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The Austrian consumption income relationship

Recently, there has emerged renewed interest for the study of seasonal fluctuations in economic activity. Traditionally, this kind of variation has been considered as "nuisance" component which only obscures the more important features of a series. In applied work, it was either removed by seasonal adjustment procedures before starting the analysis or captured by including seasonal dummies.

After the publication of the seminal paper by Nelson and Plosser (1982) the problem of nonstationarity in economic time series has received increasing attention. However, the nonstationarity tests which were proposed in the beginning in the relevant literature, search only for a unit root, which corresponds to a zero-frequency peak. Furthermore, it was assumed that there are no other unit roots in the series. A similar research strategy was adopted in tests for cointegration. For example, it was standard practice to estimate the cointegrating parameter α in the bivariate case by regressing $x_{1\,t}$ on $x_{2\,t}$, which was called the cointegrating regression. If the resulting residual z_t was stationary, the two variables were said to be cointegrated.

Many economic time series, however, exhibit strong seasonality which can be characterized by seasonal unit roots corresponding to peaks at the seasonal frequencies in the spectrum. In this situation, the above procedure is inappropriate for a cointegration test, even at the zero frequency. As a consequence, the concepts of integration and cointegration have to be extended to incorporate seasonal integration and seasonal cointegration. Such extensions are proposed in Hylleberg et al. (1990) and Engle et al. (1993), in the following abbreviated as HEGY (1990) and EGHL (1993), respectively.

Seasonal integration and cointegration

The spectrum of a seasonal series has distinct peaks at the seasonal frequencies $\omega_s = 2\,\pi\frac{j}{s}$, $j = 1, 2, \ldots, \frac{s}{2}$, where s is the number of time periods per year. In this paper, quarterly data will be analyzed so that we concentrate on $s = 4$.

Two classes of time series models are generally employed to model seasonality in economic time series:

(i) a purely deterministic model,

(ii) a stochastic model.

In a deterministic model, it is assumed that the seasonality is generated by dummy variables and can be removed by a simple regression on these variables. The seasonal component can be perfectly forecast and will never change its shape. The stochastic model assumes that the seasonality is caused by an integrated process, which has unit roots at the seasonal frequencies. Whether or not such unit roots exist, can be tested empirically.

In the following, we present a short description of possible test procedures. In doing this, we closely follow the exposition in Banerjee et al. (1993).

The familiar seasonal difference operator can be written as

$$\begin{aligned}(1 - B^4) &= (1 - B)\,(1 + B + B^2 + B^3)\\ &= (1 - B)\,S\,(B).\end{aligned}$$

That is, the seasonal difference operator can be written as product of the first difference operator and the moving-average seasonal filter $S\,(B)$, which contains further roots of modulus unity, i. e.

$$\begin{aligned}S\,(B) &= (1 + B + B^2 + B^3)\\ &= (1 + B)\,(1 + B^2)\\ &= (1 + B)\,(1 - i\,B)\,(1 + i\,B).\end{aligned}$$

Thus, a quarterly seasonal unit root process has four roots of modulus unity: one at the zero frequency, one at the two-quarter (biannual) frequency, and a pair of complex conjugate roots at the four-quarter (annual) frequency. As example, consider the process $\alpha(B)\, x_t = 0$. For $\alpha(B) = (1 + B)$, we obtain $x_{t+1} = -x_t$ and $x_{t+2} = x_t$. The process returns to its original value after a cycle with a period of 2.

Engle et al. (1988) give the following formal definition of a seasonally integrated process. A variable x_t is seasonally integrated of orders d and D (denoted by $SI(d, D)$), if $(1 - B)^d\, S(B)^D\, x_t$ is stationary. Thus, for quarterly data, if $(1 - B^4)\, x_t$ is stationary, then x_t is $SI(1, 1)$ with $S(B) = (1 + B + B^2 + B^3)$. The properties of seasonally integrated processes are similar to those of ordinary integrated processes. They have "long memory", i. e. the effects of shocks will persist indefinitely, and their variance increases linearly with time. However, since seasonally integrated processes contain multiple roots of modulus unity, they will not behave like $I(1)$ processes in all respects. Above all, shocks will also change the seasonal pattern of a series and, consequently, observations for a particular quarter will evolve in different ways. Taking first differences of such a series will not produce stationarity.

Testing for unit roots at seasonal frequencies has much in common with testing for an ordinary unit root. Tests have been proposed by Hasza and Fuller (1982), Dickey, Hasza, and Fuller (1984), Osborn, Chui, Smith, and Birchenhall (1988), HEGY (1990), and EGHL (1993), among others. Since we shall employ the HEGY (1990) approach in the empirical part of this paper, we present a brief exposition of their testing strategy here.

Let x_t be a quarterly series which is generated by

$$(1) \qquad \alpha(B)\, x_t = \varepsilon_t ,$$

where ε_t is iid $(0, \sigma^2)$ and $\alpha(B)$ is a fourth-order lag polynomial. The null hypothesis, that the roots of $\alpha(B)$ lie on the unit circle, is to be tested against the alternative, that they lie outside. Defining three positive parameters δ_1, δ_2, and δ_3, $\alpha(B)$ can be written as

$$(2) \qquad \alpha(B) = (1 - \delta_1 B)(1 + \delta_2 B)(1 + \delta_3 B^2)$$

For δ_i close to 1, this can be further rewritten by using a Taylor series approximation, as

$$(3) \qquad \alpha(B) = \lambda_1 B (1 + B)(1 + B^2) - \lambda_2 B (1 - B)(1 + B^2) - \lambda_3\, i B (1 - B)(1 + B)(1 - iB) + \lambda_4\, i B (1 - B)(1 + B)(1 + iB) + \alpha^*(B)(1 - B^4),$$

where the last term is a remainder. Making the substitutions $\pi_1 = -\lambda_1$, $\pi_2 = -\lambda_2$, $2\lambda_3 = -\pi_3 + i\pi_4$, and $2\lambda_4 = -\pi_3 - i\pi_4$, and grouping terms in π_3 and π_4, the expression for $\alpha(B)$ can be written as

$$(4) \qquad \alpha(B) = -\pi_1 B (1 + B + B^2 + B^3) + \pi_2 B (1 - B + B^2 - B^3) + (\pi_3 B + \pi_4) B (1 - B^2) + \alpha^*(B)(1 - B^4),$$

Substituting this expression into *(1)* and rearranging, we obtain

(5) $\alpha^* (B)(1 - B^4) x_t = \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_4 z_{3,t-1} + \pi_3 z_{3,t-2} + \varepsilon_t$,

where

$$z_{1t} = (1 + B + B^2 + B^3) x_t,$$

$$z_{2t} = -(1 - B + B^2 - B^3) x_t,$$

$$z_{3t} = -(1 - B^2) x_t.$$

The z_i's are transformations of the original series x_t, from which unit roots at specific frequencies have been filtered out. For quarterly data, the frequencies of interest are $\omega = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, for a cycle of length 2π. To $\omega = \frac{1}{2}$ corresponds a frequency of 2 cycles per year (biannual frequency), and to $\omega = \frac{1}{4}$ and $\frac{3}{4}$ one of 1 cycle per year (annual frequency). We see that z_{1t} can have a unit root only at the zero frequency ($\omega = 0$) so that $(1 - B) z_{1t}$ becomes stationary. Similarly, z_{2t} has a unit root at frequency $\omega = \frac{1}{2}$, i. e. at the biannual frequency for quarterly data. $(1 + B) z_{2t}$ is stationary. Finally, z_{3t} has a pair of complex unit roots at the annual frequency $\left(\omega = \frac{1}{4}\right)$ so that $(1 + B^2) z_{3t}$ becomes stationary.

Eq. *(5)* can be estimated by OLS, possibly with added lags of the dependent variable to whiten the residual error. The resulting estimates of the π_i's can then be employed for test purposes. For the null, that there exists a unit root at the zero frequency, we require $\lambda_1 = 0$ what corresponds to $\pi_1 = 0$. For a unit root at the biannual frequency, we test whether $\lambda_2 = 0$ corresponding to $\pi_2 = 0$. For a unit root at the annual frequency, finally, we investigate whether λ_1 or $\lambda_4 = 0$, each of which requires a joint test that π_3 and π_4 are equal to zero. Critical values for these tests are tabulated by HEGY. Rejection of all these null hypotheses implies stationarity of the process.

The concept of cointegration, as proposed in Engle and Granger (1987), concentrates exclusively on unit roots at the zero frequency and, moreover, it is assumed that there are no other unit roots in the system. But, since it is evident that seasonal fluctuations are an important feature in economic time series, it is necessary to extend the original concept of cointegration to include the possibility of unit roots at seasonal frequencies other than zero. This leads to the idea of seasonal cointegration as proposed by HEGY (1990) and EGHL (1993).

Before describing a testing procedure, we shall give a formal definition of seasonal cointegration. Let each component of a vector of time series, x_t, be seasonally integrated of order 1, i. e. $x_{it} \sim SI(1, 1)$. The components of x_t are then said to be fully cointegrated, denoted by $x_t \sim CI(1, 1)$, if there exists a vector α ($\neq 0$) so that $z_t = \alpha' x_t$ is stationary, i. e. $z_t \sim I(0)$. The implications of seasonal cointegration are not immediately obvious but are not unsimilar to those of ordinary cointegration. In particular, seasonal cointegration implies that an innovation has only a temporary effect on the seasonal behaviour of $z_t = \alpha' x_t$, while it will have a permanent effect on the seasonal pattern of x_t. The close relationship between cointegration and error correction models allows it to cope with this special situation by appropriate error correction mechanisms.

In testing for seasonal cointegration, we use a procedure which was proposed in EGHL (1993). The strategy behind this method is rather simple. The original Engle-Granger type 2-step testing procedure for cointegration is applied to appropriately filtered series. For example, in order to test for cointegration at the zero (long-run) frequency in the Austrian consumption income relation, we first run the regression

$$(6) \qquad c_{1\,t} = \alpha\, y_{1\,t} + \beta\, D_t + u_t\,,$$

where

$$c_{1\,t} = S(B)\, c_t = (1 + B + B^2 + B^3)\, c_t\,,$$

$$y_{1\,t} = S(B)\, y_t = (1 + B + B^2 + B^3)\, y_t\,,$$

and c_t and y_t symbolize the logs of real consumption and real disposable income, respectively, while D_t denotes a set of deterministic components (intercept, trend, seasonal dummies). In a second step, we test whether the residual u_t is stationary.

The test for cointegration at the biannual frequency is formally identical, that for cointegration at the annual frequency is slightly more complicated. Details can be found in EGHL (1993).

Empirical results

We now apply the above concepts to analyze observed data from the Austrian National Accounts, namely real consumption expenditure on nondurables and services and real disposable income covering the time period 1956.1 to 1992.4.

A graph of the natural logarithms of these series (in the following small letters always denote logarithms of a series) is given in fig. 1. The logarithmic transformation is chosen in order to stabilize the variance. Both series have a distinct seasonal pattern with strong peaks and troughs in the fourth and first quarters, respectively. The fourth-quarter peak in income is caused by massive bonus payments.

Figure 1

Log of real consumer expenditure on nondurables and services, c_t, and log of real disposable income, y_t

1956.1 to 1992.4

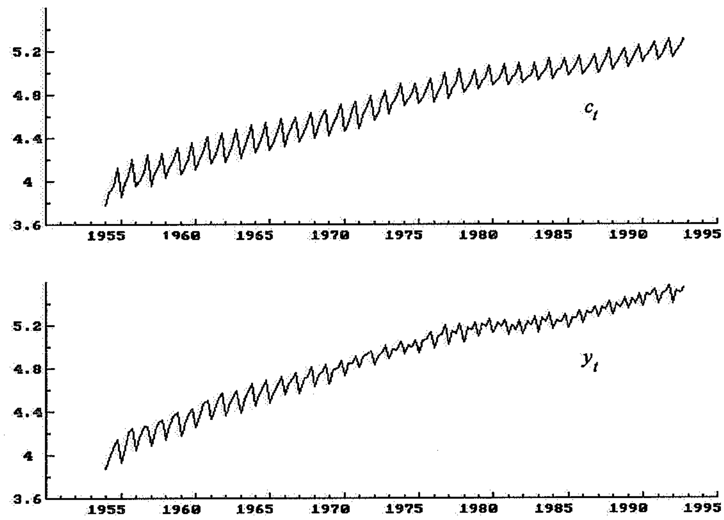


Figure 2

Quarterly change in the log of real consumer expenditure and in the log of real disposable income

1956.1 to 1992.4

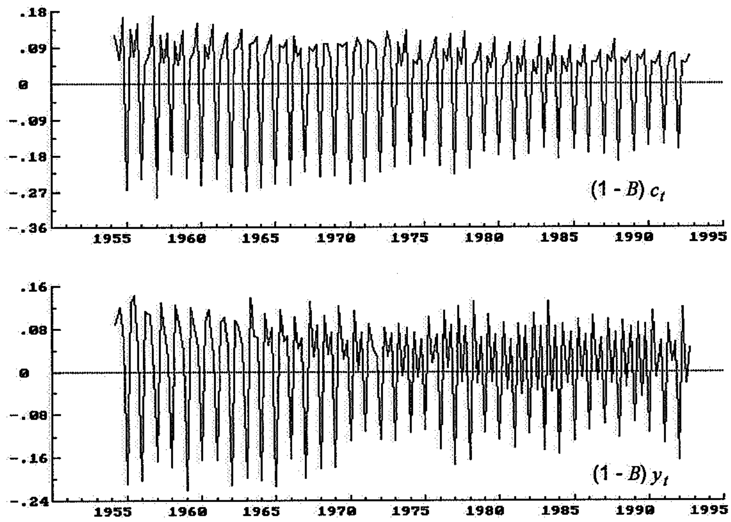
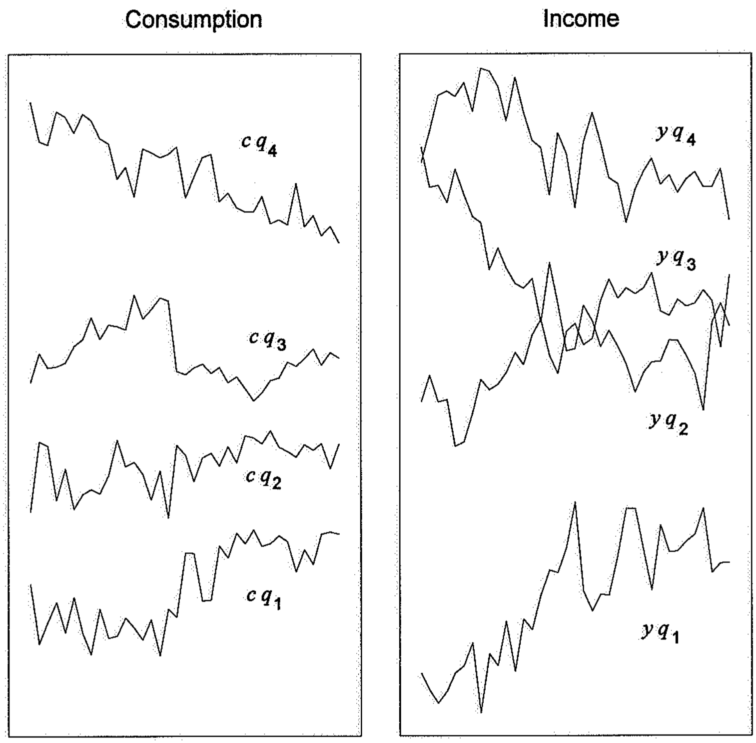


Figure 3

Log of real consumer expenditure ($c\ q\ (i), i = 1, 2, 3, 4$) and log of real disposable income ($y\ q\ (i), i = 1, 2, 3, 4$)

i th quarter value minus the average of the calendar year

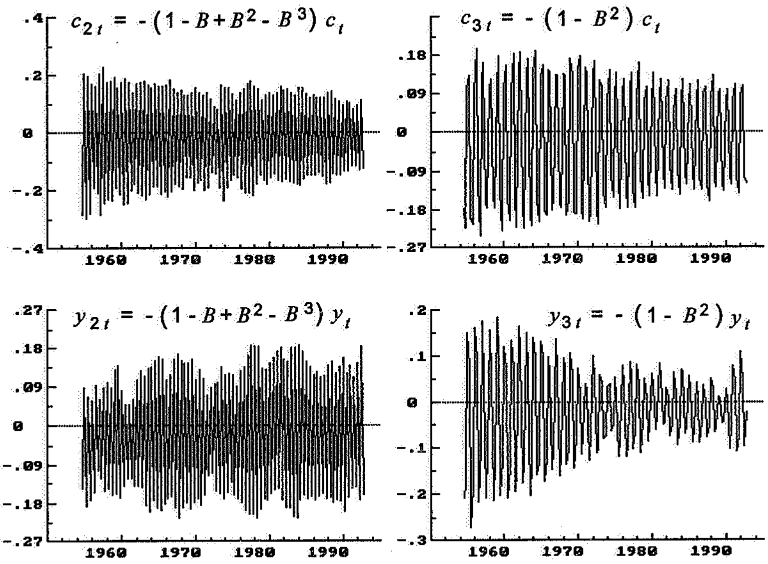


The first differences of the two series, which are depicted in fig. 2, clearly show that the seasonal pattern of both series changes over time. This becomes even more evident from fig. 3, which shows graphs for the different quarters of each series, and from fig. 4, which contains the unit root transformations $-(1 - B + B^2 - B^3)$ and $-(1 - B^2)$ of both c_t and y_t. Especially, the income series has a changing seasonal pattern where "spring" becomes "summer" since $y\ q_2$ series crosses the $y\ q_3$ series.

Figure 4

Unit root transformations of the log of real consumer expenditure and the log of real disposable income

1956.1 to 1992.4



Tests for seasonal integration

The outcome of formal tests for seasonal integration is given in table 1. These results provide evidence that both series, consumption and income, are integrated of order 1 at all frequencies $\omega = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ implying that the seasonal component of these series is stochastic and not deterministic. Thus, since consumption and income have similar univariate time series properties, cointegration cannot be ruled out.

From the results in table 1, we can draw first tentative conclusions about the existence of cointegrating relations in the Austrian consumption income data. A possible candidate for a cointegrating vector common to all frequencies would be [1, –1], which would imply that the log of the consumption income ratio, i. e. $c_t - y_t$, is $I(0)$ at the frequencies $\omega = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. Since the cointegrating vector is not estimated, the distributions given in HEGY (1990) apply. The results of table 1 cannot reject a unit root at the zero frequency, while they provide some evidence for rejecting unit roots at the other seasonal frequencies.

Table 1

Tests for seasonal integration

1956.1 to 1992.4

Variable	Auxiliary regression Determistic components[1])	Augmentation	t_{π_1}	t_{π_2}	t_{π_3}	t_{π_4}	$F_{\pi_3 \cap \pi_4}$
c_t	*I, TR*	1, 2, 3, 4	– 0.632	– 1.569	– 1.434	– 0.234	1.058
	I, SD, TR	1, 3, 4	– 0.536	– 2.273	– 2.748	– 0.677	4.052
y_t	*I, TR*	1, 4	– 1.412	0.003	– 1.761	– 1.111	2.195
	I, SD, TR	1, 4	– 1.243	– 2.597	– 2.969	– 1.553	5.727
$c_t - y_t$	*I, TR*		– 2.209	– 3.540*	– 1.930*	– 0.011	1.862
	I, SD, TR	3	– 2.689	– 3.078*	– 4.777*	– 1.067	12.133*

* . . . Significant at 5% level. Critical values are from HEGY (1990). – [1]) *I* . . . intercept, *SD* . . . seasonal dummies, *TR* . . . trend.

Table 2

Tests for cointegration at frequency 0: the long run[1])

1956.1 to 1992.4

Regressand	Cointegrating regression: Regressor $y_{1\,t}$	Cointegrating regression: Deterministic components included	R^2	Auxiliary regression: Deterministic components	Auxiliary regression: Augmentation	Tests for unit roots in residuals "ADF" t_{π_1}
$c_{1\,t}$	0.7074 *(0.0211)*	*I, TR*	0.9980	.	1, 2, 3, 4	– 1.963
$c_{1\,t}$	0.8906 *(0.0040)*	*I*	0.9970	.	1, 2, 3, 4	– 2.229
$c_{1\,t}$	0.9576 *(0.0006)*	.	0.9999	.	1, 3, 4	– 2.234
$c_{1\,t}$	1.0000 *(fixed)*	.	.	*I, TR*	1, 3, 4	– 2.204
$c_{1\,t}$	1.0000 *(fixed)*	.	.	*I, SD, TR*	1, 3, 4	– 2.178

[1]) The tests are based on the augmented Dickey-Fuller regression $u_t = \pi_1 u_{t-1} + \sum_{j=1}^{k} b_j u_{t-j} + e_t$, where u_t is the residual from the cointegrating regression $c_{1\,t}$ = deterministic component + $\alpha\, y_{1\,t} + u_t$. Critical values of the t_{π_1} statistic, when the cointegrating regression is estimated, can be found in MacKinnon (1991), those for the fixed cointegrating vector in Fuller (1976).

Table 2 contains the outcome for the zero frequency case where the cointegrating regression is run with intercept and trend, with intercept only, and without any deterministic component. In all three cases, the "Dickey-Fuller" tests based on the residuals show that a unit root cannot be rejected implying noncointegration at the long-run frequency.

As already mentioned above, a similar results is obtained if the cointegrating vector is fixed at [1, –1]. We note that the results for $c_{1\,t} - y_{1\,t}$, shown in table 2, and the "t_{π_1}" column for $c_t - y_t$ in table 1 are quite similar. This is no surprise because only the adjustment for seasonal unit roots is different in the regressions of the tables 1 and 2. In the regressions of table 2, we adjust for seasonal unit roots by prefiltering the data, while in the regressions of table 1 we adjust by including the proper variables in the regression.

Table 3

Tests for cointegration at frequency $\frac{1}{2}$: biannual[1])

1956.1 to 1992.4

Regressand	Cointegrating regression: Regressor $y_{2\,t}$	Cointegrating regression: Deterministic components included	R^2	Auxiliary regression: Deterministic components	Auxiliary regression: Augmentation	Tests for unit roots in residuals "ADF" t_{π_2}
$c_{2\,t}$	– 0.1722 *(0.0957)*	*I, SD*	0.9573		1, 2, 3, 4	– 2.937
$c_{2\,t}$	1.0647 *(0.0277)*	*I*	0.9071		1, 2, 3, 4	– 2.438
$c_{2\,t}$	1.0617 *(0.0274)*	.	0.9078		1, 2, 3	– 2.436
$c_{2\,t}$	1.0000 *(fixed)*	.	.	*I, SD*	1, 2, 4	– 3.044*
$c_{2\,t}$	1.0000 *(fixed)*	.	.	*I*	1, 2, 4	– 3.176*
$c_{2\,t}$	1.0000 (fixed)	.	.	.	1, 2, 4	– 3.264*

1) The tests are based on the augmented Dickey-Fuller regression $(v_t + v_{t-1}) = \pi_2\,(-v_{t-1}) + \sum_{j=1}^{k} b_j\,(v_{t-j} + v_{t-j-1}) + e_t$, where v_t is the residual from the cointegrating regression $c_{2\,t}$ = deterministic component + $\beta\, y_{2\,t} + v_t$. The t statistic is distributed as the "Dickey-Fuller" described in MacKinnon (1991), those for the fixed cointegrating vectors in Fuller (1976).

Based on the residuals from the cointegrating regression of $c_{2\,t} = -(1 - B + B^2 - B^3)\,c_t$ on $y_{2\,t} = -(1 - B + B^2 - B^3)\,y_t$ cointegration at the biannual frequency must also be rejected, as will be seen from the results in table 3. Including deterministic components does not influence the outcome. In all cases, we cannot reject the null hypothesis of a unit root at the frequency $\frac{1}{2}$.

However, if the cointegrating vector is fixed at [1, −1], the results clearly reject a unit root implying cointegration at the biannual frequency. Again, we observe a close correspondence between the results for $c_{2\,t} - y_{2\,t}$, as shown in table 3, and the "t_{π_2}" column for $c_t - y_t$ in table 1.

Table 4

Tests for cointegration at frequency $\frac{1}{4}\left(\text{and } \frac{3}{4}\right)$: annual[1)]

1956.1 to 1992.4

Regressand	Cointegrating regression Regressors $y_{3\,t}$	$y_{3\,t-1}$	Deterministic components included	R^2	Auxiliary regression Augmentation	Tests for unit roots in residuals "HEGY" t_{π_3}	t_{π_4}	$F\,\pi_3 \cap \pi_4$
$c_{3\,t}$	0.4549	0.1193	*I, SD*	0.9695	3	– 4.528*	– 0.969	10.779*
	(0.0469)	*(0.0467)*						
$c_{3\,t}$	1.2522	0.4601	*I*	0.8736	3	– 4.393*	– 2.438*	17.998*
	(0.0418)	*(0.0413)*						
$c_{3\,t}$	1.2230	0.4298	.	0.8636	1, 3	– 3.336*	– 1.841	7.372*
	(0.0428)	*(0.0423)*						

1) The tests are based on the auxiliary regression $(w_t + w_{t-2}) = \pi_3\,(-w_{t-2}) + \pi_4\,(-w_{t-1}) + \sum_{j=1}^{k} b_j\,(w_{t-j} + w_{t-j-2}) + e_t$, where w_t is the residual from the cointegrating regression $c_{3\,t}$ = deterministic component + $\gamma_1 y_{3\,t} + \gamma_2 y_{3\,t-1} + w_t$. The distribution of the joint F test for $\pi_3 \cap \pi_4 = 0$ and the t test on π_3 and π_4 are given in EGHL (1993).

Finally, based on the residuals from the cointegrating regression $c_{3\,t} = -(1 - B^2)\,c_t$ on $y_{3\,t} = -(1 - B^2)\,y_t$ and $y_{3\,t-1}$, we can test for unit roots at the annual frequencies $\frac{1}{4}\left(\text{and } \frac{3}{4}\right)$. The results are presented as table 4. The regression is run with an intercept and seasonal dummies, with just an intercept, and without any deterministic component at all. The F values allow a clear rejection of a seasonal unit root at the annual frequencies. A similar, although less unequivocal result is obtained by use of the t statistics on π_3 and π_4. All in all, the possibility of cointegration between consumption and income at the annual frequency is not ruled out.

Conclusions

In this paper, we apply concepts, which were proposed by EGHL (1993), to test for seasonal integration and cointegration in Austrian consumption and income data. The tests are applied to the logs of real consumer expenditure and real disposable income covering the time period from 1956.1 to 1992.4. The results show that both series are integrated of order 1 at the long-run frequency and also at the seasonal frequencies. This implies that both series are nonstationary, and that the seasonality is stochastic with the seasonal pattern changing significantly during the sample period. This variation in the seasonal pattern is especially pronounced for the income series. The consumption series has a more regular seasonal pattern but, here too, the seasonality is far from being deterministic.

The tests for seasonal cointegration show no indication of cointegration at the long-run frequency, neither with cointegrating vector [1, –1] nor with an estimated cointegrating vector. At the other seasonal frequencies, we find signs of cointegration. This result is very similar to findings of EGHL (1993) for the Japanese consumption function.

A really convincing economic rationalization for these findings is still missing. The role of bonus payments, which are paid out twice a year in Austria, might be a possible explanation. If consumers have the habit of using these payments, when they occur, to finance their summer and winter holidays, respectively, then seasonal cointegration may result. Especially, since we concentrate in this paper on consumer expenditure for nondurables and services, this hypothesis might be of some relevance.

Be that as it may, in the light of these findings, it seems doubtful whether it is good practice to consider seasonality as nuisance component, which only obscures the more important features of a series and should be removed by adequate adjustment procedures. Just to the contrary, our findings seem to indicate that seasonality is an important feature of economic time series. A careful study of these seasonal components can produce deeper insight into the generating mechanisms of economic time series. Valuable information can be obtained from an analysis of seasonally unadjusted data, which would be lost if seasonally adjusted series are studied.

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