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Monthly economic time series are often subject to calendar effects. Two forms of these effects can be distinguished. First, the levels of economic activity may vary between different days of a week. Since the composition of days of the week changes from month to month and year to year, the observed levels of economic activity will be affected. Such effects are referred to as trading day effects in the relevant literature. Second, there exist traditional festivals, on which the levels of economic activity are much lower than on ordinary working days. To complicate the situation even more, some of these festivals are set according to lunar calendars, and their dates may vary from year to year between adjacent months in the Gregorian calendar. Such effects are referred to as holiday effects.

The objective of the present paper is to remove calendar effects from the index of Austrian industrial production by identifying and estimating a ARIMA model, which includes information about these effects as deterministic input series. A discrimination between trading day and holiday effects is not feasible for the Austrian industrial production. The relatively large number of religious festivals in Austria makes it impossible to model each holiday effect separately, but that would be necessary to capture it adequately. We try to eliminate both forms of calendar effects simultaneously by designing a calendar which reflects a composition of days of the week that is typical for the Austrian situation.

ARIMA Models with Calendar Variation

Cleveland and Grupe (1981), Hillmer, Bell, and Tiao (1981), Hillmer (1982), Bell and Hillmer (1983) and Liu (1986) proposed ARIMA models in order to handle calendar effects. Assuming that Y_t is an appropriately transformed time series, which is subject to calendar effects, a ARIMA model can be written as:

$$(1) \quad Y_t = TD_t + \frac{\theta(B)}{D(B)\,\phi(B)} a_t ,$$

$$a_t \approx \text{iid } N(0, \sigma_\alpha^2),\ t = 1, 2, \ldots, n$$

where B is the backshift operator (i. e. $B\,a_t = a_{t-1}$), TD_t the total calendar effects at time t, $\theta(B)$ the moving-average, $\phi(B)$ the autoregressive, and $D(B)$ are the difference operators. These latter operators can be of simple or multiplicative form.

For the purpose of model identification, it is more convenient to consider an alternative form of model *(1)*:

$$(2) \qquad D(B)\,Y_t = D(B)\,TD_t + N_t\,,$$

$$N_t = \frac{\theta(B)}{\phi(B)}\,a_t\,,$$

where N_t is referred to as the noise of the model.

A Trading Day Effect Model

We assume that a trading day effect can be approximated by a deterministic model. Let TD_t denote the trading day effect for month t. Under our assumption, TD_t will be a function of the number of distinct types of days in month t. Thus, we have

$$(3) \qquad TD_t = \sum_{i=1}^{7} \omega_i X_{it}$$

where X_{it}, $i = 1, \ldots, 7$, are respectively the number of Mondays, Tuesdays, Wednesdays, Thursdays, Fridays, Saturdays, and Sundays in month t, and the ω_i's are parameters. The model *(3)* can be rewritten as

$$(4) \qquad TD_t = \sum_{i=1}^{6} (\omega_i - \bar{\omega})(X_{it} - X_{7t}) + \bar{\omega} \sum_{i=1}^{7} X_{it}$$

$$= \sum_{i=1}^{7} \beta_i T_{it}$$

where $\bar{\omega} = \frac{1}{7}\sum_{i=1}^{7} \omega_i$, $\beta_i = \omega_i - \bar{\omega}$, and $T_{it} = X_{it} - X_{7t}$ for $i = 1, \ldots, 6$, $\beta_7 = \bar{\omega}$, and $T_{7t} = \sum_{i=1}^{7} X_{it}$ denotes the length of month t. The interpretation of β_7 depends on the form of a model. For stationary time series, this parameter represents the average of daily effects and can be used to adjust for the length of a month. A similar interpretation holds if the regular difference operator $(1 - B)$ is present in a model. However, when a model contains the seasonal difference operator $(1 - B^{12})$, the parameter β_7 captures a possible leap-year effect. The parameterization *(4)* is more convenient than *(3)* because estimates of the β_i's will be less correlated. Moreover when making inferences, the differential effects $\beta_1, \ldots, \beta_6$ are of more interest than the ω_i.

Identification of the ARIMA Model for the Noise

A stepwise identification method for the ARIMA model of the noise, in the presence of calendar variation, is proposed in Liu and Hanssens (1982) and Liu et al. (1986). First the parameters in a tentative model of Y_t are estimated:

$$(5) \quad Y_t = \text{const.} + \sum_{i=1}^{7} \beta_i T_{it} + \frac{1}{(1-\phi_1 B)(1-\phi_{12} B)} a_t \, .$$

Here the multiplicative AR(1) and AR(12) noise model only serves as first approximation of the "true" model. If the disturbance is close to white noise, both ϕ_1 and ϕ_{12} will be close to zero. However, if ϕ_1 and ϕ_{12} are close to 1, the series Y_t should be differenced, and the following model estimated:

$$(6) \quad (1-B)(1-B^{12})\, Y_t = \text{const.} + (1-B)(1-B^{12}) \sum_{i=1}^{7} \beta_i T_{it} + \frac{1}{(1-\phi_1 B)(1-\phi_{12} B^{12})} a_t \, .$$

After a tentative model for Y_t has been estimated, the entire noise series N_t can be obtained either from

$$(7) \quad \hat{N}_t = Y_t - \widehat{\text{const.}} - \sum_{i=1}^{7} \hat{\beta}_i T_{it}$$

or

$$(8) \quad \hat{N}_t = (1-B)(1-B^{12})\, Y_t - \widehat{\text{const.}} - (1-B)(1-B^{12}) \sum_{i=1}^{7} \hat{\beta}_i T_{it}$$

depending whether a specification in levels or differences is used. Once the estimated noise series $\hat{N}_t$ is obtained, a final model for the noise can be identified using the sample autocorrelation function (ACF) and the partial autocorrelation function (PACF).

The multiplicative AR(1) and AR(12) terms in the eqs. *(5)* and *(6)* serve two purposes. First, the parameters ϕ_1 and ϕ_{12} in *(5)* provide information whether regular and/or seasonal differencing is appropriate. Using this information will reduce the danger of overdifferencing. Second, the precision of the estimates of the calendar effect parameters will be greatly improved when ϕ_1 and ϕ_{12} are present in the model. More reliable estimates of the noise series will then be the result.

Empirical Results

We now use the above approach to adjust the index of the Austrian industrial production for calendar effects. We have monthly observations of this series covering the period 1962.1 till 1993.7. A graph of the original series is given as fig. 1. We see that a rather stochastic seaonality is present, and that the variability increases with the level of the series. In order to remedy the latter problem, we work with the natural logarithms of the series in the following.

Figure 1

Index of Austrian Industrial Production

1962.1 to 1993.7

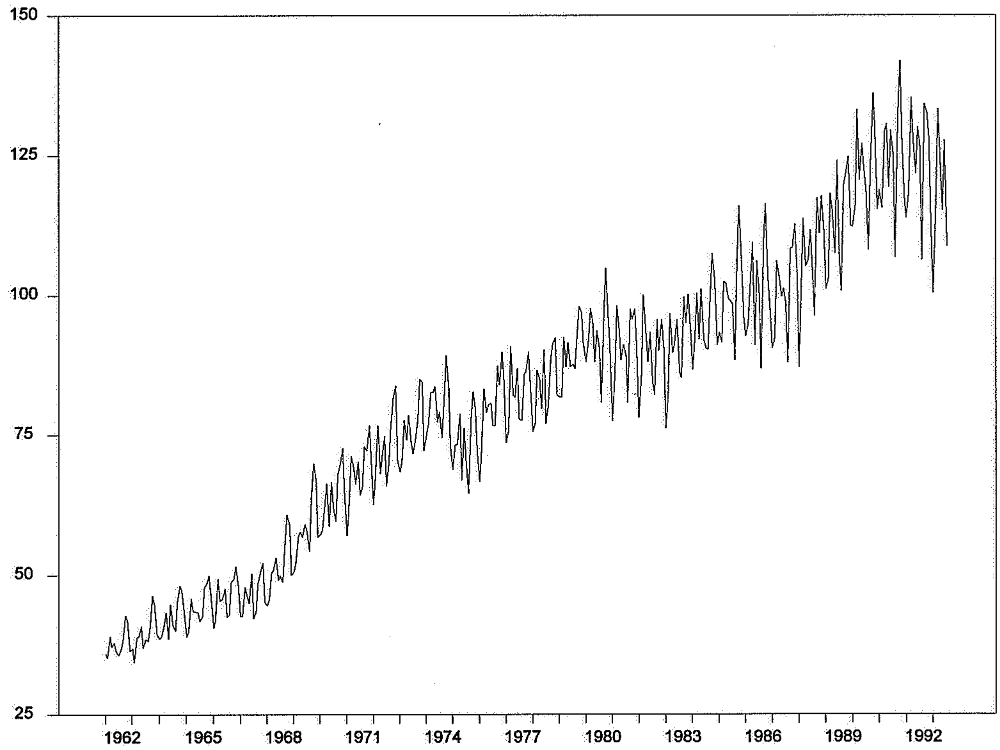
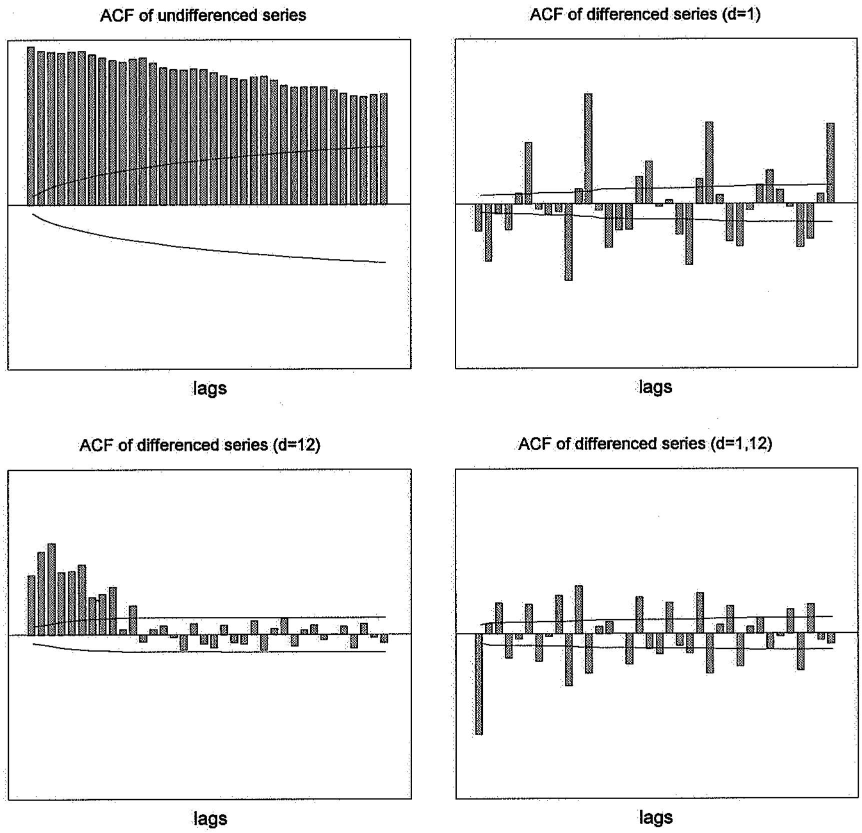


Fig. 2 and 3 display the autocorrelation and partial autocorrelation functions for the log of the original series and for various differenced series. These functions are very complex and, unfortunately, not very informative. Nevertheless, a number of conclusions can be drawn from an analysis of the sample ACF's. It is clear from the ACF for the undifferenced series that the log of industrial production is non-stationary. The ACF for first differences indicates a need for an additional seasonal difference. The sample ACF of seasonally differenced series provides an indication of the need for an additional first difference once a seasonal difference of the data has been taken. But taking regular and seasonal differences does not produce the expected result, at least not at the first glance. The sample ACF for this case is a confused pattern, which seems to indicate massive overdifferencing. From past experience (see Liu (1980), Tsay (1984), and Salineas and Hillmer (1987)) it is known, however, that such a pattern may be due to the influence of trading day effects.

Figure 2

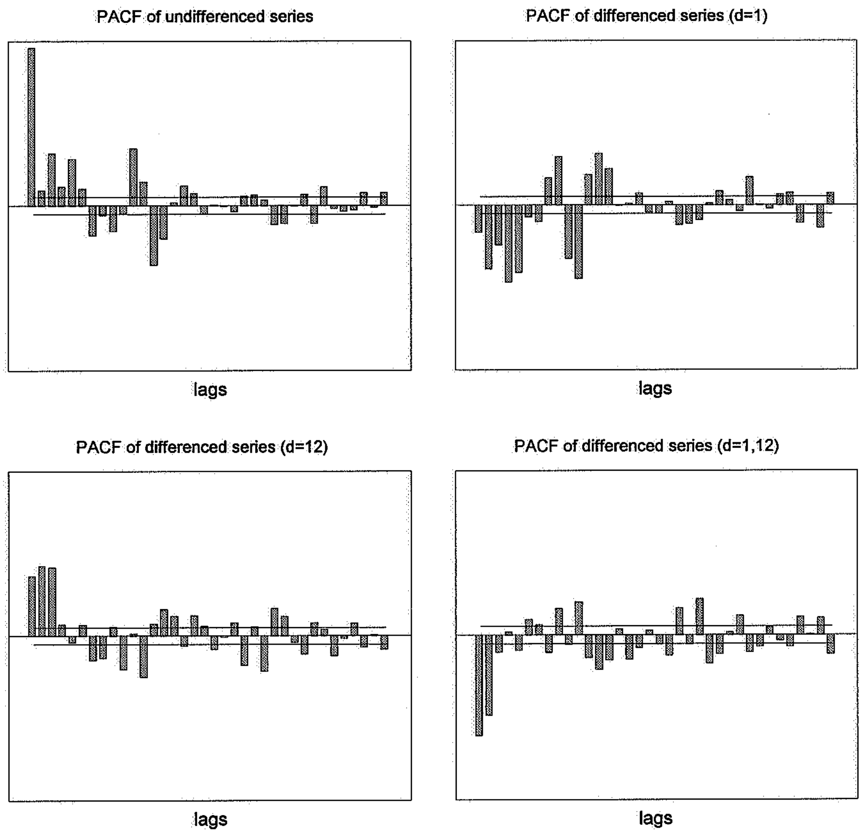
Autocorrelation Functions for Industrial Production



Therefore, we investigate next whether trading day effects are present in Austrian industrial production. Following Cleveland and Devlin (1980), we use spectral analysis for this purpose. If trading day effects are present, one should observe a spike in the spectrum of the series at frequency 0.696 π. Fig. 4 shows the log spectrum of detrended industrial production. It has a pronounced spike at the frequency 0.696 π. We take this as fairly clear indication for the presence of trading day effects in our series.

Figure 3

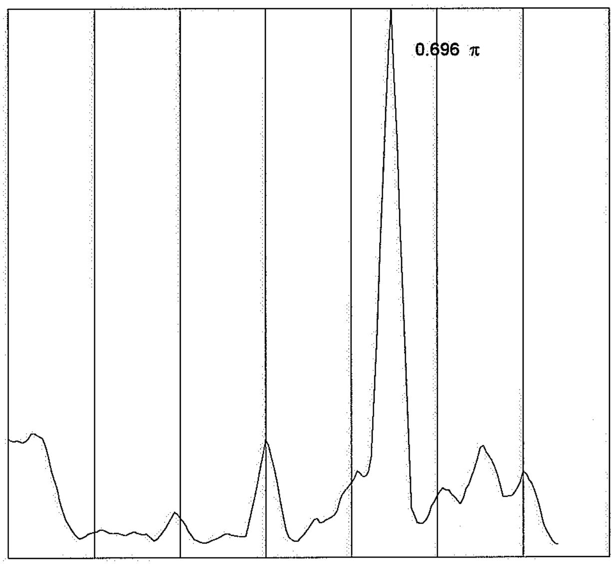
Partial Autocorrelation Functions for Industrial Production



After we have spotted trading day effects, we shall try to remove them by including deterministic variables into a ARIMA model. For this purpose, we need information about the composition of the days of the week for each month in our sample period. The SCA statistical package, which is used for the computations of this paper, provides a facility to generate seven variables containing the number of Mondays, Tuesdays, . . ., Sundays in a month for a given period of time. Including these variables into a ARIMA model and estimating improves the goodness of fit substantially. But, we still find evidence that the trading day effects are not removed completely. This happens because the facility for generating the trading day variables does not provide information about festivals. Festivals play an improtant role in Austria. In order to remove their effects on industrial production, we simply convert them into Sundays. This leads to a further improvement of our estimation results. Finally, there remains one problem which has to be solved. The festivals in May and June fall, with one exception, always on Thursdays. Therefore, it is very tempting for workers to take off the adjacent Fridays also, in order to enjoy a long weekend. Taking this fact also into account we end up, after some trial and error, with a composition of the days of the week which seems to reflect the Austrian situation accurately. A small portion of it, is shown in table 1.

Figure 4

Log Spectrum



Having accumulated the necessary calendar information to eliminate trading day effects, we proceed with the identification of the ARIMA model for the noise. This is done by estimating the tentative models *(5)* and *(6)* for the log of Austrian industrial production. Parameter estimates for these models are given in table 2. We notice that the estimates of ϕ_1 and ϕ_{12} in model *(5)* are close to 1 implying that regular and seasonal differences have to be taken. Thus, model *(6)* is appropriate in this situation. From this model, we can derive an estimate of the entire noise series N_t (not only of the residuals a_t) using eq. *(8)*.

This estimated noise series $\hat{N}_t$ can be used to identify the final model for the noise. The sample autocorrelation function and the partial autocorrelation function are given in fig. 4. These functions show clearly that the noise series follows a multiplicative moving-average model. Thus, the following final ARIMA model is estimated for the log of Austrian industrial production:

$$(9) \qquad \ln PROD_t = \sum_{i=1}^{7} \beta_i T_{it} + \frac{(1-\theta_1 B)(1-\theta_{12} B^{12})}{(1-B)(1-B^{12})} a_t .$$

Table 1

Calendar Information for Industrial Production in Austria

Year		X_{1t}	X_{2t}	X_{3t}	X_{4t}	X_{5t}	X_{6t}	X_{7t}	Sum
	.								
	.								
1992	Jan	4	3	4	4	6	4	6	31
	Feb	4	4	4	4	4	5	4	29
	Mar	5	5	4	4	4	4	5	31
	Apr	3	4	4	4	5	5	5	30
	May	4	4	4	3	3	6	7	31
	Jun	5	4	4	3	4	4	6	30
	Jul	4	4	5	5	5	4	4	31
	Aug	5	4	4	4	4	4	6	31
	Sep	4	5	5	4	4	4	4	30
	Oct	3	4	4	5	5	5	5	31
	Nov	5	4	4	4	4	4	5	30
	Dec	4	4	4	3	3	6	7	31
1993	Jan	4	3	3	4	4	4	9	31
	Feb	4	4	4	4	4	4	4	28
	Mar	5	5	5	4	4	4	4	31
	Apr	3	4	4	4	5	5	5	30
	May	4	4	4	3	4	4	8	31
	Jun	5	4	5	3	4	4	5	30
	Jul	4	4	4	5	5	5	4	31
	.								
	.								

In this equation, we denote the natural logarithm of the index of Austrian industrial production by ln $PROD_t$. The definition of the trading day variables T_{it} is given above already. To illustrate the importance of the trading day effects, the following model is also considered:

$$\text{(10)} \quad \ln PROD_t = \frac{(1-\theta_1 B)(1-\theta_{12} B^{12})}{(1-B)(1-B^{12})} a_t .$$

These two models are estimated by a maximum likelihood method. The resulting parameter estimates are given in table 3. The difference between the models is striking. The removal of trading day effects not only improves the goodness of fit substantially, but it also reduces the autocorrelation in the residuals significantly. Our final model *(9)* shows no signs inadequacy and seems to be a good approximation to the data. The coefficients of all trading day variables are highly significant. Their pattern looks plausible. The effects increase continuously from Monday till Thursday, drop sharply on Friday, and become negative for Saturday. We also observe a significant leap-year effect.

Table 2

Parameter Estimates for the Tentative ARIMA Models *(5)* and *(6)*

Parameter	Model *(5)*		Model *(6)*	
const.	4.4787	(7.50)	–0.0004	(–0.73)
β_1	0.0050	(5.49)	0.0067	(6.77)
β_2	0.0083	(8.58)	0.0077	(7.24)
β_3	0.0080	(8.39)	0.0082	(8.13)
β_4	0.0106	(11.58)	0.0095	(9.06)
β_5	0.0031	(3.36)	0.0036	(3.33)
β_6	–0.0090	(–13.32)	–0.0087	(–12.24)
β_7	0.0362	(9.56)	0.0385	(7.91)
ϕ_1	0.9069	(37.95)	–0.2373	(–4.41)
ϕ_{12}	0.9715	(72.71)	–0.4496	(–9.17)
s. e.	0.0201		0.0181	

The values in parentheses are *t*-statistics.

Figure 5

Autocorrelation and Partial Autocorrelation Function for the Estimated Noise Series $\hat{N}_t$

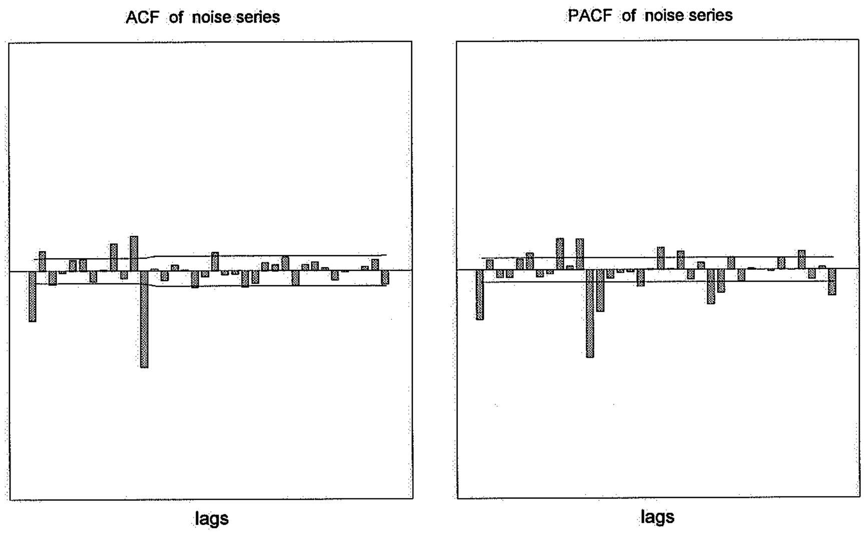


Table 3

Parameter Estimates for the ARIMA Models *(9)* and *(10)*

Parameter	Model *(9)*		Model *(10)*	
β_1	0.0063	(6.38)		
β_2	0.0077	(7.44)		
β_3	0.0084	(8.14)		
β_4	0.0094	(9.29)		
β_5	0.0035	(3.44)		
β_6	–0.0086	(–12.19)		
β_7	0.0358	(8.01)		
θ_1	0.2524	(4.96)	0.7314	(20.70)
θ_{12}	0.6008	(14.43)	0.8676	(31.14)
s. e.	0.0169		0.0414	
Q (24)	34.5		293	

The values in parentheses are t-statistics. The Q-statistic is distributed as χ^2 with $Q_{0.05}$ = 36.4.

Conclusions

In the present paper, we attempt to remove trading day effects from the Austrian industrial production by identifying and estimating a ARIMA model, which includes information about these effects as deterministic input series. The outcome of this attempt is very promising. Including this type of information not only improves the goodness of fit of the estimated ARIMA model, but it also reduces the autocorrelation in the residual series. The estimated trading day parameters could be used in a further step to compute an adjusted series of industrial production. This series might serve as input for a model-based approach of seasonal adjustment. More stable estimates of the seasonal component would certainly be obtained.

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