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MODELING ECONOMIC TIME SERIES VIA STATE SPACE APPROACH

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ABSTRACT

This paper introduces a state space approach for modeling economic time series, which incorporates many ideas from systems and control theory. In order to allow for system identification, a canonical innovation state space model has been chosen as the basic prototype model. Based on this representation, a singular value decomposition based structure determination and parameter estimation method is given. After the canonical model is specified, it is transformed into an "internal balanced" form which has been proved to have many desirable properties. This approach is applied to the well-known Lydia Pinkham data to illustrate its use and its forecasting accuracy is compared with other time series approaches.

INTRODUCTION

The state space model, one of the most often used tool in system theory, has led up to the development of many modern control techniques. For example, Kalman filtering, state-feedback, state-estimator designs. While the advantages of using the state space representation also for an analysis of business and economic systems have been advocated by several control

scientists and economists[1,2], this approach does not enjoy widespread use until recently[3,4,5]. Models in state space representation are found less often in the literature than those in the popular ARMA type representation.

This paper develops a new state space modeling technique in which many ideas are borrowed from system and control theory. Numerical linear algebra, especially the singular value decomposition, plays a central role in it.

STATE SPACE MODEL AND ITS INNOVATION REPRESENTATION

The state space model provides a description of the internal and external characteristics of a linear, finite-dimensional system. The state of a system represents the maximum amount of information on the past behavior of the system. It is therefore sufficient to predict the future response of the system. The state space model of a linear, finite-dimensional stochastic system is described by the pair of equations

$$x_{k+1} = Ax_k + w_k \tag{1a}$$

$$y_k = Cx_k + u_k \tag{1b}$$

where x_k is an unobservable state vector that describes the state of the system at time k (dim x_k=n); y_k is an observation vector (dim y_k = m); and u_k, w_k are zero-mean white noises with covariance matrices R_1 and R_2.

In model (1), except for the observation vector y_k and its order m, which are known, other components, such as n, A, C, R_1, and R_2 are unknown. Identifiability of such a model is not given. To make model (1) identifiable, we can consider its innovation representation

$$x_{k+1|k} = Ax_{k|k-1} + Bv_k \tag{2a}$$

$$y_k = Cx_{k|k-1} + v_k \tag{2b}$$

where $x_{k|k-1}$ is the optimum linear estimate of x_k; B is the steady-state optimum filter gain; v_k is the zero-mean innovation process with unknown covariance matrix Q; and A, C are assumed to take the following canonical forms

$$A = \begin{bmatrix} A_{11} & & & \\ . & . & & 0 \\ . & & . & \\ . & & & . \\ A_{m1} & . & . & A_{mm} \end{bmatrix}$$

$$A_{\substack{ii \\ (n_i \times n_i)}} = \begin{bmatrix} 0 & & & & I \\ \beta_{ii0} & \beta_{ii1} & \beta_{ii2} & . \; . \; . & \beta_{ii,n_i-1} \end{bmatrix}$$

$$A_{\substack{ij \\ (n_i \times n_j) \\ i>j}} = \begin{bmatrix} & & & 0 & \\ \beta_{ij0} & \beta_{ij1} & \beta_{ii2} & . \; . \; . & \beta_{ii,n_j-1} \end{bmatrix}$$

$$c_i^T = [0,...,0,1,0,...,0], \quad n_i > 0,$$

where the element 1 in c_i^T is in the column $(1+n_1+...+n_{i-1})$, and

$$c_i^T = [\beta_{i10},...,\beta_{i1,n_1-1},\beta_{i20},...,\beta_{i2,n_2-1},...,$$
$$\beta_{i,i-1,n_{i-1}-1}, 0, ..., 0], \quad \text{if } n_i = 0 .$$

The indices $\{n_i\}$ and the entries $\{\beta_{ijk}\}$ entering into the above canonical matrices will be related to output statistics, which can be computed using an identification algorithm proposed by Tse and Weinert[6] and the numerically reliable singular value decomposition technique.

THE COMPUTATION OF SYSTEM MATRICES

Let $R_\sigma=E\{y_{k+\sigma}y_k^T\}$. If $r_{ij}(\sigma)$ is the i, jth element of R_σ, it can be shown that the following relation is valid by making use of the structures of A and C

$$r_{ij}(n_i+\tau)=\left\{\begin{array}{l}\sum_{l=1}^{i}\sum_{k=0}^{n_l-1}\beta_{ilk}r_{lj}(k+\tau),\ n_i>0\\ \\ \sum_{l=1}^{i-1}\sum_{k=0}^{n_l-1}\beta_{ilk}r_{lj}(k+\tau),\ n_i=0\end{array}\right\},\ \tau=1,2,\ldots \quad (3)$$

This relation can be used successively for i=1,2,...,m in order to obtain equations for calculating the unknown

parameters of the innovation model. For example, for i=1 and $\tau=1,2,\ldots,n_1$, Eq.(3) is equivalent to the matrix equation

$$r_1 = \Phi_1(n_1)\beta_1 \quad (4)$$

where

$$r_1 = [r_{1j}(n_1+1),\ldots,r_{1j}(2n_1)]^T$$

$$\beta_1 = [\beta_{110},\ldots,\beta_{11,n_1-1}]^T$$

$$\Phi_1(k) = \begin{bmatrix} r_{1j}(1) & r_{1j}(2) & \dots & r_{1j}(k) \\ r_{1j}(2) & r_{1j}(3) & \dots & r_{1j}(k+1) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ r_{1j}(k) & r_{1j}(k+1) & \dots & r_{1j}(2k-1) \end{bmatrix}$$

From the definition of the index n_1, it follows that the matrix $\Phi_1(k)$ is nonsingular for k=1, 2,..., n_1 and is singular for $k>n_1$. This shows that if the output correlation sequences $r_{ij}(\sigma)$, $\sigma=1,2,...$, are used for forming the set of matrices $\Phi_1(k)$, then the correct value of the index n_1 can be determined by setting it equal to the rank of the matrix. Once n_1 is found it is then possible to compute β_1 from the Eq. (4).

For i=2,3,...,m, n_i and β_i can be computed in an analogous manner. For example, if i=2, write (3) for $\tau=1,2,...,n_1+n_2$ as

$$r_2 = \Phi_2(n_2)\beta_2 \qquad (5)$$

where

$$r_2 = [r_{2j}(n_2+1),...,r_{2j}(2n_2+n_1)]^T$$

$$\beta_2 = [\beta_{210},...,\beta_{21,n_1-1},\beta_{220},...,\beta_{22,n_2-1}]^T$$

$$\Phi_2(k) = \begin{bmatrix} & \Phi_1(n_1) & & r_{2j}(1) & \cdots & r_{2j}(k) \\ & & & r_{2j}(2) & \cdots & r_{2j}(k+1) \\ r_{1j}(n_1+1) & \cdots & r_{1j}(2n_1) & \cdot & & \cdot \\ r_{1j}(n_1+2) & \cdots & r_{1j}(2n_1+1) & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ r_{1j}(n_1+k) & \cdots & r_{1j}(2n_1+k-1) & r_{2j}(n_1+k) & \cdots & r_{2j}(n_1+2k-1) \end{bmatrix}$$

As before, n_2 can be determined from the rank of the matrix $\Phi_2(k)$; and β_2 can be obtained from the Eq. (5). The procedure can continue in a similar manner for j=2,3,...,m. Then $n=n_1+...+n_m$ and a canonical form for (A,C) is obtained.

For determining $n_1, n_2, \ldots, n_m$, Tse and Weinert[6] give a procedure which is based on the computation of determinants. However, this procedure is apparently very fragile from a numerical point of view[7]. In practice, the given information is often inexact, and the available observations are usually noise corrupted, which will easily prevent Tse and Weinert's method from being really effective. Moreover, in computing the parameters of the model Tse and Weinert's method only uses the sample information included in the $n_i \times n_i$ upper left submatrix of $\Phi_i(k)$, but better results could be obtained by utilizing the "most essential" sample information included in the full matrix $\Phi_i(k)$ in a manner that will be made precise below. Therefore, we need a more robust and informative approach to characterize the ranks of $\{\Phi_i(k)\}$ and compute their parameters.

Fortunately, the singular value decomposition[8], one of the basic and most important tools of modern numerical analysis, particularly numerical linear algebra, can provide exactly what we desired: the robustness and the informativeness. It is known that the singular values and singular vectors of a matrix are relatively insensitive to perturbations in the entries of the

matrix, and to finite precision errors[7]. Moreover, the decomposition displays a set of singular values, mathematically the number of nonzero singular values is corresponding to the rank of the matrix. The advantage is that the decomposition will not only display the rank but also display it in a quantitative way. Namely, the set of singular values will be used to determine the rank as well as to judge the distance of the matrix to a lower-rank one[9]. Especially, by retaining the n_i dominant singular values and corresponding singular vectors, we can compute the parameters of the model from the sample information which we call the "most essential" sample information.

In practice, A and C can be constructed from β_1, β_2 ,..., β_m. Here we only provide the algorithmic details for computing β_1, because β_2, β_3 ,..., β_m can be obtained in an analogous manner.

Formally, relation (4) can be solved for β_1 by computing the product of the inverse of $\Phi_1(n_1)$ and r_1. We can accomplish this step through the LU-factorization of $\Phi_1(n_1)$ and the forward and backward substitutions. However, as it has been pointed out above already, such a β_1 is not optimal because $\Phi_1(n_1)$ only includes partial sample information. A different algorithm is thus suggested here, which consists of the following major steps.

Step 1: Compute the singular value decomposition of $\Phi_1(k)$ and arrange the singular values of $\Phi_1(k)$ in nonincreasing order, i.e.,

$$\Phi_1(k) = U \Lambda V^T = [U_1 \;\; U_2] \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix},$$

where the $n_1 \times n_1$ matrix Λ_1 contains the dominant singular values and the $(k-n_1)\times(k-n_1)$ matrix Λ_2 contains the smaller singular values. The n_1 rank approximation is obtained by retaining the n_1 dominant singular values and the corresponding singular vectors, i.e.,

$$\Phi_1'(k) = U_1 \Lambda_1 V_1^T$$

Making use of the well-known Eckart and Young Theorem[8], it can be shown that the optimal approximation errors in the spectral norm and Frobenius norm are respectively

$$\min_{rank(\Phi_1'(k))=n_1} \|\Phi_1(k)-\Phi_1'(k)\|_S = \sigma_{n_1+1} ,$$

and

$$\min_{rank(\Phi_1'(k))=n_1} \|\Phi_1(k)-\Phi_1'(k)\|_F = \sum_{i=n_1+1}^{k} \sigma_i^2 .$$

Step 2: Consider the matrix equation

$$r_1^* = U_1 \Lambda_1 V_1^T \beta_1^* ,$$

where

$$r_1^* = [r_{1j}(n_1+1),...,r_{1j}(n_1+k)]^T$$

$$\beta_1^* = [\beta_{110},...,\beta_{11,k-1}]^T$$

Then β_1^* can be computed from the relation

$$\beta_1^* = (U_1 \Lambda_1 V_1^T)^+ r_1^* = V_1 \Lambda_1^{-1} U_1^T r_1^*$$

where + denotes the Moore-Penrose pseudoinverse.

Step 3: Determine β_1 as the vector consisting of the first n_1 elements of β_1^*.

Besides the numerically stable singular value decomposition, the proposed algorithm only requires simple matrix multiplications, so it can be expected that the algorithm is highly numerically reliable. Moreover, the computed β_1 is optimal because the most essential sample information has been used.

β_2, β_3,...,β_m can be computed in an analogous manner and finally the canonical form (A,C) is obtained.

Once n, A and C are determined, it remains to compute the unknown covariance Q of the innovation sequences v_k and the optimum filtering gain B. Let Σ denote the covariance matrix of the states in model (2), then (2) implies

$$\Sigma = A\Sigma A^T + BQB^T$$

$$R_0 = C\Sigma C^T + Q$$

$$R_\sigma = CA^{\sigma-1}S, \quad \sigma > 0$$

where

$$S = A\Sigma C^T + BQ$$

Note that Σ is also unknown. S can be computed from the relation

$$\begin{bmatrix} r_{1j}(1) \\ \cdot \\ r_{1j}(n_1) \\ r_{2j}(1) \\ \cdot \\ r_{mj}(n_m) \end{bmatrix} = \begin{bmatrix} c_1^T \\ \cdot \\ c_1^T A^{n_1-1} \\ c_2^T \\ \cdot \\ C_m^T A^{n_m-1} \end{bmatrix} s_j = \Delta s_j , \quad j=1,...,m.$$

If the canonical form described in the above section is used, then it is easily seen that $\Delta=I$ and thus s_j can be obtained by lagged correlations.

A real Schur decomposition approach is suggested here for calculating B and Q which has its origin in Laub[10]. This approach can be described as follows. First, a 2n×2n matrix Φ is constructed, where n is the dimension of the matrix A:

$$\phi = \begin{bmatrix} F-GF^{-T}J & GF^{-T} \\ -F^{-T}J & F^{-T} \end{bmatrix}$$

where auxiliary n×n matrices are defined by

$$F = A^T - C^T R_0^{-1} S^T, \; G = C^T R_0^{-1} C, \; and \; J = S R_0^{-1} S^T.$$

Second, the matrix Φ is transformed into the real Schur form

$$U^T \phi U = H = \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix},$$

Third, the eigenvalues with magnitudes less than one are collected into the upper left n×n blocks of the above matrix using our ordering algorithm and an ordered real Schur form is obtained

$$\hat{U}^T \phi \hat{U} = \hat{H} = \begin{bmatrix} \hat{H}_{11} & \hat{H}_{12} \\ 0 & \hat{H}_{22} \end{bmatrix}.$$

Fourth, the orthogonal matrix $\hat{U}$ is partitioned conformably into four n×n submatrices

$$\hat{U} = \begin{bmatrix} \hat{U}_{11} & \hat{U}_{12} \\ \hat{U}_{21} & \hat{U}_{22} \end{bmatrix}.$$

Finally, the covariance matrix Σ of the states is obtained by computing the product of submatrix $\hat{U}_{21}$ and the inverse of submatrix $\hat{U}_{11}$

$$\Sigma = U_{21} U_{11}^{-1} .$$

Correspondingly B and Q can be obtained from

$$B = (S - A\Sigma C^T)(R_0 - C\Sigma C^T)^{-1} .$$

and

$$Q = R_0 - C\Sigma C^T$$

The main stumbling block with using this approach is the ordering problem with the real Schur form[10]. An efficient ordering algorithm has been developed to overcome this obstacle[11].

BALANCED REALIZATION OF MODEL

The state space model obtained in the above has a canonical representation which is most parsimonious in the number of parameters. To have a system representation with the least possible number of parameters has been recognized as important in system identification

problems. However, a possible disadvantage of such a representation is that it is more susceptible to numerical errors[3,9,12]. To overcome this deficit, it is necessary to convert the canonical model into a model which has a good numerical behavedness before it can be used for forecasting. For this reason, the theory of *balanced realization*, originally proposed by Moore[12] and generalized by others, is introduced here.

The main idea of balancing is to find a coordinate system in the state space where each state is controllable as well as observable. The most important application of balancing is in model reduction where the states of the balanced form which are least controllable and observable are truncated. However, the problem of model reduction is not of real interest for us because our canonical model has been in a minimal dimension and generally needs not to be reduced. What we are concerned with are other useful properties of balanced form such as the numerical robustness[9,12,13]. The most commonly used algorithm for state space balancing is that of Laub[14]. Because of lack of space, we omit the algorithmic details here and refer the reader to Laub[14].

A SUMMARY OF THE STATE SPACE MODELING METHODOLOGY

Having introduced the details of state space modeling for time series, we now outline this methodology, which consists of the following main steps:

(a) Transformation of the data into stationary time series: This step involves differencing (regular and seasonal) of each time series and the computation of the autocorrelation functions to test the series for stationality. The procedure is similar to that of Box and Jenkins[15]. In many economic time series, it is also advantageous to log transform the series before differencing.

(b) Identification of the canonical state space model: This step involves the determination of model order n and then the computation of system matrices A, C, B and covariance matrix Q. The value of n can be determined using the numerically reliable singular value decomposition technique, and A and C can be computed using a singular value decomposition based algorithm. B and Q can be obtained by the real Schur decomposition method.

(c) Balanced realization of model: This step accomplishes a coordinate transformation using Laub's algorithm[14] to get an internally balanced model with good numerical characteristics.

(d) Diagnostic checks on model adequacy: To see whether the estimated model adequately represents the given set of data, diagnostic checks should be performed. These tests can be done based on some error measure criteria or on an analysis of the cross-correlation matrices of the residuals[16]. If any inadequacy is detected, the above iterative cycle is repeated until a suitable representation is found.

(e) Forecasting: Once the state space model has been identified and identified, it can be used to forecast all of the time series into the future. This can be accomplished by first calculating $x_{N+1|N}$ from

$$x_{k+1|k} = (A-BC)x_{k|k-1} + By_k, \quad x_{0|-1}=0, \quad k=0,1,...,N,$$

where N is the number of observation data used to build model. Then the predicted values are generated by

$$y_{N+k|N} = Cx_{N+k|N} = CA^{k-1}x_{N+1|N}, \quad k=1,2,....$$

A CASE STUDY

In this section the well-known Lydia Pinkham advertising and sales series are used to illustrate the ideas presented in this paper. Several alternative models were proposed to describe these data. A Box-Jenkins time series approach is suggested by Helmer and Johansson[17] in which univariate models and two candidate transfer functions are modeled on the annual data. A joint model approach is used by Heyse and Wei[18] in which a bivariate ARMA model is identified, estimated and checked. Here, we use the state space approach to model the same data. For comparison with the results of Helmer and Johansson and Heyse and Wei, the first 40 annual observations (1907-1946) are used for the identification and estimation procedure. The remaining 14 are held back to study the forecasting performance of models.

Like Helmer and Johansson and Heyse and Wei, we have also chosen to model the first differences of the data using the state space approach. Applying the previously discussed computation algorithms we obtain the following estimates for the system matrices(For completeness, both canonical and balanced forms are given) Canonical representation:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -0.4075 & 0.0060 & 0 \\ 0.0094 & 0.3445 & 0.1046 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.2607 & 0.5147 \\ -0.4985 & 0.1820 \\ 0.0385 & 0.4841 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Q = 10^4 \times \begin{bmatrix} 3.9506 & 2.2355 \\ 2.2355 & 5.2126 \end{bmatrix}.$$

Balanced representation:

$$\hat{A} = \begin{bmatrix} 0.1925 & 0.7425 & -0.1015 \\ -0.5460 & -0.1641 & -0.0849 \\ 0.1378 & 0.3276 & 0.0821 \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} -0.3921 & 0.7271 \\ -0.5885 & -0.0493 \\ -0.1758 & -0.3583 \end{bmatrix},$$

$$\hat{C} = \begin{bmatrix} 0.8050 & -0.1586 & 0.2189 \\ 0.4355 & -0.2253 & 0.4363 \end{bmatrix}, \quad \hat{Q} = Q.$$

In order to check for model adequacy, we compute the cross-correlation matrices of the residuals for lags 1 to 12. All residual cross-correlations lie inside the range of ±2 standard derivations(= ±0.32), Therefore, the estimated model is considered as an adequate representation of the data under analysis.

For the 14 one-step-ahead advertising forecasts, summary statistics for the forecasting errors are shown in Table 1 for Helmer and Johansson's univariate ARMA model (UARMA), Heyse and Wei's bivariate ARMA model (BARMA), and our state space model(SP).

For the 14 one-step-ahead sales forecasting, summary statistics for forecasting errors are listed in Table 2, where TF1 and TF2 are the two transfer function models suggested by Helmer and Johansson.

CONCLUSION

An innovation state space approach to the modeling of economic time series is shown. The numerically reliable singular value decomposition technique is used to determine the structures and parameters of models. This approach is applied to the well-known Lydia Pinkham data, showing that its forecasting accuracy is comparable to other time series approaches.

Table 1. Summary statistics for errors in forecasting advertising

Advertising	UARMA	BARMA	SP
Number of forecasts	14	14	14
Mean squared error	10247.00	9224.00	8361.00
Maximum error	171.00	162.00	164.39
Median error	-44.50	-35.00	-21.23
Minimum error	-191.00	-147.00	-142.89
Mean error	-42.10	-6.60	-10.65

Table 2. Summary statistics for errors in forecasting sales

Sales	TF1	TF2	BARMA	SP
Number of forecasts	14	14	14	14
Mean squared error	15177.00	14044.00	15021.00	13905.00
Maximum error	212.00	221.00	227.00	184.11
Median error	-50.00	-58.50	3.00	-20.34
Minimum error	-213.00	-195.00	-208.00	-220.99
Mean error	-49.10	-43.80	-25.20	-51.08

Unlike the computationally-demanding maximum likelihood method, the model-building procedure introduced in this paper is easy to carry out computationally. It is nothing but a summary expression of the data in a mathematically consistent representation. It only uses little more than linear algebra, and is therefore not too difficult to be understood. Because of the ease of model building by this approach, the approach may be used in other ways, for example, as a way of generating initial guesses to be further improved by the maximum likelihood method.

REFERENCES

1. Harvey, A. C., Time Series Models, Philip Allan Publishers Limited, Oxford, 1981.
2. Mehra,R.K.,Identification in Control and Economics, in Hazewinkel, M. and Rinnooy Kan, A.H.G. (eds.), Current Developments in the Interface: Economics, Econometrics, Mathematics, Reidel, Boston, MA, 1992, pp. 261-288.
3. Aoki, M., State Space Modeling of Time Series, Springer-Verlag, Heidelberg, 1987.
4. Harvey, A.C., Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, Cambridge, 1989.
5. Hahn, F.R. and Thury, G., Structural time series models for the Austrian and German industrial product, Working Paper 49, Austrian Institute of Economic Research, 1992.
6. Tse,E. and Weinert,H.L., Structure determination and parameter identification for multivariable stochastic linear systems, IEEE Trans. Automat. Contr., 120 (1975), pp. 603-613.

7. Wilkinson, J.H., The Algebraic Eigenvalue Problem, Clarendon Press, Oxford, 1965.
8. Golub,G.H.and Van Loan,C.F.,Matrix Computations, Second Edition, The Johns Hopkins University Press, Baltimore and London, 1989.
9. Kung,S., A new identification and model reduction algorithm via singular value decomposition, Proc. 12th Asilomar Conf. Circuits, Syst., Comput., Pacific Grove, CA, 1978, pp. 705-714.
10. Laub, A. J., A Schur method for solving algebraic Riccati equations, IEEE Trans. Automat. Contr., 24(1979), pp. 913-921.
11. Wang, L. and Libert, G., Ordering eigenvalues in the real Schur form: algorithm and applications, Technical Report, Department of Computer Science, FPMs, 1992.
12. Moore, B.C., Principal component analysis in linear systems: controllability, observability, and model reduction, IEEE Trans. Automat. Contr., 26(1981), pp. 17-32.
13. Mullis, C.T. and Roberts,R.A., Synthesis of minimum roundoff noise fixed point digital filters, IEEE Trans. Automat. Contr., 23(1976), pp. 551-562.
14. Laub, A. J., Computation of "balancing" transfor- mations, Proc. 1980 Joint Automat. Conf., San Francisco, CA, 1980, Session FA8-E.
15. Box,G.E.P. and Jenkins,G.M., Time Series Analysis: Forecasting and Control, Second Edition, Holden- Day, San Francisco, 1976.
16. Tiao, G.C. and Box, G.E.P., Modeling multiple time series with applications, J. Amer. Statist. Assoc., 76(1981), pp. 802-816.
17. Helmer,R.M. and Johansson, J.K., An exposition of the Box-Jenkins transfer function analysis with an application to the advertising-sales relationship, J. Marketing Res., 14(1977), pp. 227-239.
18. Heyse,J.F. and Wei,W.S., Modeling the advertising- sales relationship through use of multiple time series techniques, J. Forecasting, 4(1985), pp. 165-181.

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