 (-6.11)	$A1\nabla w_t$	-.210 (-4.58)	∇w_t	-.115 (-5.99)
$\nabla\nabla_4 p_t$	-.308 (-8.32)	$\nabla\nabla_4 p_t$	-.370 (-3.48)	$\nabla\nabla p_t$	-.419 (-4.83)
ECM_{t-4}	-.350 (-11.29)	$S(L)\ ECM_{t-1}$	-.092 (-8.58)	ECM_{t-1}	-.132 (-8.37)
p.e.v.	8.70×10^{-5}		11.20×10^{-5}		8.75×10^{-5}
σ_ε^2					4.17×10^{-6}
σ_ω^2					1.64×10^{-5}
R^2	.722		.641		.994
Normality	.303		.633		3.088
H(46)	.877		.629		.641
Q(1)	7.484**		5.151*		-
Q(2)	9.010*		5.755		3.478
Q(3)	9.313*		5.755		3.705
Q(4)	10.364*		7.801		6.550
Q(8)	17.623		16.365*		9.067

See note under Table 1 for a description of test statistics.

Table 6. Equation Stability

(a) 1961:1 - 1980:4

Test Statistic	ECM	ECM/AGG	ECM/SEAS
$Q(1)$	0.0119	0.1536	-
$Q(2)$	5.4364	2.5241	4.1476*
$Q(3)$	5.9841	3.3703	6.5661*
$Q(4)$	7.1759	4.1190	8.8716*
$Q(8)$	13.1925	11.2865	12.8333
p.e.v.	8.7888×10^{-5}	13.2232×10^{-5}	10.6728×10^{-5}
σ^2_ω			2.3401×10^{-5}
σ^2_ε			0
Chow Test	0.8934 [.60]	0.5755 [.91]	0.7204 [.79]
Parameter Stability	1.5816 [.09]	0.7678 [.74]	0.5864 [.91]

(b) 1961:1 - 1985:4

Test Statistic	ECM	ECM/AGG	ECM/SEAS
$Q(1)$	2.2347	1.1211	-
$Q(2)$	4.6565	2.0669	3.3309
$Q(3)$	4.8688	2.5579	4.6310
$Q(4)$	5.6902	3.2907	6.8059
$Q(8)$	15.0202*	12.2189	10.0224
p.e.v.	8.5984×10^{-5}	11.9028×10^{-5}	9.2015×10^{-5}
σ^2_ω			1.9017×10^{-5}
σ^2_ε			1.7203×10^{-6}
Chow Test	1.2557 [.23]	0.8999 [.59]	0.6827 [.83]
Parameter Stability	1.4606 [.12]	1.3519 [.17]	1.2828 [.22]

Table 6 (continued)

(c) 1961:1 - 1990:4

Test Statistic	ECM	ECM/AGG	ECM/SEAS
$Q(1)$	6.3621*	4.0165*	-
$Q(2)$	8.2876*	5.1782	4.1017*
$Q(3)$	8.6423*	5.2319	4.2904
$Q(4)$	10.6359*	7.4192	7.1410
$Q(8)$	15.9218*	15.0597*	10.2212
$p.e.v.$	9.3863×10^{-5}	12.6336×10^{-5}	9.8280×10^{-5}
σ_{ω}^{2}			1.7985×10^{-5}
σ_{ε}^{2}			$6.2802 \quad 10^{-6}$
Chow Test	1.5255 [.09]	1.3458 [.17]	1.3636 [.16]
Parameter Stability	0.5700 [.92]	0.4228 [.98]	0.4009 [.99]

Figure 1. Consumption and Income
(natural logarithms)

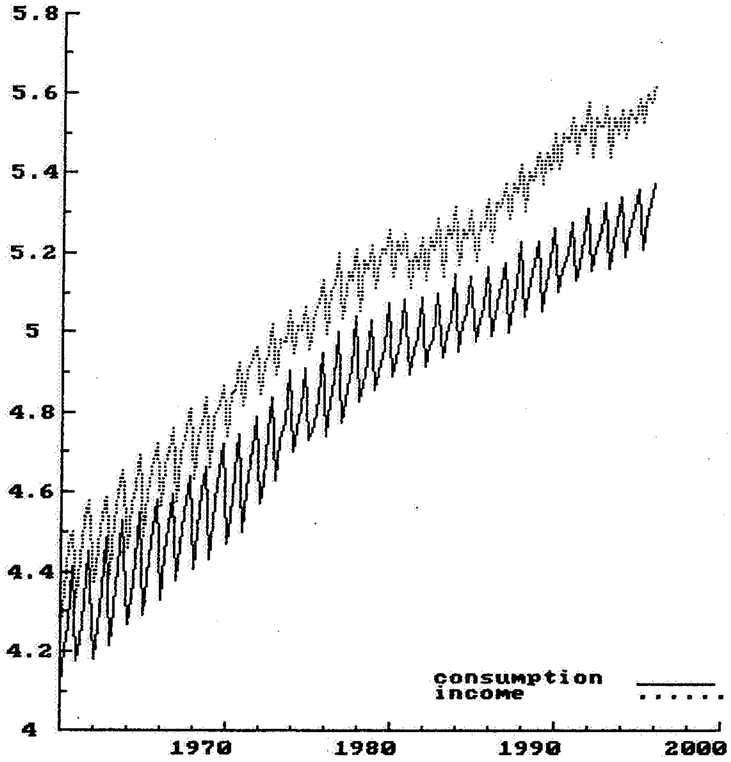

Figure 2a. Seasonal Component of Consumption by Quarter

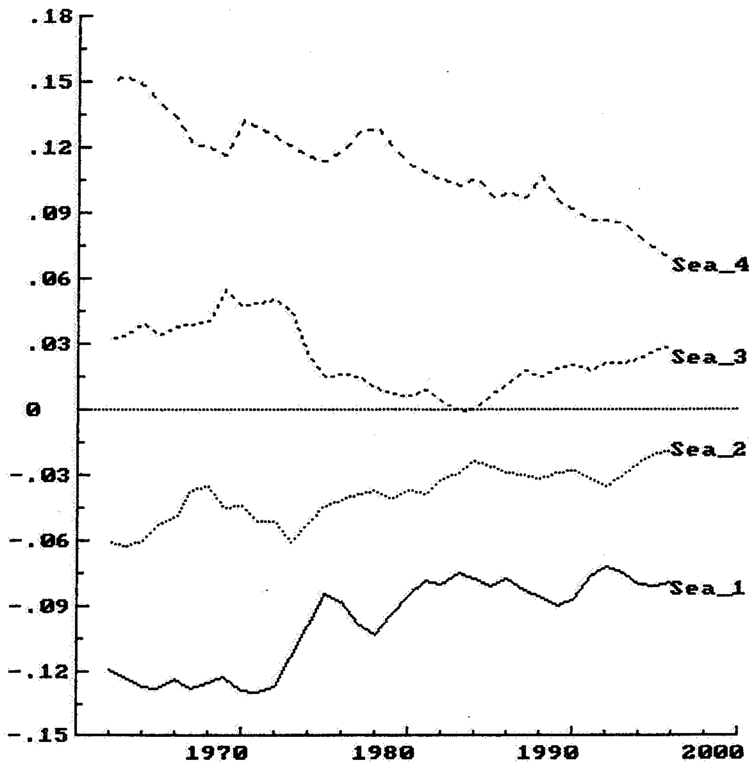

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Figure 2b. Seasonal Component for Disposable Income by Quarter

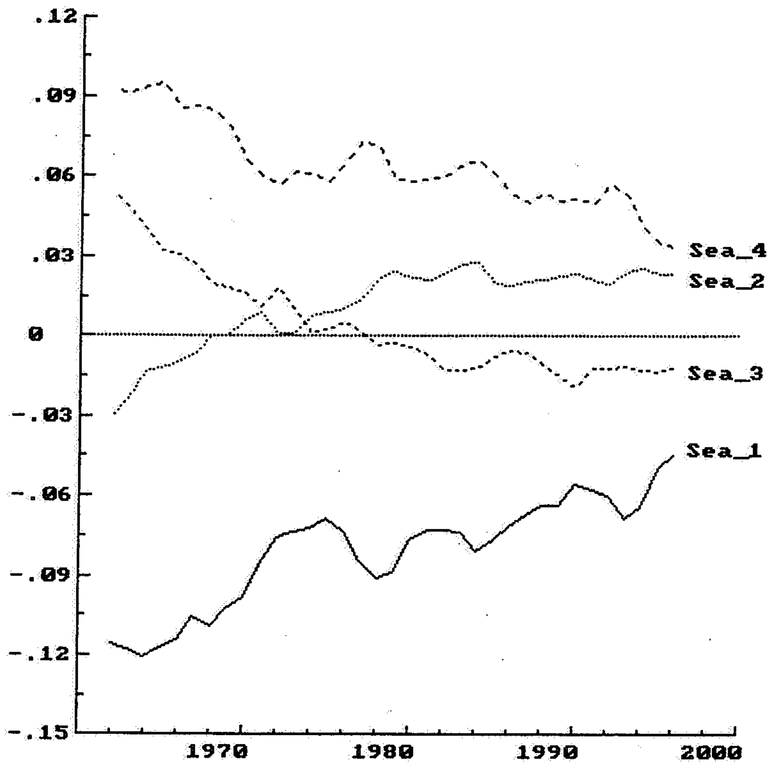

Figure 3. Basic Structural Model for Consumption

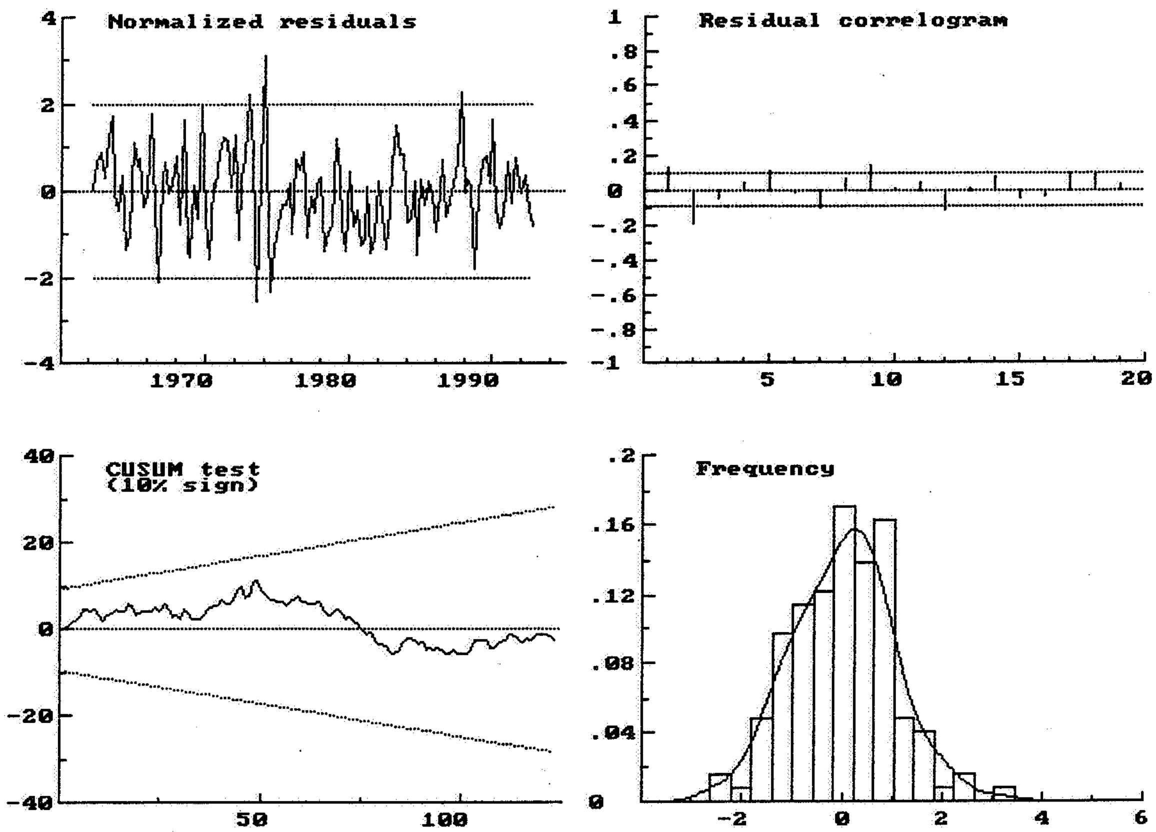

Figure 4. Unobserved Components for Consumption

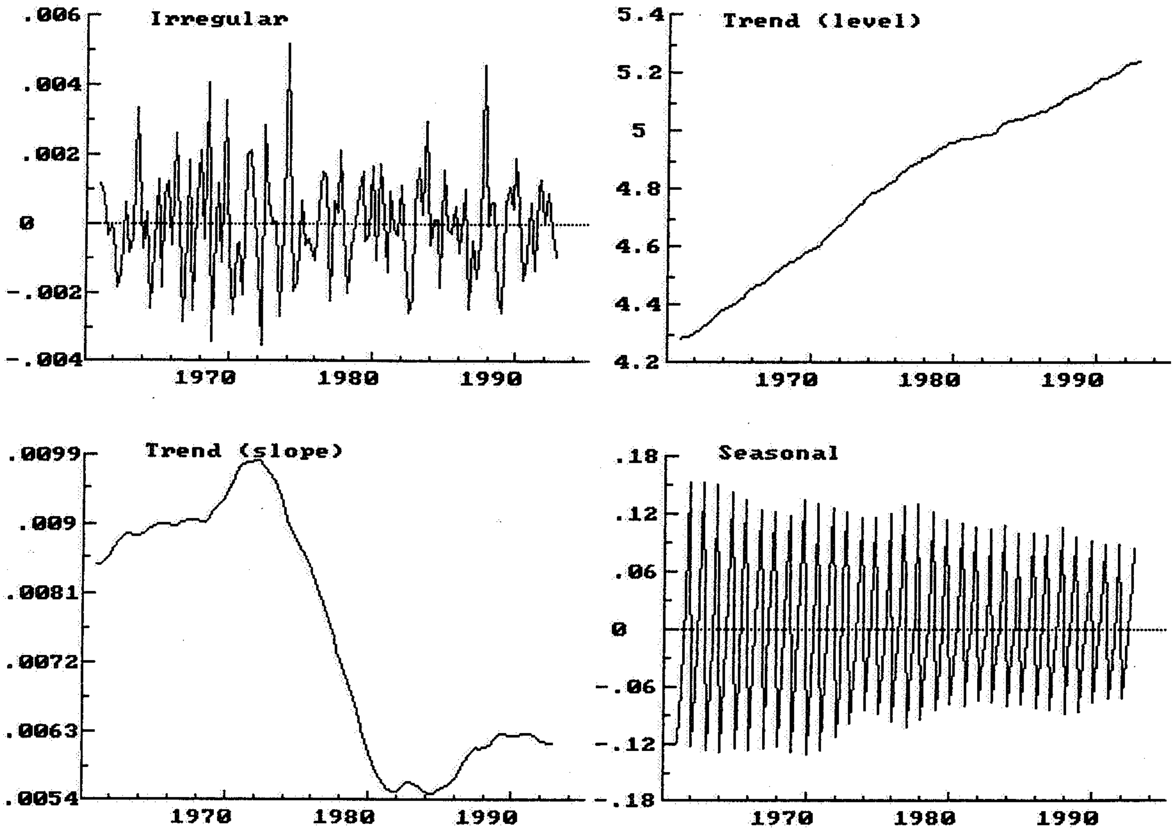

Figure 5a. One-Step Ahead Forecasts for Consumption

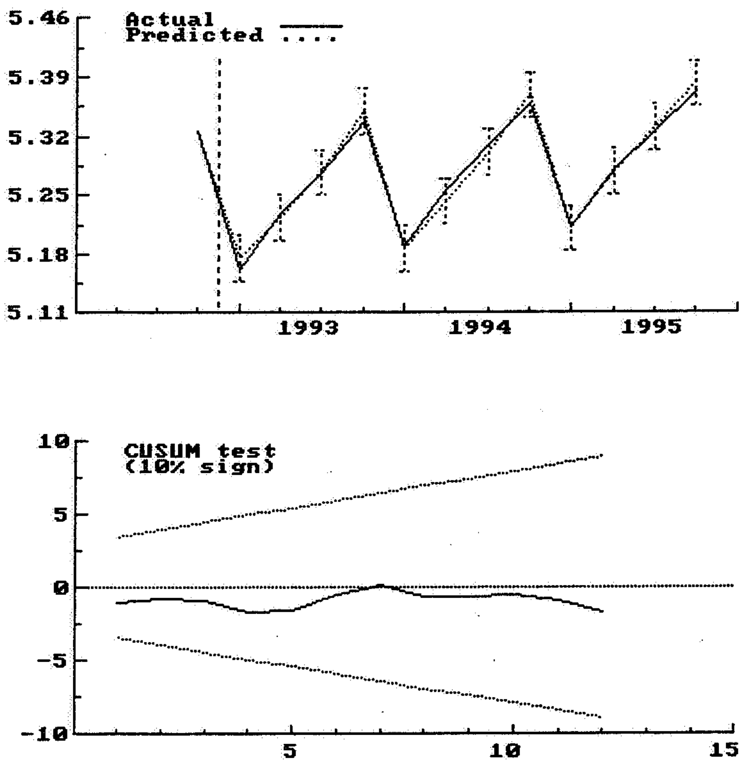

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Figure 5b. Extrapolations for Consumption

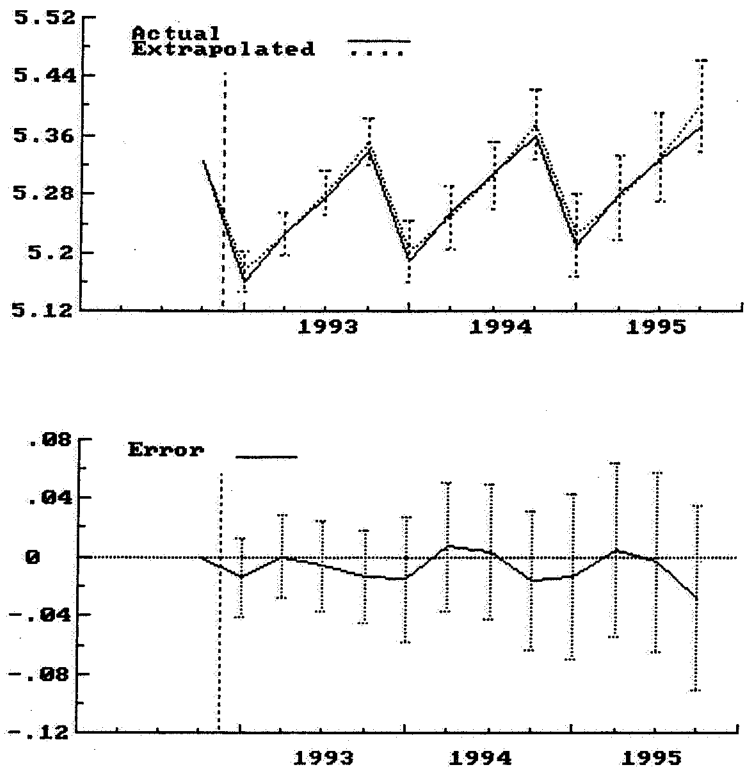

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Figure 6. One-Step Ahead Forecasts for Disposable Income

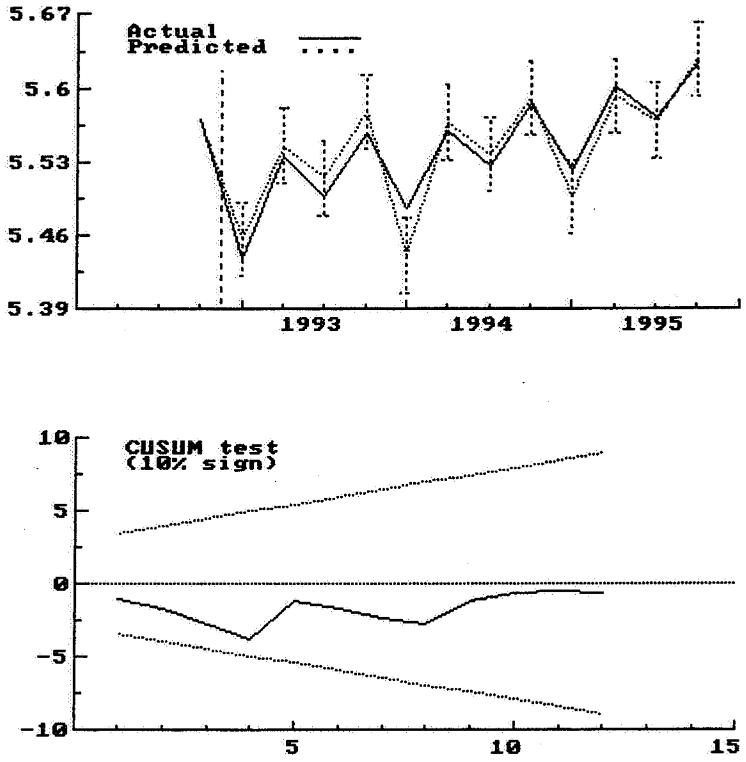

Figure 7. Unobserved Components for Disposable Income

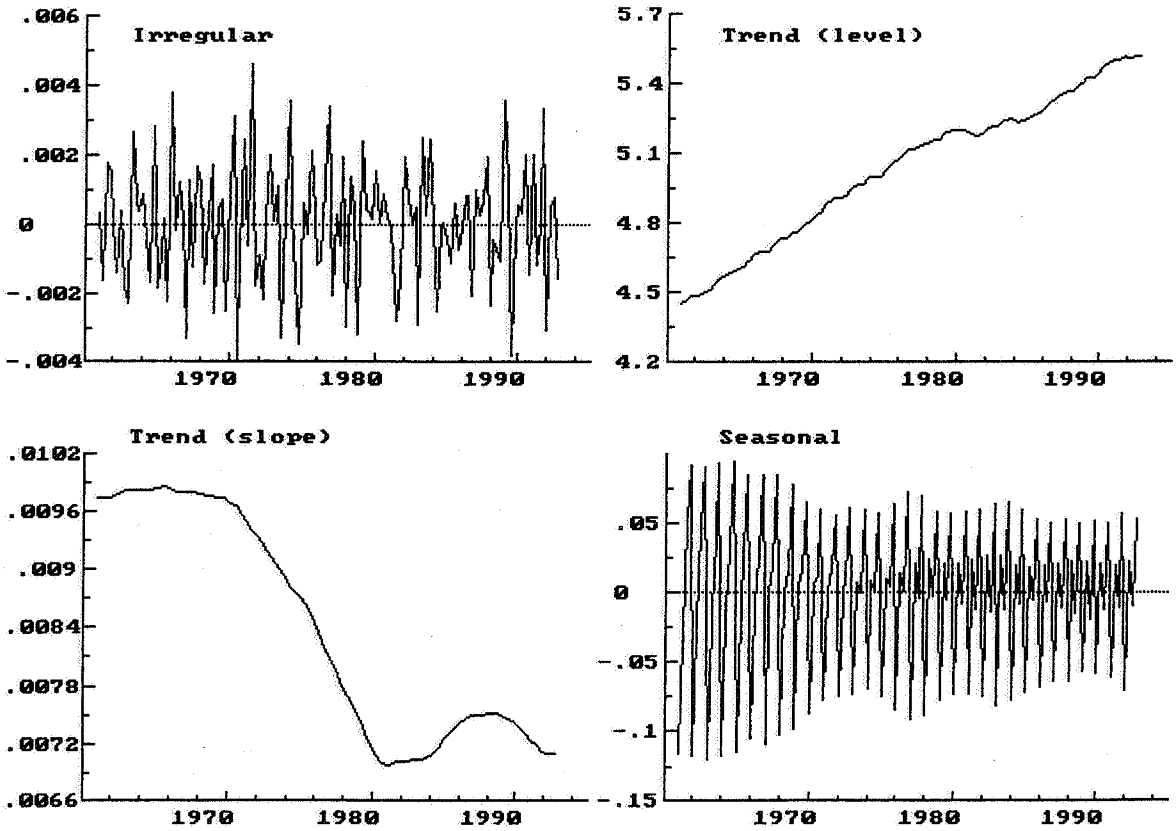

Figure 8a. One-Step Ahead Forecasts for Disposable Income

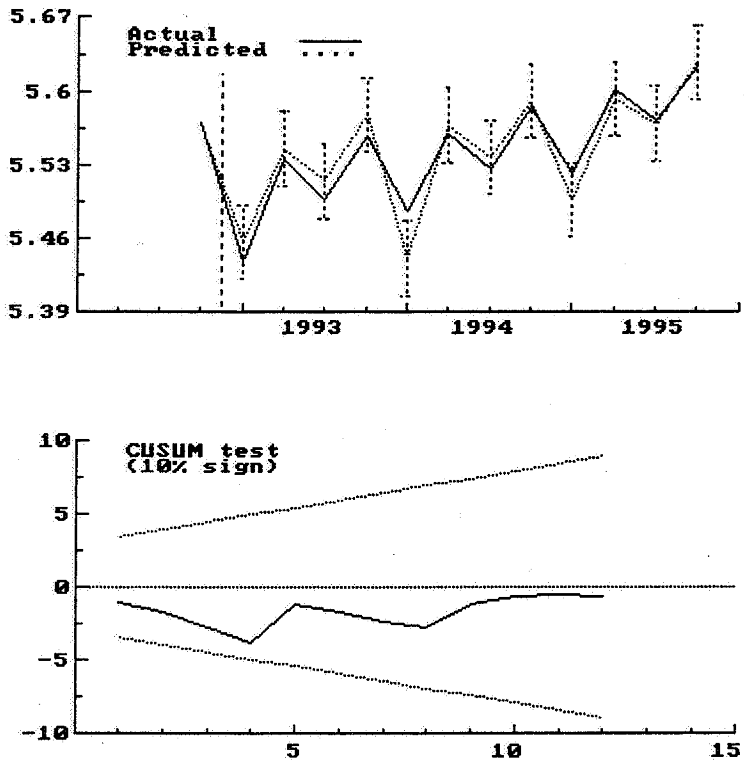

Figure 8b. Extrapolations for Disposable Income

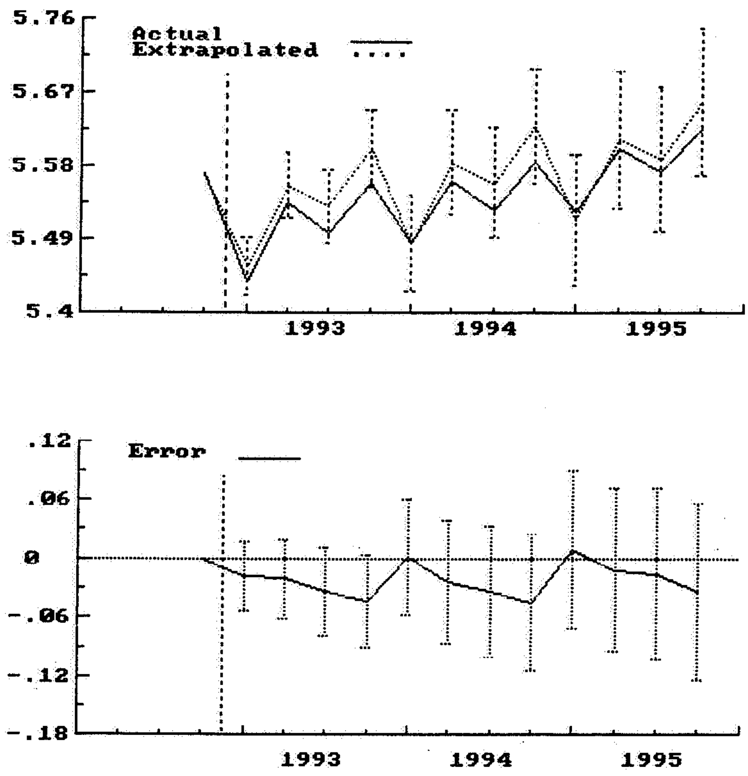

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Figure 9a. Bivariate Model for Consumption and Income

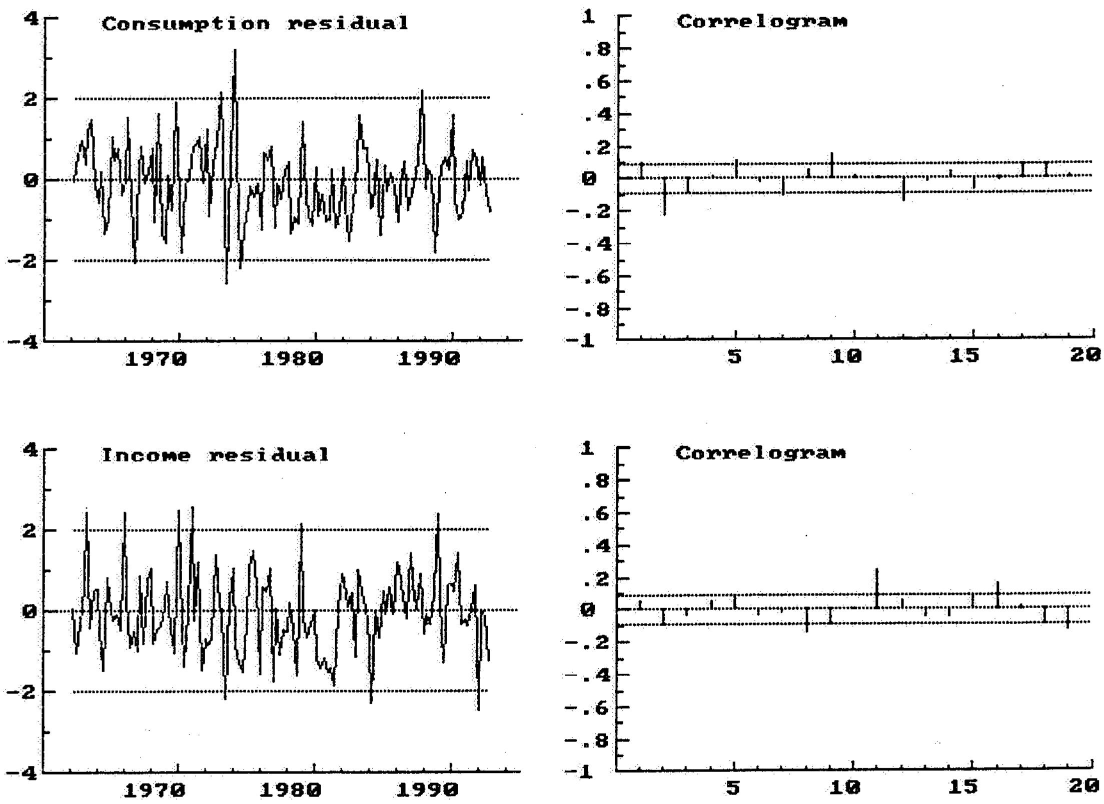

Figure 9b. Bivariate Model for Consumption and Income

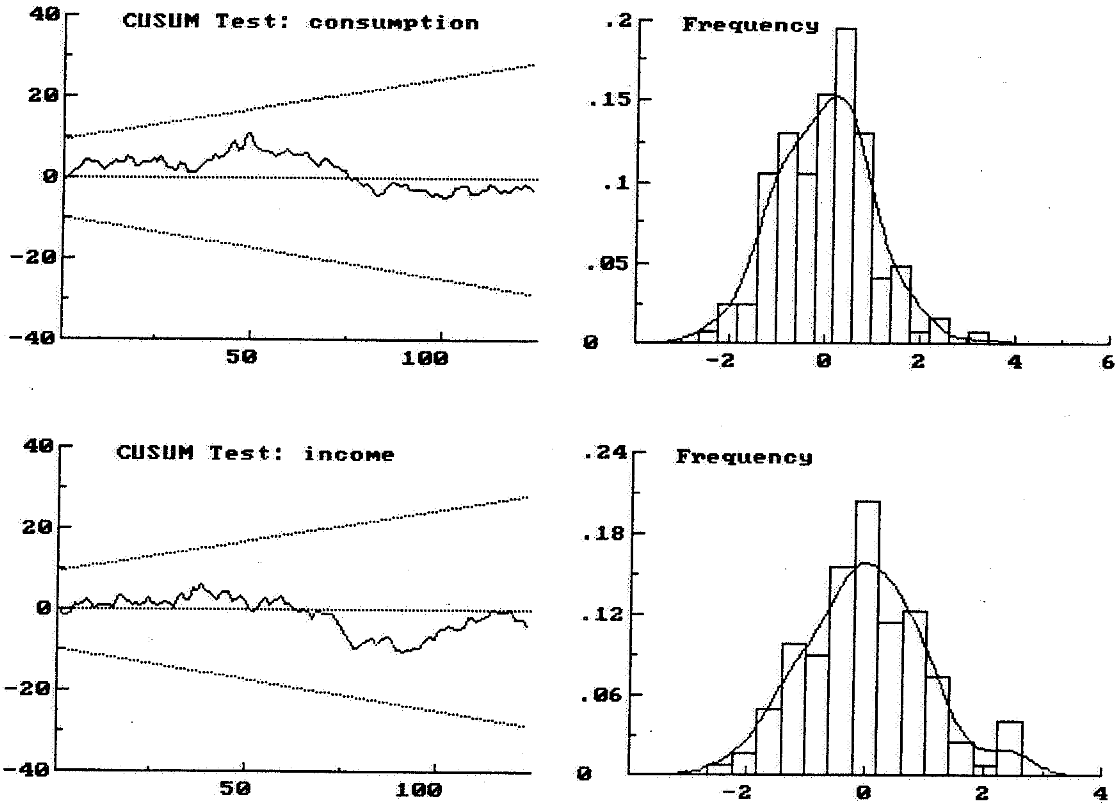

Figure 10a. Components of Consumption from a Bivariate Model

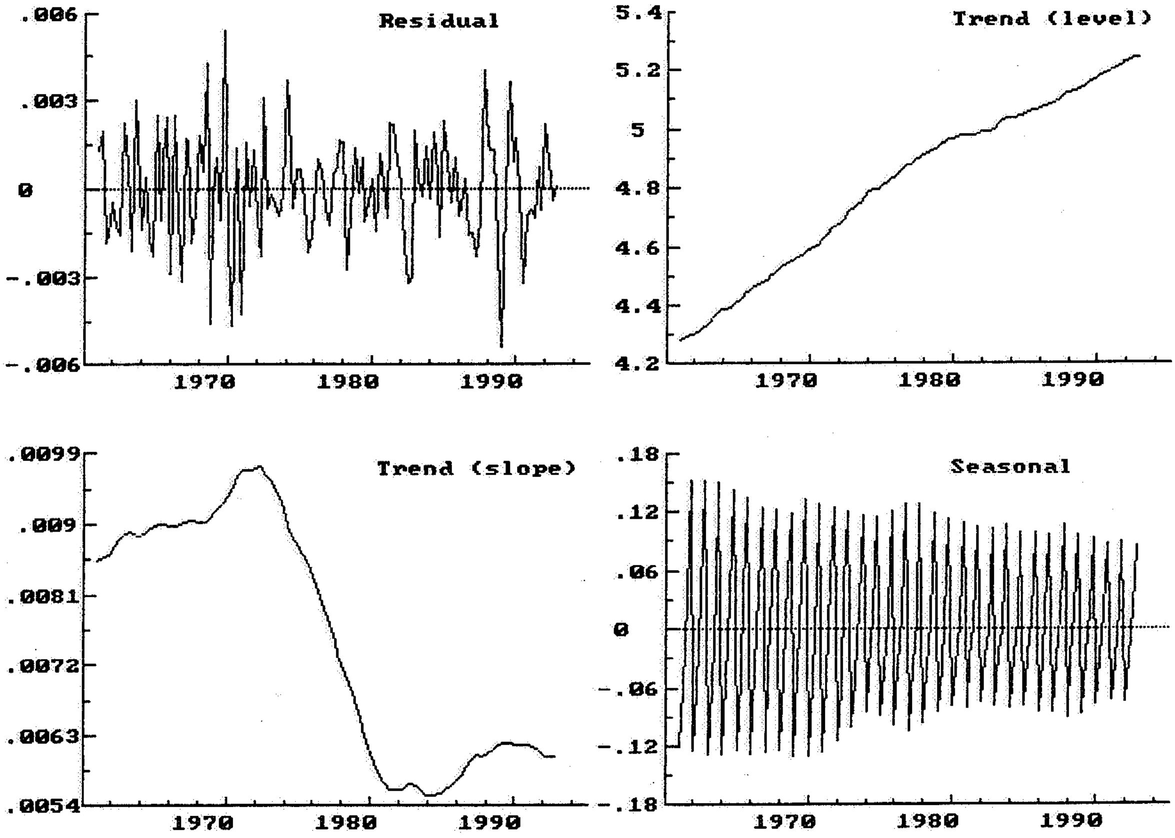

Figure 10b. Components of Income from a Bivariate Model

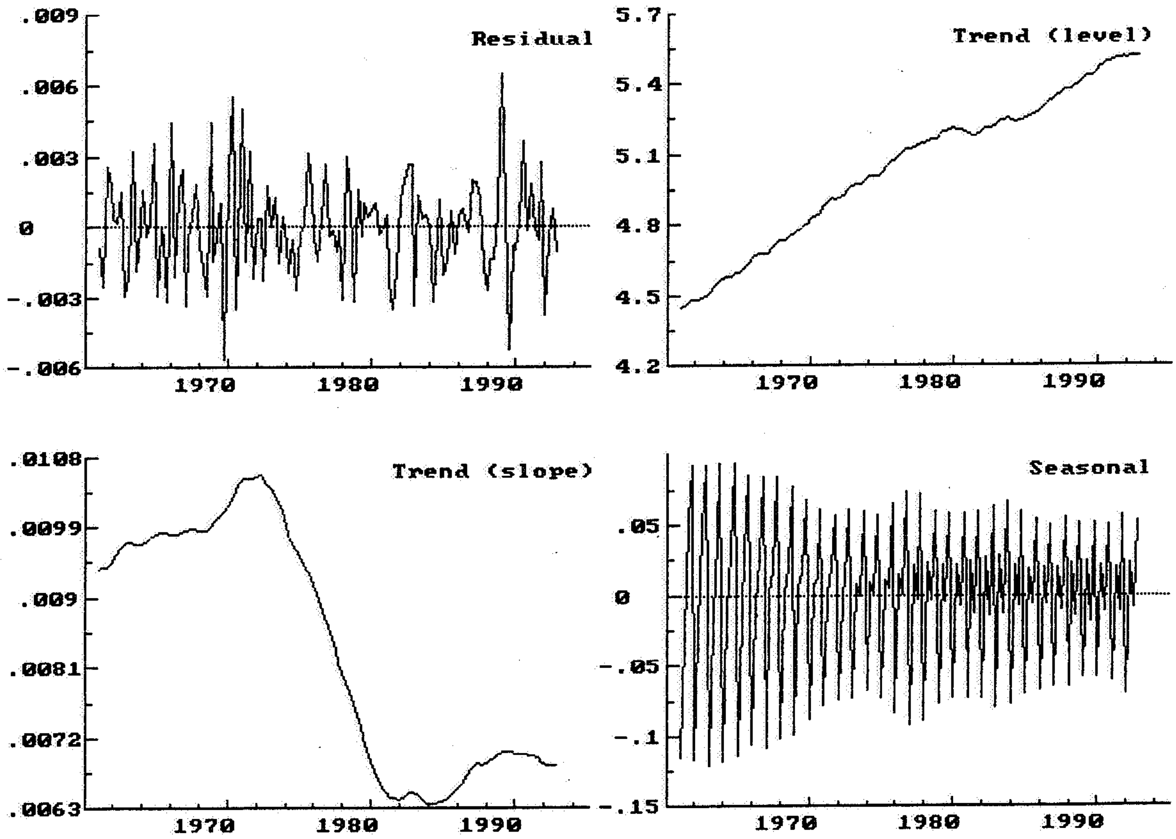

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Figure 11a. One-Step Ahead Forecasts for Consumption and Income

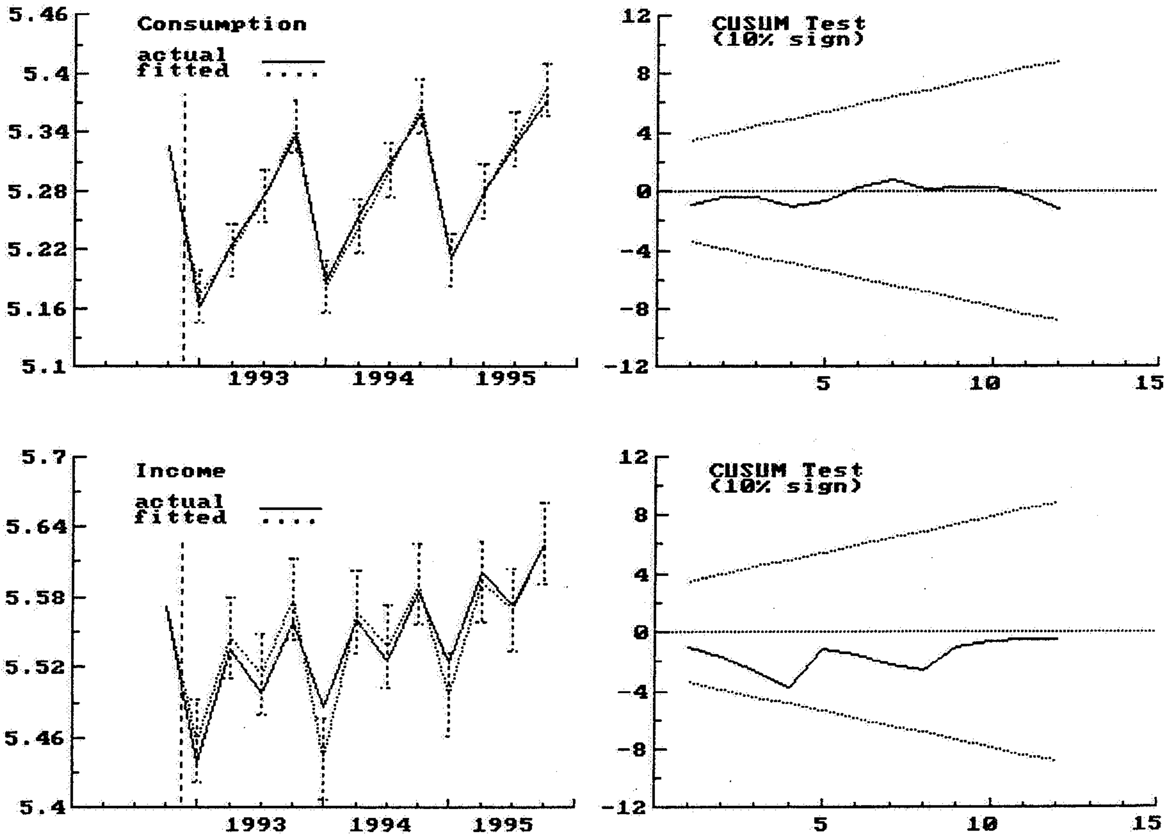
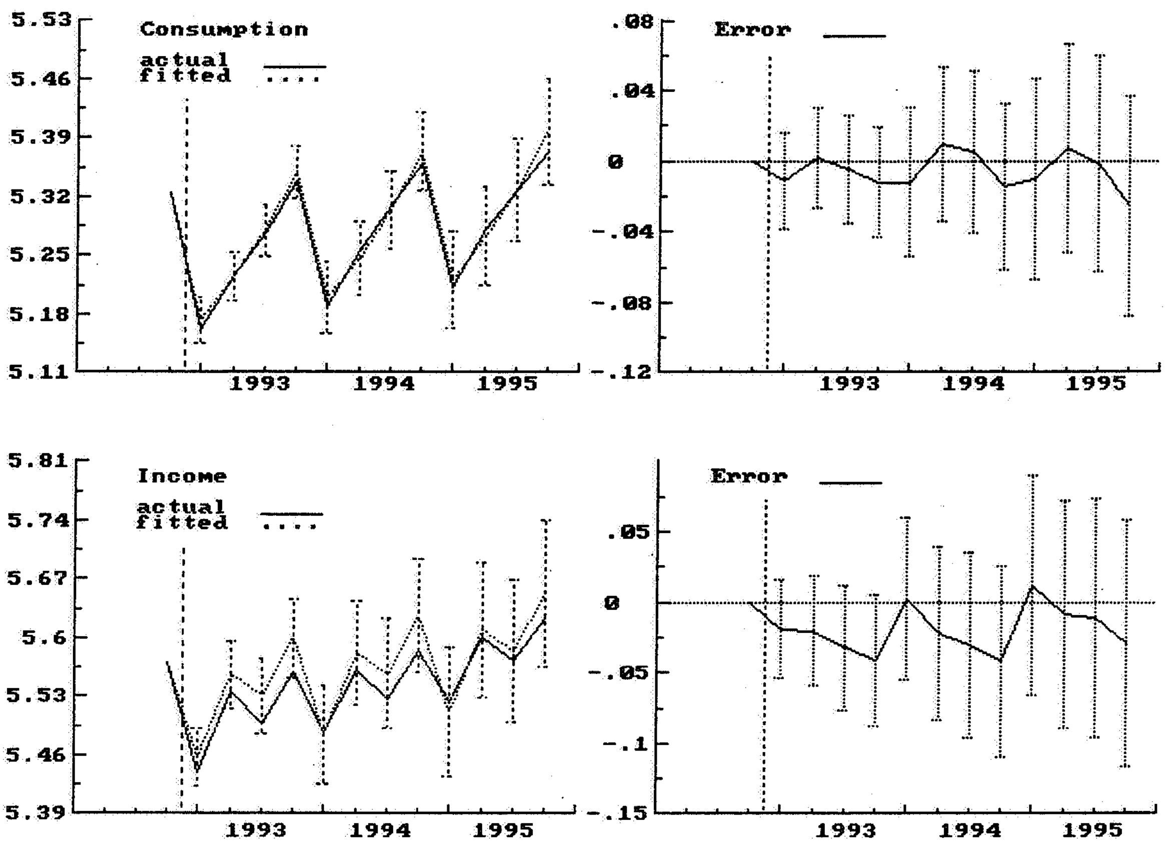


Figure 11b. Extrapolations of Consumption and Income



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