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Following *Hendry – Doornik* (1994), *Hendry – Mizon* (1993) and *Hendry – Neale – Srba* (1988) we formulate a linear dynamic 3-equation model for consumption of nondurables and services, disposable income and inflation rate in Austria. Applying a system approach results in a heavier modeling burden than specification of single equations. In order to cope with this problem, we use PcFiml 8 (see *Doornik – Hendry,* 1994) as a modeling tool.

1. Linear Dynamic Systems

Restricting the analysis to linear models, the basic form of the joint density function is a vector autoregressive representation with deterministic variables. Assuming a maximum lag length of ℓ periods, it can be written as

$$(1) \qquad \mathbf{x}_t = \sum_{j=1}^{\ell} \Pi_j \mathbf{x}_{t-j} + \mathbf{F}\,\mathbf{q}_t + \varepsilon_t ,$$

where $\mathbf{x}_t \approx \mathrm{IN}\,[0, \Omega]$.

In equation *(1)* $\mathbf{x}_t$ is a vector of n observable real random variables $\mathbf{x}_t = (\mathbf{x}_{1\,t}, \ldots, \mathbf{x}_{n\,t})'$ and $\mathbf{q}_t$ a set of m deterministic conditioning variables (such as intercept, seasonals, trend and zero/one dummy variables). $\mathrm{IN}\,[0, \Omega]$ denotes an independent normal density with mean zero and covariance matrix Ω. We assume further that the parameters $(\Pi_1, \ldots, \Pi_\ell, \mathbf{F}, \Omega)$ are constant in the postulated representation.

When the data $\{\mathbf{x}_t\}$ are $I\,(1)$, a useful reformulation of the above system is to error correction form (see *Hendry – Pagan – Sargan,* 1984, *Engle – Granger,* 1987, *Johansen,* 1988, and *Banerjee et al.,* 1994):

(2) $$\Delta \mathbf{x}_t = \sum_{j=1}^{\ell-1} \Pi_j^* \Delta \mathbf{x}_{t-j} + \pi \mathbf{x}_{t-1} + \mathbf{F} \mathbf{q}_t + \varepsilon_t .$$

No restrictions are imposed by this transform. However, when $\mathbf{x}_t$ is $I(1)$ then $\Delta \mathbf{x}_t$ is $I(0)$ and the system specification is balanced only if $\pi \mathbf{x}_t$ is $I(0)$. It follows that π cannot be full rank, so let rank $(\pi) = r < n$. Then $\pi = \alpha \beta'$ where α and β are $n \times r$ matrices of rank r, and $\beta' \mathbf{x}_t$ must contain r cointegrating $I(0)$ relations allowing the restricted $I(0)$ representation:

(3) $$\Delta \mathbf{x}_t = \sum_{j=1}^{\ell-1} \Pi_j^* \Delta \mathbf{x}_{t-j} + \alpha (\beta' \pi \mathbf{x}_{t-1}) + \mathbf{F} \mathbf{q}_t + \varepsilon_t .$$

2. Empirical Results

In the following we apply a general to specific research strategy. An excellent summary of the advantages of this approach can be found in *Hendry – Doornik* (1994).

2.1 The Variables and their Univariate Time Series Properties

For our planned application, the vector $\mathbf{x}_t$ is given by standard consumption theory as

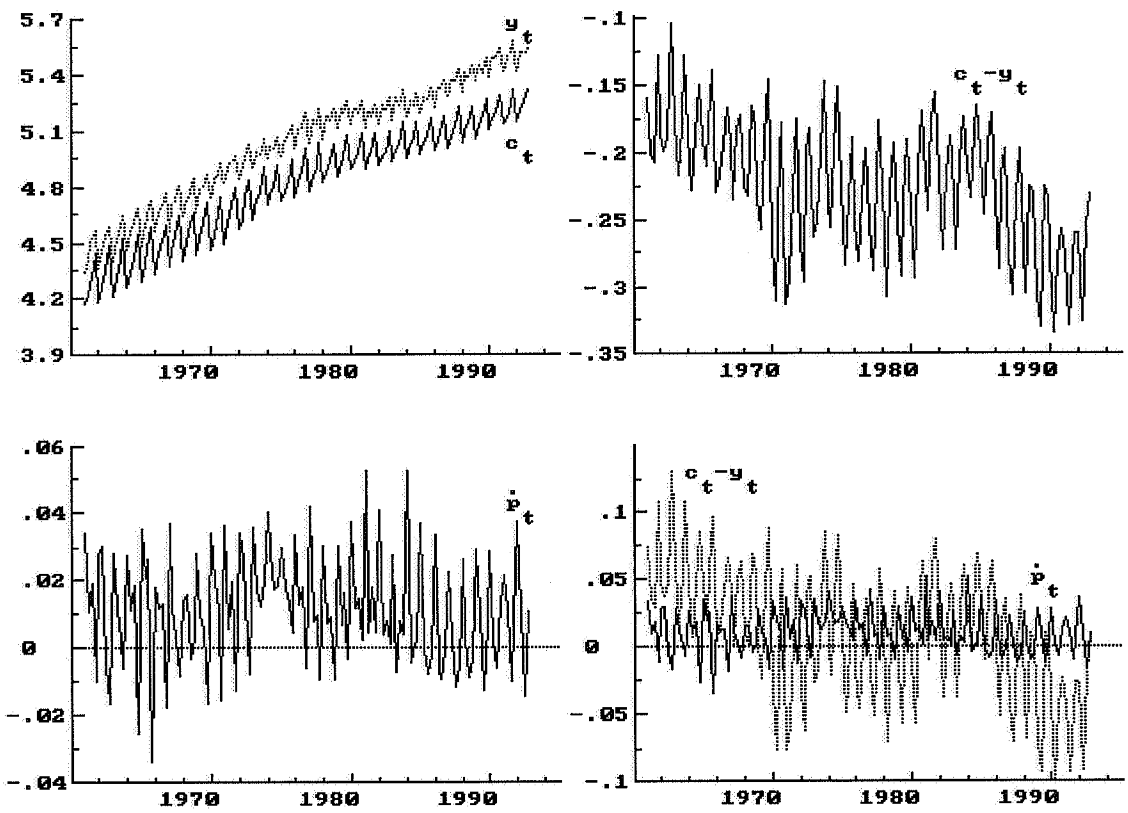
(4) $$\mathbf{x}_t = (c_t, y_t, p_t)',$$

where c_t is the log of real consumer expenditure on nondurables and services, y_t is the log of real disposable income, and p_t denotes the log of the implicit consumption deflator. It is shown in *Maravall* (1993) that, in general, seasonally adjusted data will not give inverible AR or VAR models. Therefore, we use seasonally unadjusted data. The sample period covers the time span 1960:1 to 1992:4. In a previous study (*Thury – Wüger,* 1994) we found out that it is better to replace the log level of the price index by its rate of change. Thus, we work here with $\dot{p}_t = \Delta p_t$ and address it as inflation rate in the following. Figure 1 shows the graphs of these basic variables. This graph is only meant to give a rough impression of their development over time. We note that income and consumption are trending together. Thus, the possibility of cointegration cannot be ruled out. We see further that income and consumption are strongly seasonal whereby, above all, the seasonal pattern in consumption seems to be very stable.

Visual inspection of the data can be quite helpful at the outset of an analysis, but should be replaced by formal testing later on. Therefore, we turn now to an analysis of the univariate time series properties of our basic variables. Since the publication of the seminal paper by *Nelson – Plosser* (1982) the problem of nonstationarity in economic time series has been receiving steadily increasing attention in applied research. However, the nonstationarity tests, which were proposed in the relevant literature, search only for a unit root corresponding to a zero-frequency peak. In addition, it was assumed that there are no other unit roots present in the series. This assumption may be too restrictive for economic time series which often exhibit strong seasonality. To determine the degree of integration for such series, it is mandatory to amend the standard terminology. Following *Osborn et al.* (1988) we call a series $\mathbf{x}_t$ to be integrated of order (d, D), denoted by $\mathbf{x}_t \approx I(d, D)$, if it becomes stationary after first differencing d times and seasonally differencing D times.

Figure 1

Basic Variables



Seasonality may have both deterministic and stochastic components. We assume that these components can be separated. Thus, if the log of the observed series is y_t, we write

$$(5)\quad y_t = \mathbf{x}_t + k_q ,$$

where $\mathbf{x}_t$ is purely stochastic and k_q is the deterministic component for season q. For quarterly data, the deterministic seasonality is removed by a prior regression of the log level series on four quarterly dummy variables. The residuals from this regression are then treated in the subsequent analysis as if they were the true $\mathbf{x}_t$.

Table 1

Testing Orders of Integration

Test	c	y	p	$\dot{p}$	$c-y$	5 percent critical values
ADF (ℓ)	– 0.54 (8)	– 1.00 (5)	– 0.52 (8)	– 3.12 (3)	– 2.47 (4)	– 2.89
DHF (ℓ)	– 1.34 (7)	– 3.44 (5)	– 1.41 (5)	– 4.70 (4)	– 4.30 (8)	– 4.11
EGHY						
't' π_1 (ℓ)	– 0.34 (7)	– 0.66 (5)	– 0.49 (5)	– 3.28 (0)	– 1.98 (3)	– 2.96
't' π_2	– 0.84	– 2.52	– 1.15	– 2.64	– 2.02	– 2.95
't' π_3	– 1.05	– 1.66	– 0.81	– 4.42	– 3.68	– 3.51
HF (ℓ)	1.03 (8)	2.83 (5)	1.35 (4)	22.32 (6)	18.75 (4)	3.56
OCSB						
't' β_1	– 0.35	– 0.14	0.23	– 4.41	– 3.70	– 1.83
't' β_2	– 0.75	– 1.99	– 1.43	– 3.92	– 3.94	– 2.03

Legend:
ADF . . . Augmented Dickey-Fuller,
DHF . . . Dickey-Hasza-Fuller,
EGHY . . . Engle-Granger-Hylleberg-Yoo,
HF . . . Hasza-Fuller,
OCSB . . . Osborn-Chui-Smith-Birchenhall.

Numerous tests for the order of integration have been proposed in the literature. Since all of them have their advantages, but also their shortcomings, we employ a whole battery of such tests. The value of the lag length ℓ is chosen as follows. We start out with $\ell = 8$ and drop insignificant coefficients, because including too many lags will reduce the power of the tests. The outcome of these tests is presented as Table 1. It is far less controversial than might have been expected according to foreign experiences. With one or two exceptions, all tests produce identical results. Consumption, income, and the implicit consumption deflator are all unambiguously $I\,(1, 1)$. Somewhat surprisingly, taking first differences of this deflator gives an $I\,(0, 0)$ series. The outcome for the consumption/income ratio is also slightly controversial. ADF and EGHY statistic indicate the presence of a regular unit root in $(c_t - y_t)$, while the other tests classify it as $I\,(0, 0)$. Since it is known that unit root tests are not powerful against "nearly stationary" alternatives, it seems legitimate to assume that it is $I\,(0, 0)$, indeed. The conclusion that the average propensity to consume is nonstationary, would be too uncomfortable from an economic point of view.

Table 2 summarizes the classification for the basic variables. These results imply that consumption cannot move far away from income. Thus, the possibility of cointegration in the long run is not ruled out. We shall analyze this aspect systematically later on.

Table 2

Orders of Integration

Variable	Order of Integration
c	$I(1, 1)$
y	$I(1, 1)$
p	$I(1, 1)$
$\dot{p}$	$I(0, 0)$
$c-y$	$I(0, 0)$

Table 3

VAR Order Criteria

Order ℓ	Final Prediction Error	Akaike	Hannan-Quinn	Schwarz
0	86,010.82	– 1.8629	– 1.8629	– 1.8629
1	35.4309	– 2.6423	– 2.6287	– 2.6087
2	13.6485	– 2.7378	– 2.7105	– 2.6705
3	4.7717	– 2.8430	– 2.8020	– 2.7422
4	0.2216	– 3.1503	– 3.0956	– 3.0158
5	0.0897*	– 3.2412*	– 3.1729*	– 3.0731*
6	0.0978	– 3.2331	– 3.1511	– 3.0314
7	0.1024	– 3.2294	– 3.1338	– 2.9941
8	0.1008	– 3.2321	– 3.1228	– 2.9632

* . . . minimum.

2.2 *The Initial System*

We begin with an analysis of a system in the three stochastic variables $(c_t, y_t, \dot{p}_t)$ with a large number of deterministic variables, such as an intercept, seasonals, an Easter dummy and several fiscal policy dummies. For choosing the maximal length ℓ in the VARs several information criteria are computed: final prediction error, Akaike, Hannan-Quinn, and Schwarz. The outcome is given in Table 3. All four criteria indicate that a length of $\ell = 5$ should be sufficient. For quarterly, seasonally unadjusted data this seems to be an appropriate choice.

Table 4

Residual Correlations

	c	y	$\dot{p}$
y	0.43	–	–
$\dot{p}$	–0.17	–0.17	–

Lag Length and Dynamics

	c	y	$\dot{p}$
$F_{s=1}$ (3, 102)	3.04*	10.94**	29.83**
$F_{s=2}$ (3, 102)	0.85	0.18	3.26
$F_{s=3}$ (3, 102)	5.76**	0.22	0.42
$F_{s=4}$ (3, 102)	23.49**	15.13**	2.87*
$F_{s=5}$ (3, 102)	1.52	3.63*	3.86
$\lvert\lambda_a\rvert$	0.50	0.02	0.02
$\lvert\lambda_b\rvert$	0.86	0.86	0.78
	0.98	0.98	0.86
	0.86	0.60	0.60
	0.01	0.60	0.60
	0.92	0.92	0.99

Table 5

Goodness of Fit and Evaluation

Statistic	c	y	$\dot{p}$	VAR
$\hat{\sigma}$	1.22%	1.76%	0.80%	
F_{ar} (5, 99)	0.47	0.25	1.26	
F_{arch} (4, 96)	1.14	0.47	0.57	
F_{het} (30, 73)	0.79	1.56	0.85	
(χ^2_{nd}) (2)	0.34	2.27	4.76	
F^V_{ar} (45, 259)				1.02
F^V_{het} (180, 409)				0.94
(χ^{2V}_{nd}) (6)				8.45

The residual correlations and summary statistics for this system, when estimated by ordinary least squares, are reported in Tables 4 and 5. There $\hat{\sigma}$ denotes the residual standard deviation; $\lvert\lambda_a\rvert$ the modulus of the eigenvalues of the π matrix; $\lvert\lambda_b\rvert$ the modulus of the companion matrix of the dynamics; F_j (., .) denotes F-tests for the hypotheses of an i-period lag ($F_{s=i}$), no serial correlation (F_{ar}), no autoregressive conditional heteroscedasticity (F_{arch}), no heteroscedasticity (F_{het}); and a chi-square test for normality of the residuals (χ^2_{nd}). The system (vector) tests are denoted by a superscript V; * and ** indicate significance at the 5 percent and 1 percent levels. We observe three larger residual correlations: a positive one between consumption and income, and two negative ones between consumption and inflation rate and between income and inflation rate, respectively. Lag 2 is insignificant for all variables, and lag 5 is significant only for income at the 5 percent level. But reducing the lag length is decisively rejected by the data, however. The long-run matrix has one larger and two smaller eigenvalues (indicating $r = 1$). The companion matrix shows no roots on or outside the unit circle which might signal an explosive system. The F-tests and χ^2 tests are insignificant, for individual variables as well as for the system.

Figure 2

Actual and Fitted Values, their Cross Plots, and Scaled Residuals

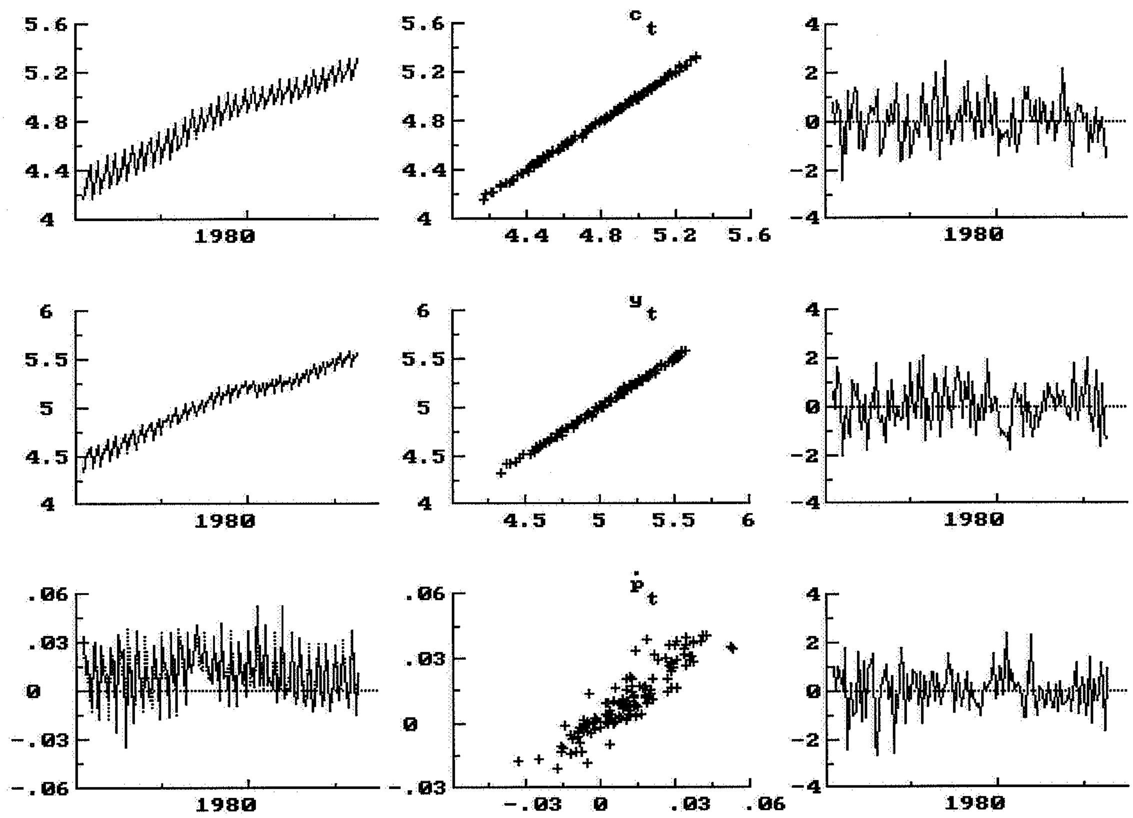
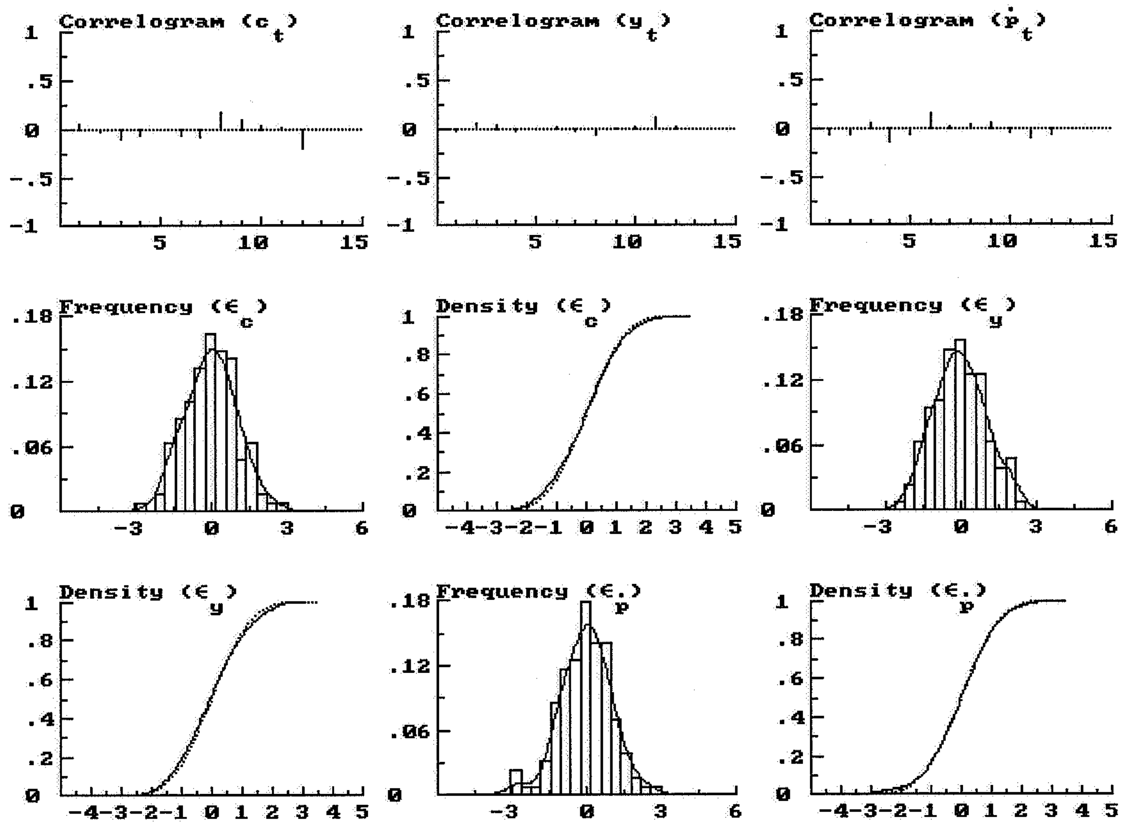


Figure 2 offers a condensed picture of the system's descriptive power, showing actual and fitted values, their cross plots, and scaled residuals. We note a significant difference in goodness-of-fit between the equations for consumption and income on the one side and that for the inflation rate on the other. But the close fits for consumption and income are a consequence of the integrated and nonstationary nature of these variables and should not be accepted as evidence for the goodness of the system representation. The residuals in the final column appear to be white noise.

Figure 3

Graphical Diagnostic Information



Some features of these residuals are investigated further in Figure 3 where residual correlograms, residual histograms, residual densities, and cumulative distributions are displayed. The white-noise nature of the residuals is loosely confirmed by the absence of any substantial autocorrelation. Normality seems a fair approximation concerning their distributional shape.

Recursive estimation provides valuable additional information, especially as far as constancy of the estimated system is concerned. The plots of 1-step residuals with ±2 $\hat{\sigma}$ in Figure 4 suggest reasonable constancy for all three equations. This impression is fully corroborated by the plots of the break-point Chow tests in Figure 5. None of these tests is significant, neither for the individual variables nor for the system. Overall, the system seems acceptable and may be used as a baseline against which restrictions can be tested.

Figure 4

System Recursive Evaluation Statistics

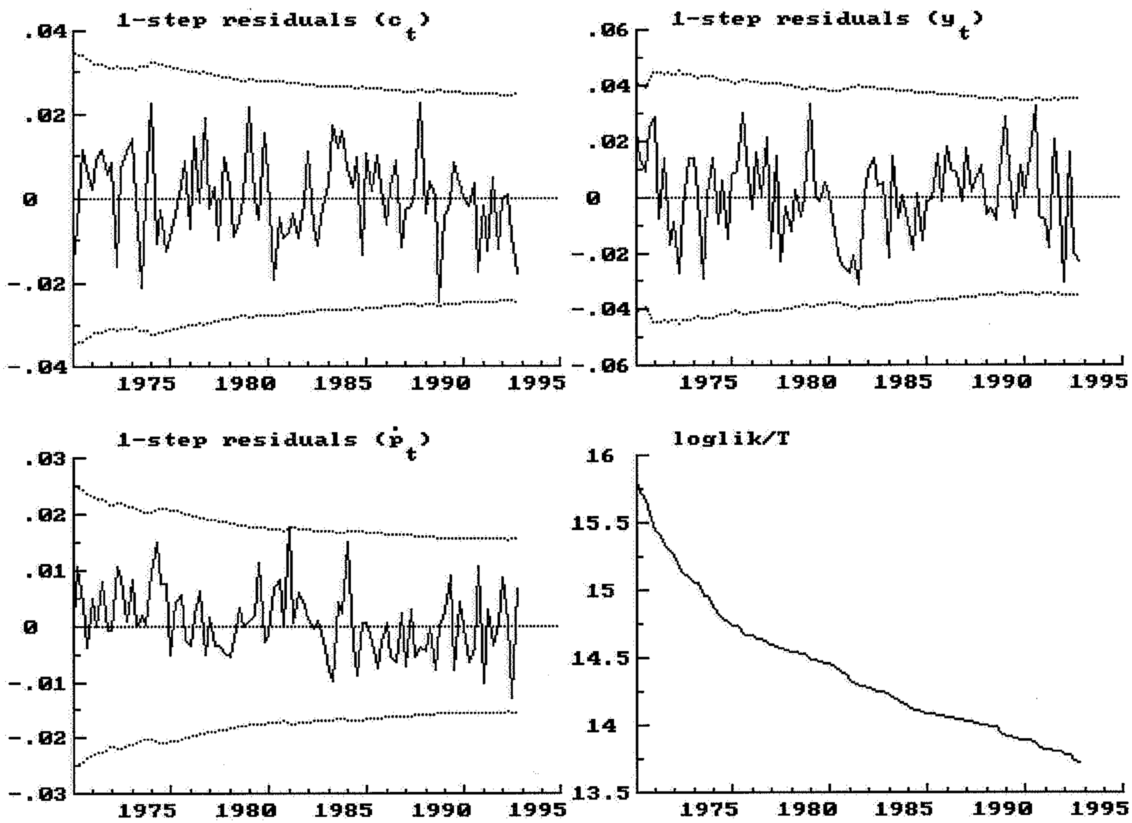
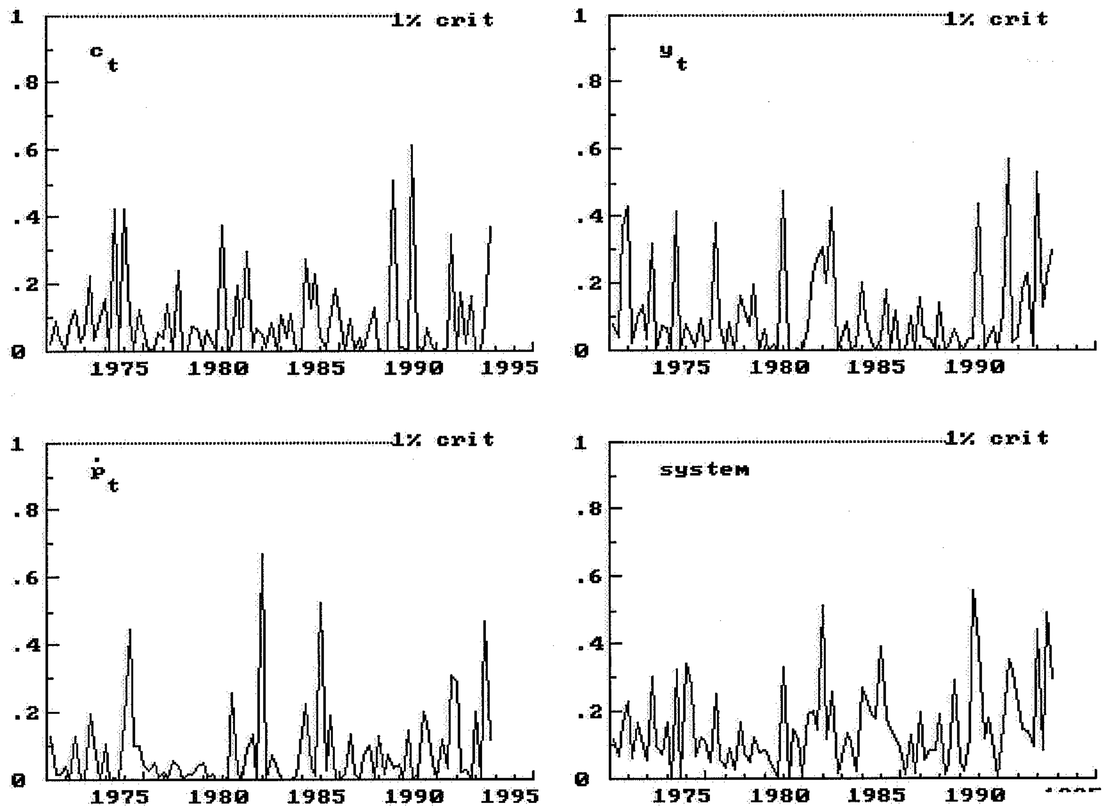


Figure 5

Break-point Chow Tests



2.3 *Cointegration Analysis*

In order to analyze cointegration properties, we apply the system-based procedures of *Johansen* (1988) and *Johansen – Juselius* (1990). These procedures are available in PcFiml 8. Their application is computationally straightforward. But, before the computations can be done, one has to decide about the treatment of regime dummies. This is a problematic aspect of the analysis which might influence the outcome significantly. We have to include a substantial number of deterministic variables: an intercept, seasonals ($Q_{i\,t}$), a trend, an Easter dummy (E_t) for calendar effects, three zero/one dummies ($D73{:}2_t$, $D73{:}4_t$, $D78{:}4_t$) to capture the effects of the introduction of VAT and other fiscal policy measures and, finally, a zero/one dummy ($D90{:}1_t$) meant to eliminate a substantial level shift in consumer spending in the first quarter of 1990. Most of these deterministic variables will only be relevant for consumer expenditure. How these variables enter the system is important. We have to develop some ideas about the likely short-run and long-run effects. The intercept, the seasonals, the Easter dummy, and $D90{:}1_t$ enter the system unrestricted. The trend must lie in the cointegration space, because there is no evidence of a quadratic trend in levels. The treatment of the fiscal policy variables, however, is an open question. After some experimentation, we find that this problem reduces to choosing the status of $D73{:}2_t$, which simultaneously captures the effect of the VAT introduction and of a substantial change in the Austrian income tax system. Whether $D73{:}2_t$ enters restricted or unrestricted has substantial consequences for the outcome. To restrict $D73{:}2_t$ to lie in the cointegration space is not implausible, because both above mentioned fiscal policy measures, the effects of which this dummy variable is supposed to capture, could have had long-run repercussions.

Table 6

Cointegration Analysis with Trend only

r	1	2	3
ℓ	1,757	1,764	1,767
μ	0.22	0.10	0.05
Max	27.7*	13.7	6.3
Tr	45.4*	20.0	6.3

$\hat{\alpha}$	1	2	3
c	– 0.49	– 0.03	0.00
y	0.07	– 0.11	0.01
$\dot{p}$	0.01	– 0.05	– 0.00

$\hat{\beta}'$	c	y	$\dot{p}$	t
1	1	– 0.84	– 0.20	0.0019
2	– 2.11	1	0.68	0.0002
3	– 7.32	– 1.30	1	0.0484

Table 7

Cointegration Analysis with Trend and $D73{:}2_t$

r	1	2	3
ℓ	1,749	1,757	1,761
μ	0.25	0.13	0.05
Max	32.2**	17.2	7.1
Tr	53.6**	24.2	7.1

$\hat{\alpha}$	1	2	3
c	– 0.20	– 0.00	0.04
y	– 0.04	0.01	0.15
$\dot{p}$	– 0.01	0.01	– 0.03

$\hat{\beta}'$	c	y	$\dot{p}$	t	*D73:2*
1	1	– 1.11	2.03	0.002	– 0.21
2	2.58	1	–63.74	– 0.030	– 0.85
3	– 0.03	– 0.31	1	0.002	0.02

The Tables 6 and 7 give the results for both cases ($D73{:}2_t$ entering unrestricted and restricted). These tables report log-likelihood values (l), eigenvalues (μ), and the associated maximum eigenvalue (Max) and Trace (Tr) statistics together with the estimated cointegrating vectors β and weighting matrix α. The test statistics are adjusted for degrees of freedom following *Reimers* (1992), and the critical values are from *Osterwald-Lenum* (1992). The results with only a trend in the cointegration space are inconclusive, as can be seen from Table 6. But, the situation can be improved substantially by adding $D73{:}2_t$. The results of Table 7 support the hypothesis that there is only one cointegrating vector. The first β vector in Table 7, for example, is recognizably a consumer demand relation with an unrestricted unit income elasticity and a very large negative long-run effect from the inflation rate. This effect decreases substantially however, when the cointegrating vector is restricted to enter only the consumption equation.

Figure 6

Cointegrating Vectors and Recursive Eigenvalues

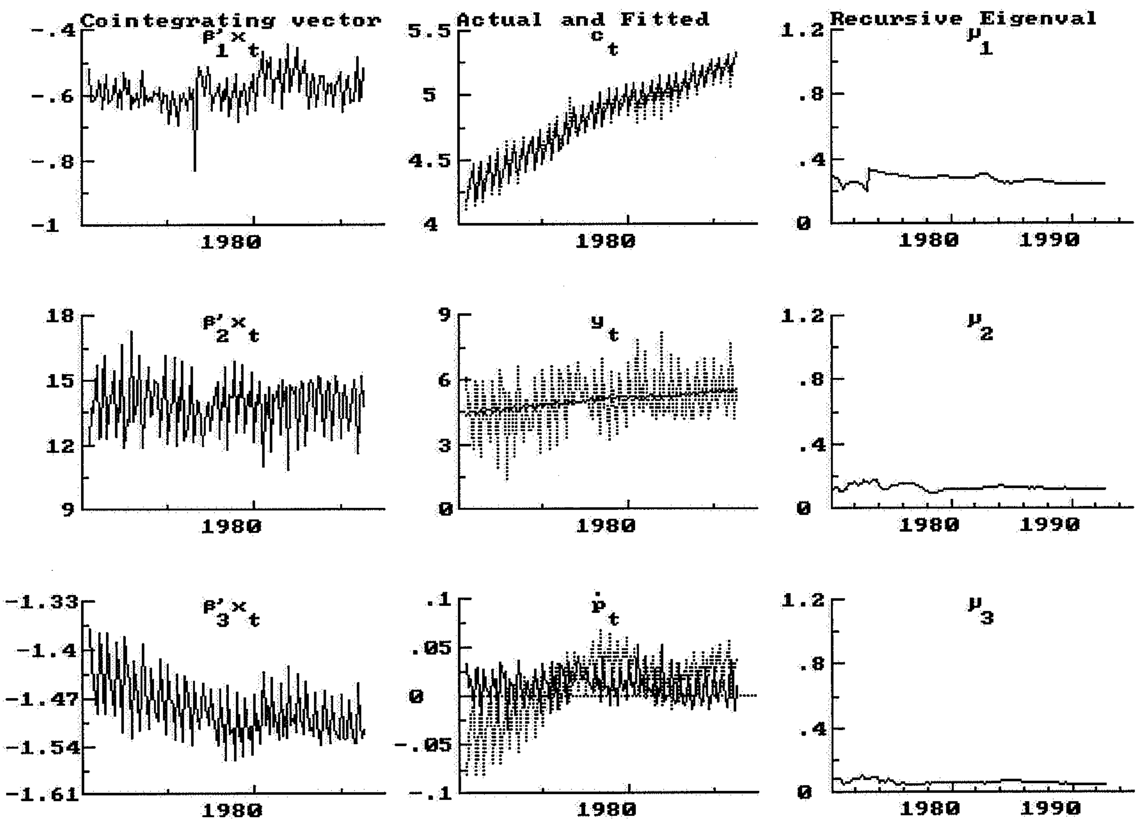


Figure 6 plots the estimated disequilibria $\beta' \mathbf{x}_t$ for this cointegrating vector, and for the remaining two nonstationary components together with their fitted and actual values and the recursively estimated eigenvalues. The disequilibria in consumer demand are small, consistent with the substantial welfare benefits attainable by adjusting. The associated eigenvalue is constant and different from zero. The remaining two components look distinctly nonstationary, and the associated eigenvalues are everywhere close to zero.

Next, we try to impose several restrictions on the first cointegrating vector in Table 7. Imposing zero trend and dummy effects is rejected by the data, giving $\chi^2(4) = 29.14$ [0.0000]. That a substantial change in the income tax system, which was the main reason for including this dummy variable, has a long-run effect, should not be surprising. We test then whether the first cointegrating vector enters only the consumption equation. This restriction is easily accepted yielding $\chi^2(2) = 0.59$ [0.7430]. Imposing this restriction causes a substantial reduction in the absolute value of the coefficient for the inflation rate. Thus, we can test jointly whether the vector (1, –1, 1, *, *)' lies in the cointegration space and is absent from the second and third system equation implying long-run weak exogeneity of income and inflation rate for the consumer demand parameters (see *Johansen,* 1992). The latter involves checking whether the weighting vector α has the from (*, 0, 0)' when β is specified as noted. The test yields $\chi^2(\approx 3) = 0.93$ [0.8187] thus very comfortably accepting the null.

2.4 Reduction to a Stationary System

As a next step in model construction, we map the data to $I(0, 0)$ series by differencing and a cointegrating combination. The available information set comprises ($\Delta_4 c_t$, $\Delta_4 y_t$, $\Delta_4 \dot{p}_t$, $\mathbf{ci}_{t-4}$) together with $\Delta_4 E_t$, $\Delta_4 D73{:}2_t$, $\Delta_4 D73{:}4_t$, $\Delta_4 D78{:}4_t$, $\Delta_4 D90{:}1_t$, seasonals and an intercept, where the cointegrating vector $\mathbf{ci}_t$ is given by

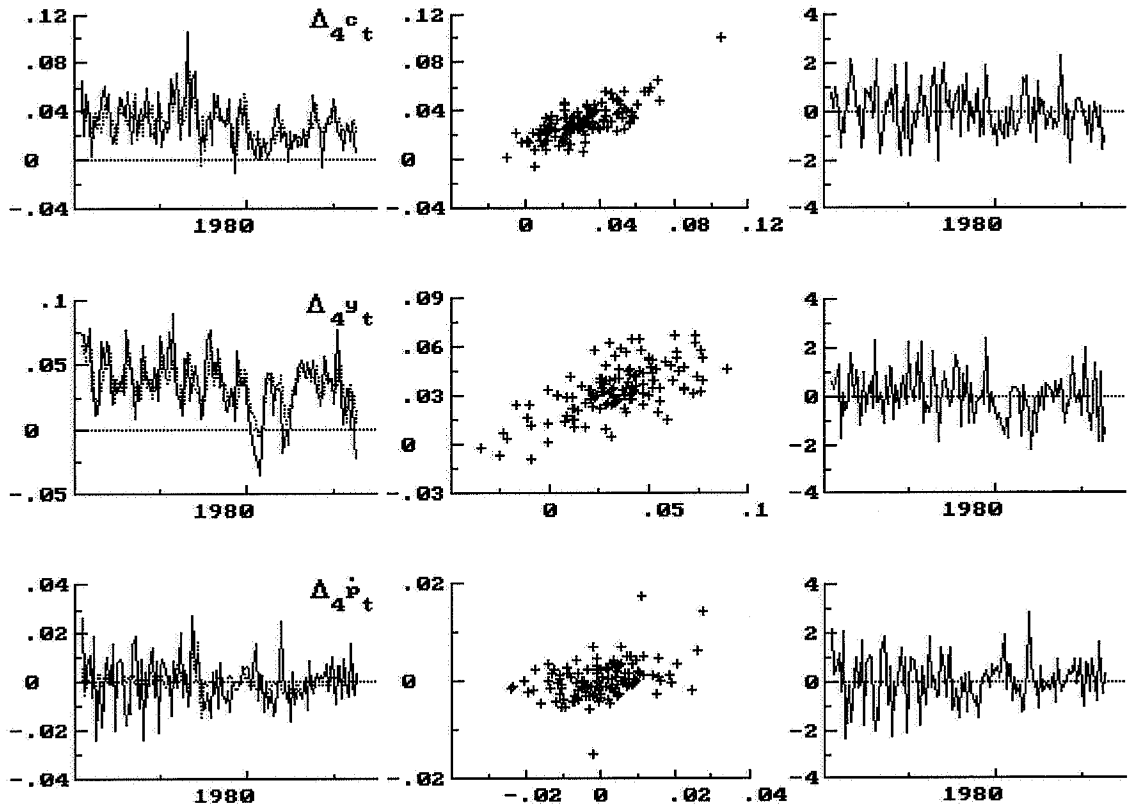
$$(6) \quad \mathbf{ci}_t \; = \; c_t - y_t + p_t - 0.1959\, D73{:}2_t + 0.000975\ \text{Trend}$$

Since we are now working with differences of order 4, we can reduce the lag length from 5 to 1 without any loss of information. These reductions (shorter lag and cointegration) lead to a decrease in the total number of parameters from 78 to 45. A rough impression of the fit of this reduced system is given by Figure 7.

Finer details cannot be discerned here, but it can be seen from the cross plots that the correlations of actual and fitted values are much lower in $I(0)$ space. These correlations for consumption, income and inflation rate are 0.77, 0.65, and 0.35, respectively. However, the equation standard errors are still very close to those obtained for the unrestricted system. There is no evidence that the outcomes are markedly affected by any influential observations, nor do the residuals manifest an excessive number of outliers.

Figure 7

Actual and Fitted Values, Cross Plots, and Scaled Residuals

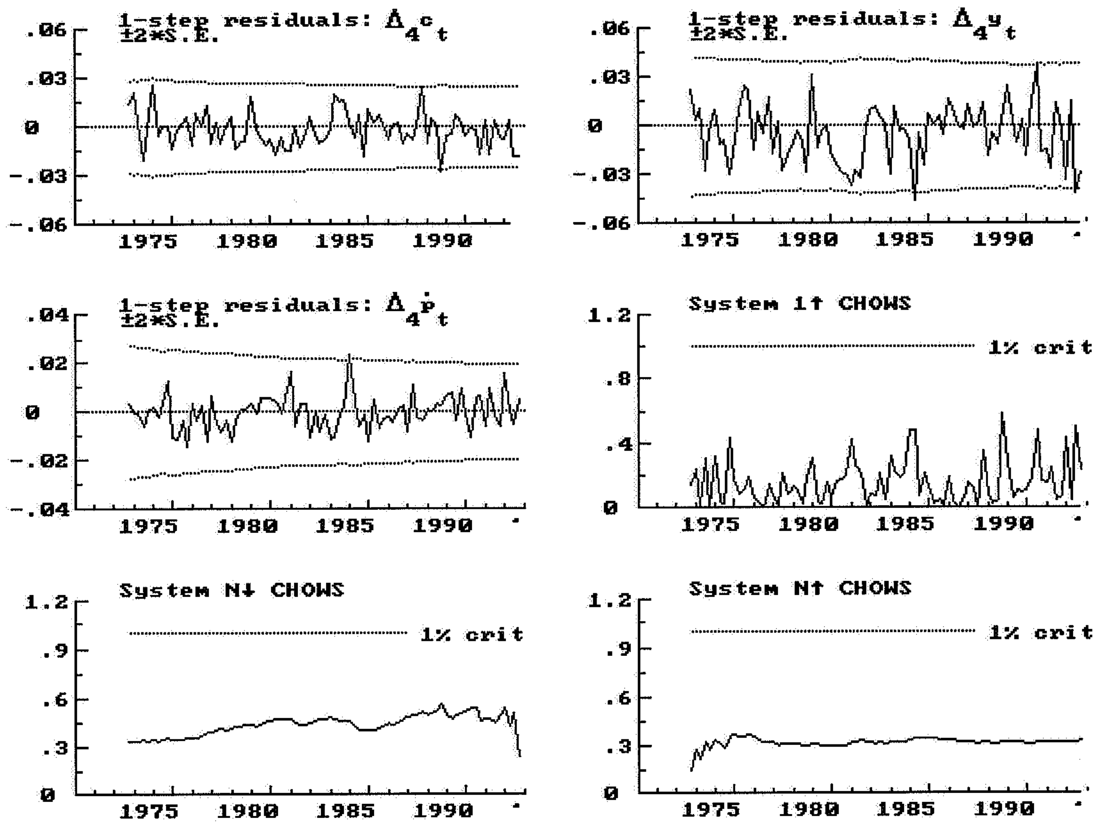


Recursive estimation provides information about the constancy of our reformulated system. The graphs of 1-step residuals and system Chow tests are presented in Figure 8. Constancy can be accepted for this reduced system. The residual standard deviations are constant over time, and the system Chow tests are anywhere insignificant at the 1 percent level. All in all, the validity of the rank reduction is confirmed by these results.

Looking at parameter estimates, we find that the cointegrating vector **ci**, does not enter the equations for income and inflation rate confirming the weak exogeneity status of these variables. Additionally, most of the dummy variables enter only the consumption equation. Thus, the annual change in income follows an AR (1) process, and that of the inflation rate is close to a random walk.

Figure 8

Recursive Constancy Statistics



This information can be used to impose restrictions on the system. The lagged inflation rate is eliminated from all three equations. The cointegrating vector enters only the consumption equation. An intercept and the lagged dependent variable are the only two explanatory variables in the income equation. Lagged income and the majority of dummy variables are removed from the equation for the inflation rate. The outcome of a full information maximum likelihood (FIML) estimation of this restricted system is given in Table 8. A variety of diagnostic information is shown in Table 9.

In particular, the test of over-identifying restrictions ($\chi^2(18) = 12.47$ [0.8219]) does not reject, matching the weak exogeneity outcome of the above cointegration tests. The equation standard errors are of a similar magnitude than those for the unrestricted system. The F-tests for serial correlation, heteroscedasticity, and a chi-square test for normality contain no evidence for inadequacy. Among the residual correlations, however, we detect a large positive correlation between consumption and income and a smaller – but still substantial – negative correlation between consumption and inflation rate, which merit further attention.

Table 8

FIML Restricted System Estimates

$$\Delta_4 c_t = \underset{(0.062)}{0.162}\,\Delta_4 c_{t-1} + \underset{(0.049)}{0.170}\,\Delta_4 y_{t-1} - \underset{(0.032)}{0.222}\,\mathbf{ci}_{t-4} + \underset{(0.003)}{0.020}\,\Delta_4 E_t + \underset{(0.008)}{0.047}\,\Delta_4 D73{:}2_t + \underset{(0.008)}{0.021}\,\Delta_4 D73{:}4_t$$

$$- \underset{(0.008)}{0.021}\,\Delta_4 D78{:}4_t + \underset{(0.008)}{0.016}\,\Delta_4 D90{:}1_t - \underset{(0.003)}{0.002}\,Q_{1\,t} - \underset{(0.004)}{0.017}\,Q_{2\,t} - \underset{(0.003)}{0.004}\,Q_{3\,t} + \underset{(0.004)}{0.002}$$

$$\Delta_4 y_t = \underset{(0.072)}{0.633}\,\Delta_4 y_{t-1} + \underset{(0.003)}{0.012}$$

$$\Delta_4 \dot{p}_t = -\underset{(0.082)}{0.106}\,\Delta_4 \dot{p}_{t-1} - \underset{(0.007)}{0.010}\,\Delta_4 D73{:}2_t + \underset{(0.007)}{0.015}\,\Delta_4 D73{:}4_t - \underset{(0.0008)}{0.0003}$$

$$\mathbf{ci}_t = \mathbf{ci}_{t-4} + \Delta_4 c_t - \Delta_4 y_t + \Delta_4 \dot{p}_t - 0.196\,\Delta_4 D73{:}2_t + 0.004$$

Table 9

Diagnostic Information on the Restricted System

Residual Correlations

	$\Delta_4 c$	$\Delta_4 y$	$\Delta_4 \dot{p}$
$\Delta_4 y$	0.46	.	.
$\Delta_4 \dot{p}$	- 0.32	- 0.15	.

System Diagnostic Tests

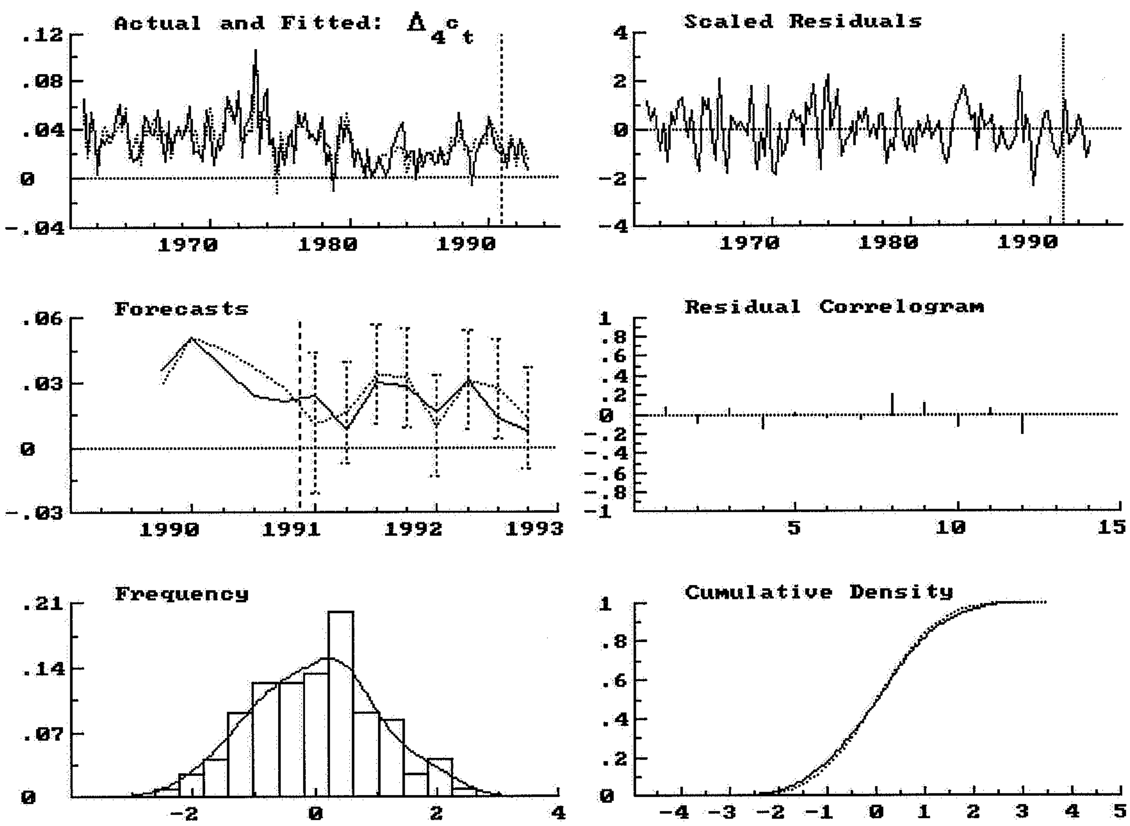
Statistic	$\Delta_4 c$	$\Delta_4 y$	$\Delta_4 \dot{p}$	System
$\hat{\sigma}$	1.25	1.90	0.96	
F^S_{ar} (45, 312)				1.40
F^S_{het} (126, 558)				0.94
F^S_{fn} (312, 390)				1.14
$(\chi^{2\,S}_{nd})$ (6)				4.21

2.5 *A Conditional Model for Consumer Expenditure*

Subject matter theory can be used to model these correlations. Consumption could depend on income (short-run propensity to consume) or vice versa (an impact of consumption on income, which seems less plausible though it has its proponents). We choose the first alternative. Additionally, we postulate that causality is running from the inflation rate to consumption and not the other way round. Based on the outcome of earlier tests on long-run weak exogeneity, we change the status of income and inflation rate correspondingly and estimate a conditional single equation model for consumer expenditure. Parameter estimates and equation diagnostics are given in Table 10. Comparing the equation standard error, $\hat{\sigma}$, with the corresponding system estimate shows that conditioning on income and inflation rate improves the goodness of fit substantially. The different diagnostic checks reveal no inadequacy of the estimated equation. The forecast Chow test provides no evidence of parameter non-constancy. We observe no excessive number of outliers, and the first four moments of the residual series are not unlike those of a normal distribution. The residuals are not autocorrelated, and are free of autoregressive conditional heteroscedasticity. The presence of heteroscedasticity due to missing squares of the regressors can also be rejected. Functional form mis-specification due to missing cross-products of the regressors or caused by specific forms of nonlinearity does not occur. The goodness-of-fit graphs in Figure 9 provide additional evidence for the adequacy of the estimated equation.

Figure 9

Goodness-of-fit Graphs for the Conditional Model



Actual and fitted values lie close together. The residual looks like normally distributed white noise. Autocorrelation is no problem as can be seen from the graph of the residual correlogram. Actual and predicted values show no tendency to deviate with an increasing forecast horizon.

Table 10

A Conditional Model for Consumer Expenditure

$$\Delta_4 c_t = \underset{(0.065)}{0.158}\,\Delta_4 c_{t-1} + \underset{(0.048)}{0.263}\,\Delta_4 y_t - \underset{(0.111)}{0.351}\,\Delta_4 \dot{p}_t - \underset{(0.036)}{0.227}\,\mathbf{ci}_{t-4} + \underset{(0.003)}{0.020}\,\Delta_4 E_t + \underset{(0.008)}{0.044}\,\Delta_4 D73{:}2_t +$$

$$\underset{(0.008)}{0.026}\,\Delta_4 D73{:}4_t - \underset{(0.007)}{0.021}\,\Delta_4 D78{:}4_t + \underset{(0.011)}{0.022}\,\Delta_4 D90{:}1_t - \underset{(0.003)}{0.003}\,Q_{1\,t} - \underset{(0.004)}{0.017}\,Q_{2\,t} - \underset{(0.003)}{0.004}\,Q_{3\,t} - \underset{(0.004)}{0.005}$$

R^2 = 0.676 $\qquad\sigma$ = 1.116% $\qquad DW$ = 1.84

$CHOW\,F\,(7, 108)$ = 0.377 [0.931]

AR 1–5 $F\,(5, 102)$ = 1.047 [0.395]	$ARCH$ 4 $F\,(4, 99)$ = 1.270 [0.287]
Normality $\chi^2\,(2)$ = 0.220 [0.896]	$X_i^2\,F\,(21, 85)$ = 0.926 [0.559]
$X_i X_j\,F\,(51, 55)$ = 0.488 [0.995]	$RESET\,F\,(1, 106)$ = 0.608 [0.437]

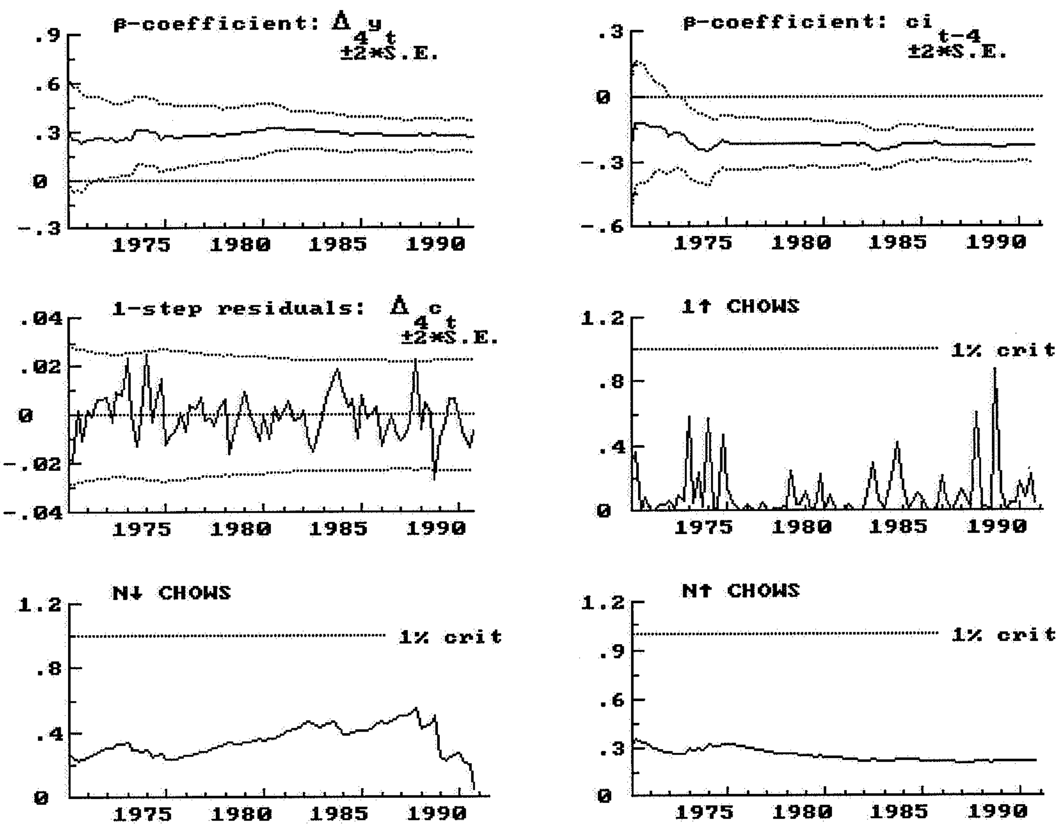
$$\mathbf{ci}_t = \mathbf{ci}_{t-4} + \Delta_4 c_t - \Delta_4 y_t + \Delta_4 \dot{p}_t - 0.196\,\Delta_4 D73{:}2_t + 0.004$$

The outcome of recursive estimation is presented in Figure 10. The estimates of important parameters hardly fluctuate and show no tendency to drift. The graphs of 1-step residuals and Chow tests highlight the constancy of the estimated consumption function. However, our attempt to resolve the above mentioned outlier and structural change problems by adding zero/one dummies is not fully successful. Especially, in the beginning of the nineties there seem to have occurred problems in the process of data collection, which would require more sophisticated treatment for their complete elimination.

The estimated conditional model has not only very appealing statistical properties, but it is also in accordance with economic theory. The specification is isomorphic to an error correction model as shown in *Engle – Granger* (1987). Short-run dynamics are modeled jointly with long-run interactions via the error correction mechanism. Unlike, for example, partial adjustment models error correction specifications do not restrict the magnitude of dynamic responses. Rather, they allow for the possibility of general dynamics, the extent of which can be determined from the data. In the above equation, the coefficients of the log differences of income and inflation rate capture the short-run reaction of consumer expenditure to changes in these variables. Even more important than these short-run dynamics are the long-run interactions between consumer expenditure and its two explanatory variables, which are reflected in the coefficient of the cointegration term. This coefficient must be negative to secure dynamic stability. It summarizes the attempts of households to correct for disequilibria between actual and planned consumer expenditure. Such discrepancies between plans and realizations can arise from errors in the households' past decisions.

Figure 10

Constancy Statistics for the Conditional Model



3. Conclusions

Since the above system provides little information about the determination of income and inflation rate, the present exercise is best seen as one of studying consumer expenditure on nondurables and services in Austria. By commencing from the joint density, we test the reductions needed to validate the traditional single equation approach.

In the cointegration analysis, we find one cointegrating vector which can be interpreted as long-run consumption function. This cointegrating vector enters only the consumption function, thus establishing long-run weak exogeneity of income and inflation rate for the parameters of the consumption function. This allows conditioning on these variables. The resulting conditional model is a data-congruent, parsimonious specification containing both differences and a differential in levels as explanatory variables.

The differences capture the short-run reaction of consumer expenditure to changes in income and inflation rate. The coefficient of the differential provides information how disequilibria between the consumption plans of households and the observed actual outcome are corrected.

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