

# W o r k i n g   P a p e r s

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for the Austrian and German  
Industrial Production

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### **1. Introduction**

In this paper we adopt the structural methodology proposed, among others, by Harvey (1989) for modeling the monthly dynamics of the Austrian and German industrial production. In a recently published paper we concluded that, for both countries, the ARIMA 'airline' model [i.e. ARIMA (0,1,1)x(0,1,1)<sub>S</sub>] seems to be a rather good representation of the dynamical behavior of the monthly industrial production (Hahn and Thury 1992). However, we also stressed in this paper that, modeling these time series, the structural approach seems to be superior to the ARIMA philosophy. The reasons why we lean towards the structural approach are the following: First, structural models allow for a decomposition of a time series into 'stylized facts' associated naturally with its dynamics. These stylized facts are trend, cycle, seasonal, and irregular component, all of which are of interest to economists in themselves. Knowing the dynamics of these components in detail is very helpful for making sound predictions on the basis of univariate time series models. In addition, within the structural framework the model selection procedure is similar to that employed for regression models in econometrics (Harvey 1989). Second, the ARIMA 'airline' model was originally designed to represent the dynamics of time series whose salient features are trend and seasonality. As production data, in particular industrial production, become more and more volatile, the cycle becomes more and more an intrinsic part of this type of time series. As for modeling the dynamics of a time series with a cyclical component, there is convincing evidence that univariate structural models are better qualified than univariate ARIMA models (see Harvey and Todd 1983, and section 3 of this paper).

The paper is organized as follows: In section 2, the basics of the structural approach as a class of unobserved component models which explicitly allow for modeling the 'stylized facts' of the dynamics of time series are introduced. In section 3, the estimation, testing and evaluation procedure applied is presented and the estimated models and their statistical properties are discussed. Section 4 presents conclusions and announcements of further research.

## 2. Modeling Unobserved Components: The Structural Approach

Loosely speaking, the structural approach refers explicitly to the traditional view that an observable economic time series  $y(t)$  can be decomposed into various unobservable components such as trend, cycle, seasonal and irregular component. Basically, structural time series models can be seen as regression models in which the explanatory variables are functions of time and the parameters are time-varying (Harvey 1989). For handling such unobserved component models from an estimation point of view, there are a number of techniques available (see, for example, Nerlove, Grether and Carvalho 1979). Within the structural framework, the 'state-space-Kalman-filter approach' is the most common and appealing one. Thereby, the structural model is first put into the state-space form with the unobserved components constituting the state of the system. Then, the Kalman filter is applied for facilitating the formulation of the likelihood function, for updating the various components and for the extrapolation of the components into the future. For a thorough introduction into structural time series models, state-space models, and the Kalman filter we refer to Harvey (1989).

As for the basic structural model (BSM), it consists of a trend, a seasonal and an irregular component. In mathematical terms, this reads as follows

$$(1) \quad y_t = \mu_t + \gamma_t + \varepsilon_t, \quad t = 1, \dots, T$$

where  $y_t$  is, in most cases, the logarithm of the observed value,  $\mu_t$  is a trend,  $\gamma_t$  is the seasonal and  $\varepsilon_t$  the irregular component, respectively. The irregular component is a white noise disturbance term with variance  $\sigma_\varepsilon^2$ .

The trend-generating process is designed as follows:

$$(2) \quad \mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \quad \eta_t \sim \text{NID}(0, \sigma_\eta^2)$$

$$(3) \quad \beta_t = \beta_{t-1} + \zeta_t, \quad \zeta_t \sim \text{NID}(0, \sigma_\zeta^2)$$

where  $\text{NID}(0, \sigma^2)$  denotes a normally distributed, serially independent, random variable with mean zero and variance  $\sigma^2$ .  $\eta_t$  and  $\zeta_t$  are assumed to be mutually uncorrelated. The trend component as modeled in (2) and (3) obviously follows a random walk with a variable drift.

The seasonal component  $\gamma_t$  is usually modeled by a series of sines and cosines. If there are  $s$  seasons in the year, then

$$(4) \quad \gamma_t = \sum_{j=1}^{s/2} \gamma_{jt}$$

and  $\gamma_{jt}$  is a non-stationary cycle

$$(5) \quad \begin{bmatrix} \gamma_{jt} \\ \gamma_{jt}^* \end{bmatrix} = \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{jt-1} \\ \gamma_{jt-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{jt} \\ \omega_{jt}^* \end{bmatrix}, \omega_{jt}, \omega_{jt}^* \sim \text{NID}(0, \sigma_\omega^2)$$

with  $\lambda_j = 2\pi j/s$ ,  $j=1, \dots, s/2$ .  $\omega_{jt}$  and  $\omega_{jt}^*$  are taken to be uncorrelated,  $\gamma_{jt}^*$  appears by construction (see Harrison and Akram 1983).

In the literature, the variances  $\sigma_\varepsilon^2$ ,  $\sigma_\eta^2$ ,  $\sigma_\zeta^2$  and  $\sigma_\omega^2$  are usually referred to as hyperparameters.

Harvey (1989) shows that the ARIMA 'airline' model and the BSM have a lot in common in terms of statistical properties. In fact, under certain restrictions ( $\sigma_\zeta^2 = 0$  and  $\sigma_\omega^2 = 0$ ), the airline model is equivalent to the BSM (for details, see Harvey 1989). More important from a practitioner's point of view, both models are obviously designed to approximate time series whose dynamics are driven by trend and seasonality only. In this respect, it comes as no surprise that the fit of the BSM and the ARIMA 'airline' model, applied to our data set, is basically the same (for details, see the following section). However, as we will show in the upcoming section, for both countries the BSM extended by an additive cyclical component proves to be the better, if not the best, univariate candidate for representing the dynamical structure of monthly industrial production. The cycle-generating process used to enhance the BSM in our approach is modeled as follows:

$$(6) \quad \begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda_c & \sin \lambda_c \\ -\sin \lambda_c & \cos \lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}, \kappa_t, \kappa_t^* \sim \text{NID}(0, \sigma_\kappa^2)$$

where  $\psi_t$  is the cyclical component,  $\kappa_t$  and  $\kappa_t^*$  are assumed to be uncorrelated. The parameters  $0 \leq \lambda_c \leq \pi$  and  $0 \leq \rho \leq 1$  represent the frequency of the cycle and the damping factor of the amplitude, respectively.

### 3. Estimation, Testing, and Model Evaluation

As already mentioned, data on industrial production belong to that category of economic time series, for which cyclical components are of considerable importance. Therefore, the estimation of structural time series models should be an appropriate approach for modeling these series. These models will provide information, which is not available from standard decomposition methods, where no attempt is made to identify cyclical components.

### 3.1 *Hyperparameters of the models*

Table 1 presents estimates for the hyperparameters of structural time series models for the production in the manufacturing industry of Austria and Germany<sup>1)</sup>.

These models are estimated by the estimation procedure as outlined in the previous section, that is to say, by exact maximum likelihood on the state-space-Kalman filter basis. The data base consist of calendar adjusted monthly data covering the period 1962 to 1991. With noisy monthly data, it is obviously too optimistic to believe that 'the data can speak for themselves'. Unrestricted estimation gives unsatisfactory results. In order to improve the results, we have to impose some restrictions. A natural thing to do is to set the variance of the trend level equal to zero, what guarantees smoothness of this component. This formulation is adopted by Kitagawa (1981) and Kitagawa and Gersch (1983). Imposing this restriction improves the results substantially. We obtain smooth trends and additive cyclical components, which look extremely plausible. The average length of these cycles is approximately 5 years. Tests reveal that the estimation results are insensitive to moderate changes in the frequency parameter  $\lambda_c$ , from which the average cycle length is computed. In order to reduce the number of parameters, which have to be estimated, we pre-specify the average cycle length with 5 years. Inspection of Table 1 reveals that two of the estimated hyperparameters, namely the variances of the trend slope and of the seasonal component, are numerically rather small. This fact might explain why a simple airline model is a good description of the two production series under analysis, as it is shown in Hahn and Thury (1992). As mentioned in the previous section, an airline model is a special case of a BSM with certain hyperparameters restricted to zero. Although ARIMA models and structural models often seem to describe the data equally well, the latter have some advantages for the practioner. They provide directly information about the decomposition of a series into trend, cyclical and seasonal components, which is either not available at all from ARIMA models or is obtainable only after additional computation. Moreover, specification of an ARIMA model can be hazardeous. It is based on the inspection of the correlogram. In practice, it is often difficult to interpret the correlogram, and there may exist a wide range of models consistent with it. The problems are particularly acute when the observations have been differenced, what is generally the case with economic time series.

### 3.2 *Statistical Properties of the Estimated Models*

The coefficient of determination  $R^2$  is of little value in the context of time series models. The measures  $R^2_D$  and  $R^2_S$  provide goodness of fit criteria which contain useful information for time series data. The baseline in  $R^2_D$  is the random walk with drift, and any model which gives a worse fit, i.e. has  $R^2_D < 0$ ,

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<sup>1)</sup> The structural models are estimated with the PC-STAMP program by S. Peters, B. Pesaran and A. Harvey.

Table 1

**Hyperparameters of the estimated structural time series models for the Austrian and German industrial production**

| AUSTRIA                      |   |       |         | GERMANY                      |   |       |         |
|------------------------------|---|-------|---------|------------------------------|---|-------|---------|
| $\hat{\sigma}_\eta^2$        | = | 0     |         | $\hat{\sigma}_\eta^2$        | = | 0     |         |
| $\hat{\sigma}_\tau^2$        | = | .20   | (.10)   | $\hat{\sigma}_\tau^2$        | = | .10   | (.00)   |
| $\hat{\sigma}_\omega^2$      | = | .40   | (.00)   | $\hat{\sigma}_\omega^2$      | = | .50   | (.10)   |
| $\hat{\sigma}_\kappa^2$      | = | 73.30 | (65.00) | $\hat{\sigma}_\kappa^2$      | = | 72.50 | (51.50) |
| $\hat{\rho}$                 | = | .96   | (.02)   | $\hat{\rho}$                 | = | .97   | (.02)   |
| $\hat{\lambda}_C$            | = | .1047 |         | $\hat{\lambda}_C$            | = | .1047 |         |
| $\hat{\sigma}_\varepsilon^2$ | = | 55.10 | (15.20) | $\hat{\sigma}_\varepsilon^2$ | = | 71.00 | (16.80) |
| $\hat{\sigma}$               | = | .052  |         | $\hat{\sigma}$               | = | .052  |         |
| $R^2$                        | = | .998  |         | $R^2$                        | = | .993  |         |
| $R^2_D$                      | = | .938  |         | $R^2_D$                      | = | .915  |         |
| $R^2_S$                      | = | .437  |         | $R^2_S$                      | = | .224  |         |
| Q(24)                        | = | 13.81 |         | Q(24)                        | = | 34.02 |         |
| T                            | = | 356   |         | T                            | = | 357   |         |

Values in parentheses are standard errors. The estimates of the variances and their corresponding standard errors were multiplied by  $10^6$ .

should not be entertained. Our estimated models perform much better than a random walk with drift. But, for monthly data with strong seasonal effects present, a random walk is not a very stringent criterion. For such series, seasonal mean models proved to give quite a good fit. The measure  $R^2_s$  adopts this model as baseline. Any model, which has  $R^2_s$  negative, can be rejected, whereas if  $R^2_s$  is positive but close to zero the gain is obviously marginal. For our models, we observe substantial gains in comparison to seasonal mean models. They yield a reduction in prediction error variance of more than 40 and 20 percent, respectively.

Additional insight into the quality of a model can be gained from an analysis of the corresponding residual series. The Q statistics in Table 1 do not point to a presence of excessive autocorrelation. Tests for normality show that the hypothesis of normality cannot be rejected. Additionally, we find no signs of skewness, kurtosis or heteroscedasticity. All in all, we observe that, while both models have acceptable statistical properties, those for the model of the Austrian industrial production seem to be significantly better.

### 3.3 *Extracted Components*

Figure 1 depicts estimates for the trends in Austrian and German industrial production. Restricting the variances of the levels to zero produces the expected smooth trends for both countries.

The long-term development of industrial production seems to have been significantly different in both countries. Starting from different production levels in the early sixties, this gap is reduced completely over the years. The process of catching-up does not take place steadily over time. We observe periods of faster catching-up such as the years from 1968 till 1972 and from 1985 onwards and periods, where the gap remains unchanged.

Figure 2 shows estimates of the cyclical components which are present in Austrian and German industrial production.

As already mentioned, we work with additive cycles with an average length of five years. The extracted cyclical components are almost identical. The correlation is 0.75. This finding should be no surprise to insiders, who know about the close connections between Austrian and German industry. The numerical order of magnitude of the cyclical components is substantial. Amplitudes of a magnitude up to 10 percentage points are observed in the early seventies. In the recent past, we find indications for decreasing amplitudes of the cyclical fluctuations especially for Germany. Obviously, Germany was more successful than Austria in reducing the share of primary industries, which seem to be particularly exposed to business cycle fluctuations.



Fig.1

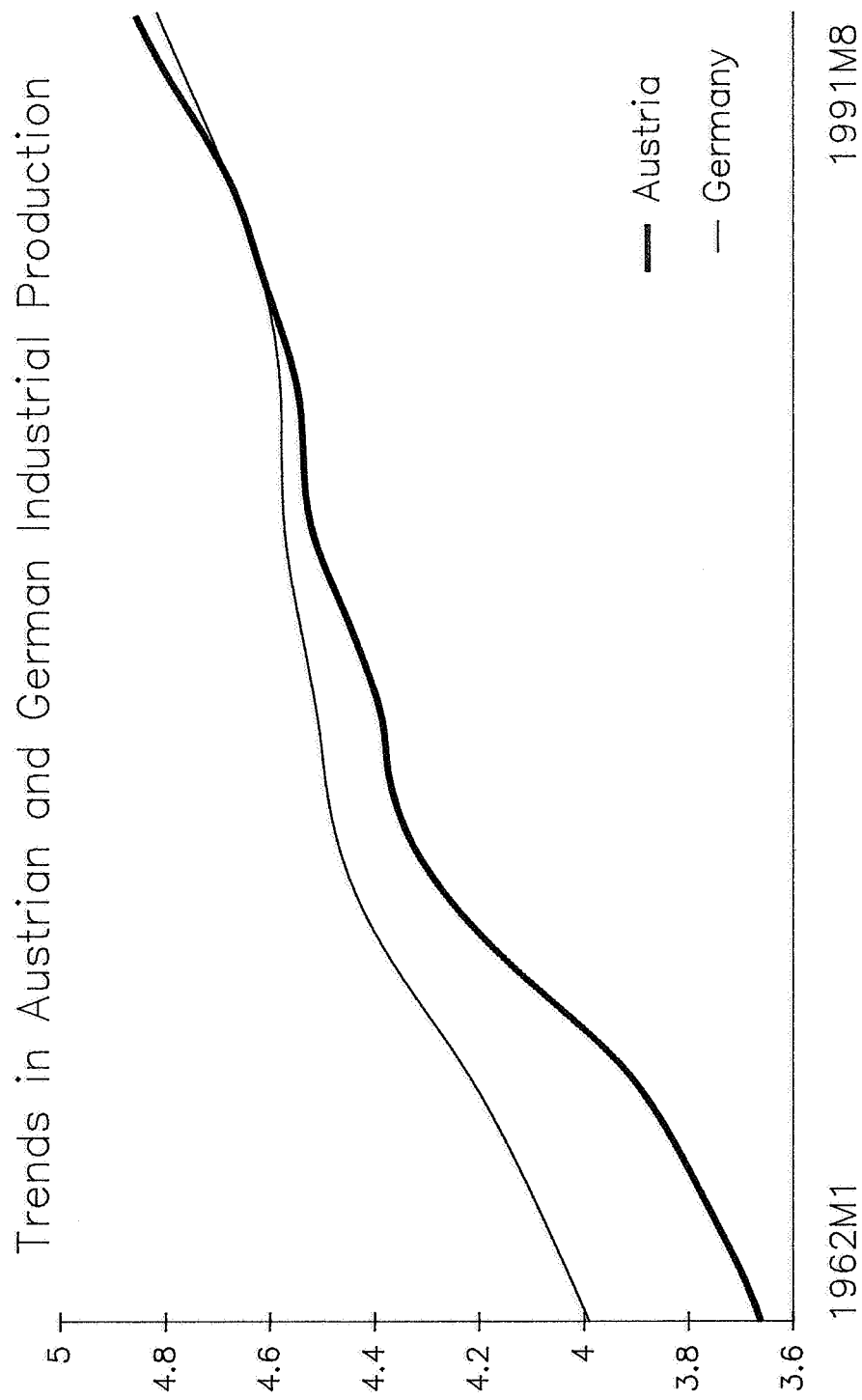


Fig.2

Cyclical components in Austrian  
and German Industrial Production

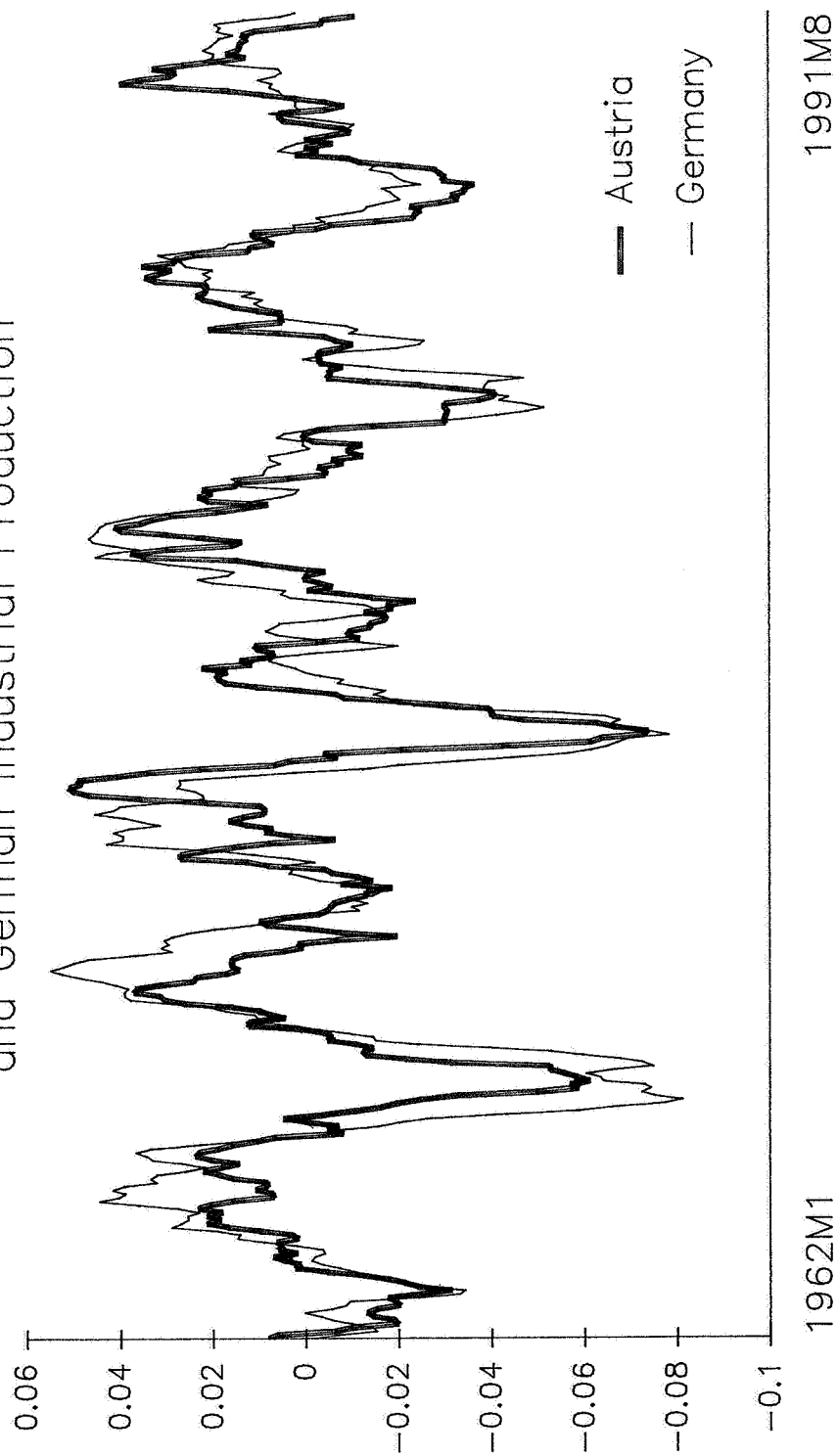


Fig.3a

Irregular components, Austria

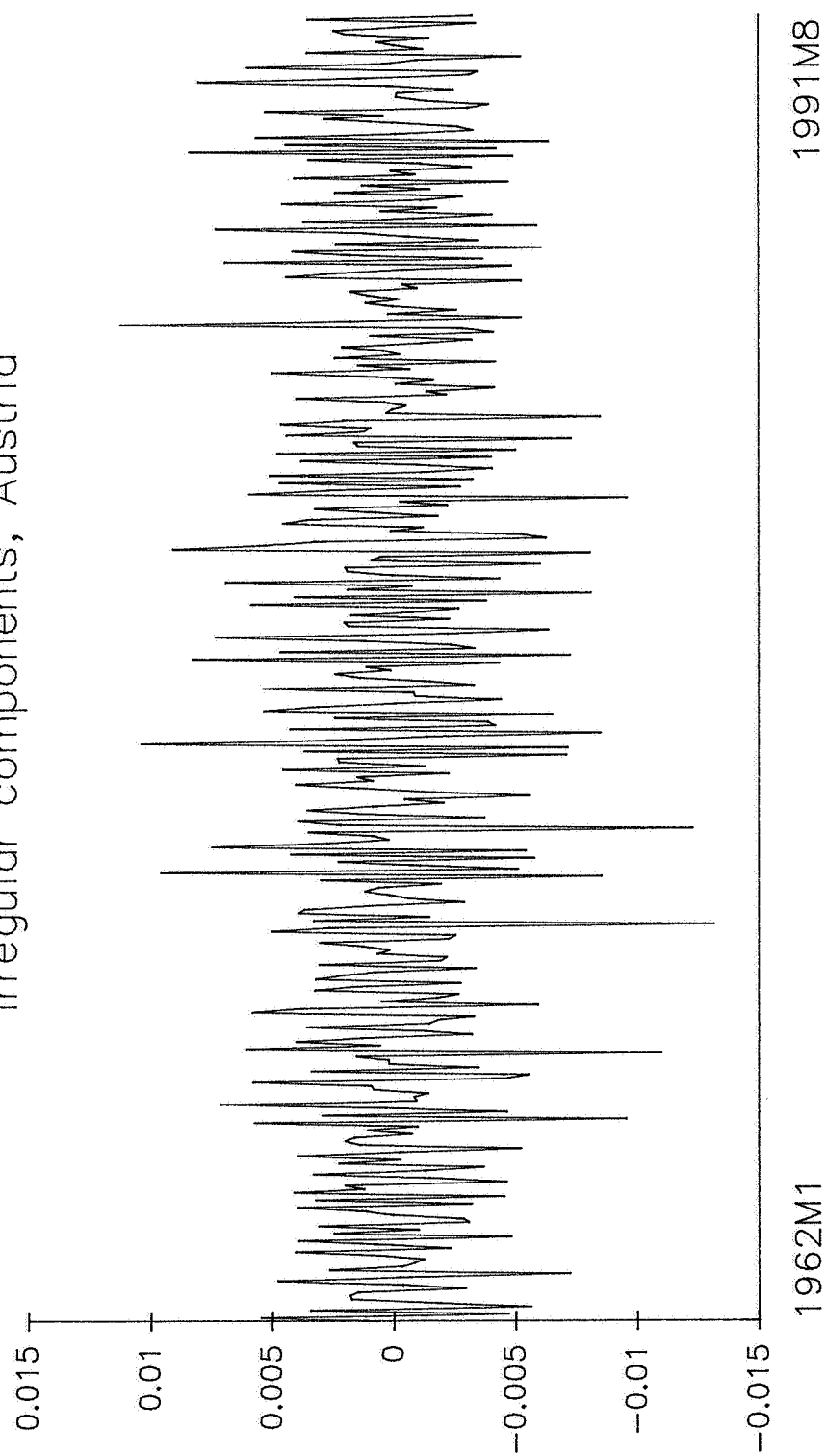
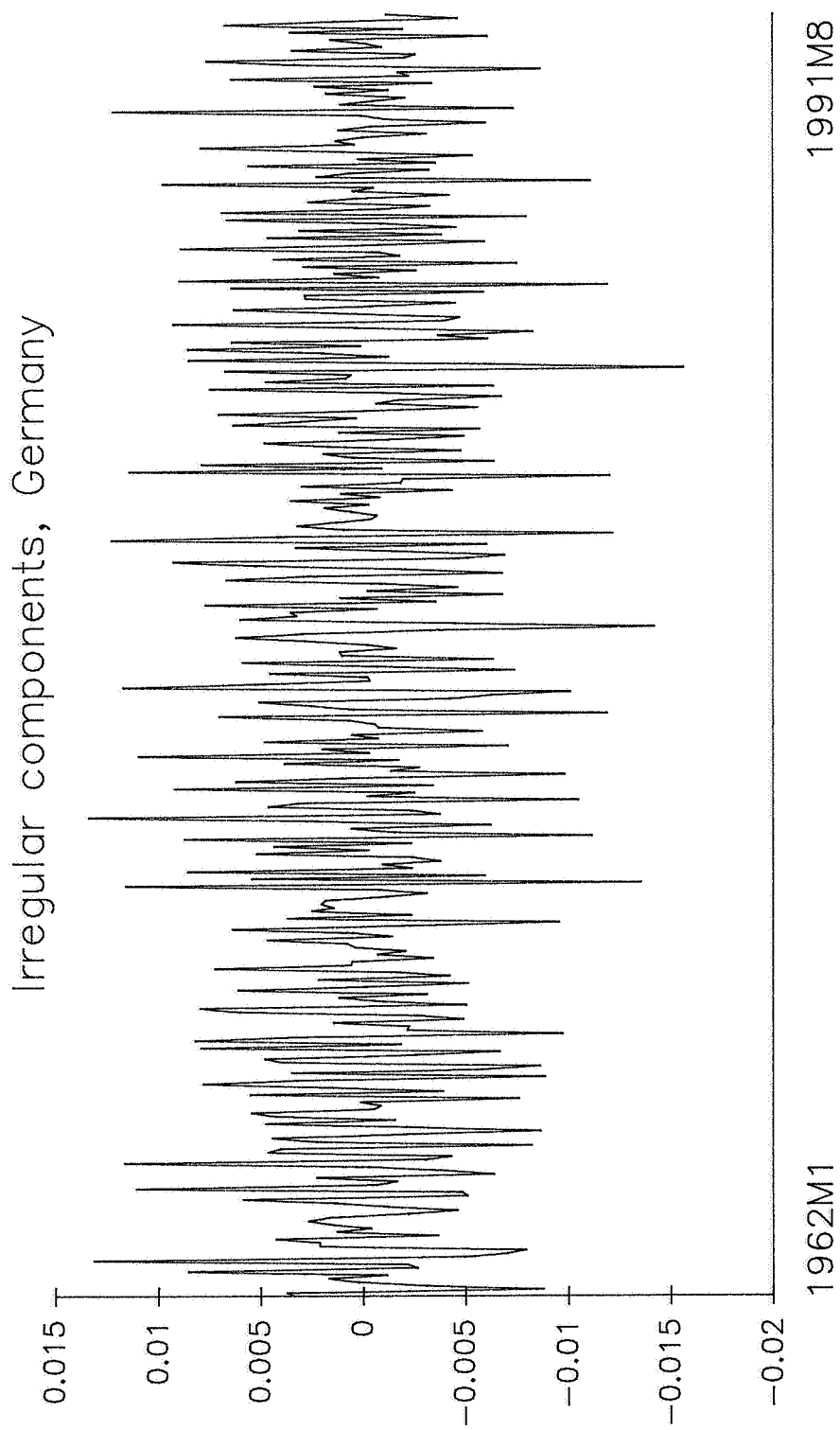


Fig.3b



Multiplicative seasonal factors for industrial production

|         | J     | F     | M      | A      | M      | J      | J     | A     | S      | O      | N      | D      |
|---------|-------|-------|--------|--------|--------|--------|-------|-------|--------|--------|--------|--------|
| AUSTRIA | .9375 | .9232 | 1.0521 | 1.0255 | 1.0511 | 1.0430 | .9524 | .8749 | 1.0256 | 1.0795 | 1.0558 | 1.0031 |
| GERMANY | .9825 | .9522 | 1.0408 | 1.0097 | 1.0247 | 1.0186 | .9313 | .9028 | 1.0014 | 1.0755 | 1.0555 | 1.0198 |

Graphs of repetitive series like seasonal components are rather unattractive. We refrain therefore from presenting graphs of the extracted seasonal components. These components evolve over time. Since the estimated variance is very small, the changes in the seasonal components stay within narrow bands. It should be sufficient to tabulate the seasonal effects in industrial for a single year, just to give an impression of the shape of the seasonal pattern. Estimating in logarithms means that the models are multiplicative. Exponentiating the estimated seasonal effects gives the following sets of multiplicative seasonal factors which are shown in Table 2.

The pattern of seasonal effects is identical in the two countries, with the amplitude of fluctuations being somewhat bigger for Austria.

Figure 3 finally shows estimates for the irregular components. We see that these irregular components are stationary white noise sequences for both countries.

### 3.4 *Forecasting Performance*

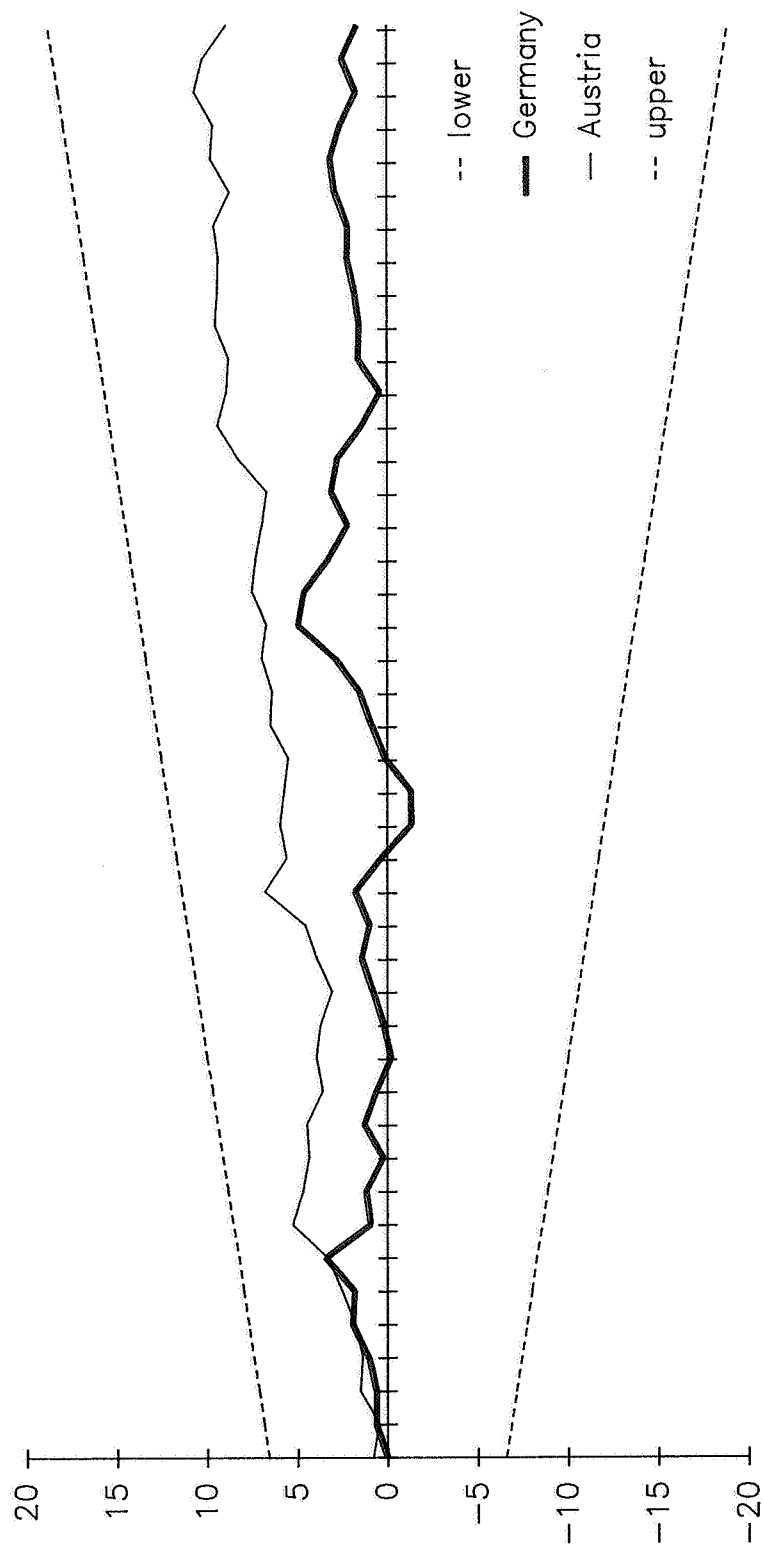
A plot of the cumulative sum (CUSUM) of standardized, one-step-ahead prediction errors is a simple device to gain insight into the forecasting performance of a model. To calculate cumulative sums of prediction errors, we estimate the models until 1987:12 and then make one-step-ahead forecasts until 1991:8. The plot of the corresponding cumulative sums is given in Figure 4.

The 5% significance lines are never crossed but, especially for the Austrian industrial production, we observe a steady rise in the cumulative sum indicating a tendency to underpredict the level of industrial production. This result is no surprise because, from 1988 onwards, Austria experienced a relatively unexpected, substantial increase in production activity.

Numerical measures of forecast accuracy provide more detailed and exacter information than cumulative sums of prediction errors. Basis for the calculation of these measures are month-to-month log-changes of the above mentioned one-step-ahead predictions.

Several numerical measures are collected in Table 3. Root mean square error (RMSE) and mean absolute error (MAE) are absolute measures, which become larger with increasing deviations between forecasts and realizations. The inequality coefficient (U) is a relative measure. It compares the performance of a particular method with naive no-change extrapolations. Values smaller than 1 indicate superiority over no-change extrapolations. The decomposition of the forecast error into a bias ( $U^M$ ), regression ( $U^R$ ), and disturbance proportion ( $U^D$ ) tells us whether this error contains systematic components, which might be reduced by applying a linear correction. For perfect forecasts, the slope coefficient in a regression of realizations on predictions should be unity. Inspection of Table 3 reveals that all calculated measures point to a satisfactory forecasting performance of the estimated structural

Fig.4  
CUSUM of standardized one-step-ahead prediction errors  
(with 5% significance lines)



*Table 3*

**Measures of Forecast Accuracy**

|                         | AUSTRIA | GERMANY |
|-------------------------|---------|---------|
| RMSE                    | .025    | .023    |
| MAE                     | .020    | .019    |
| U                       | .320    | .408    |
| UM                      | .000    | .002    |
| UR                      | .003    | .091    |
| UD                      | .997    | .907    |
| CORRELATION COEFFICIENT | .947    | .921    |
| REGRESSION COEFFICIENT  | 1.020   | .882    |



Fig.5a

Forecast for trend, Austria

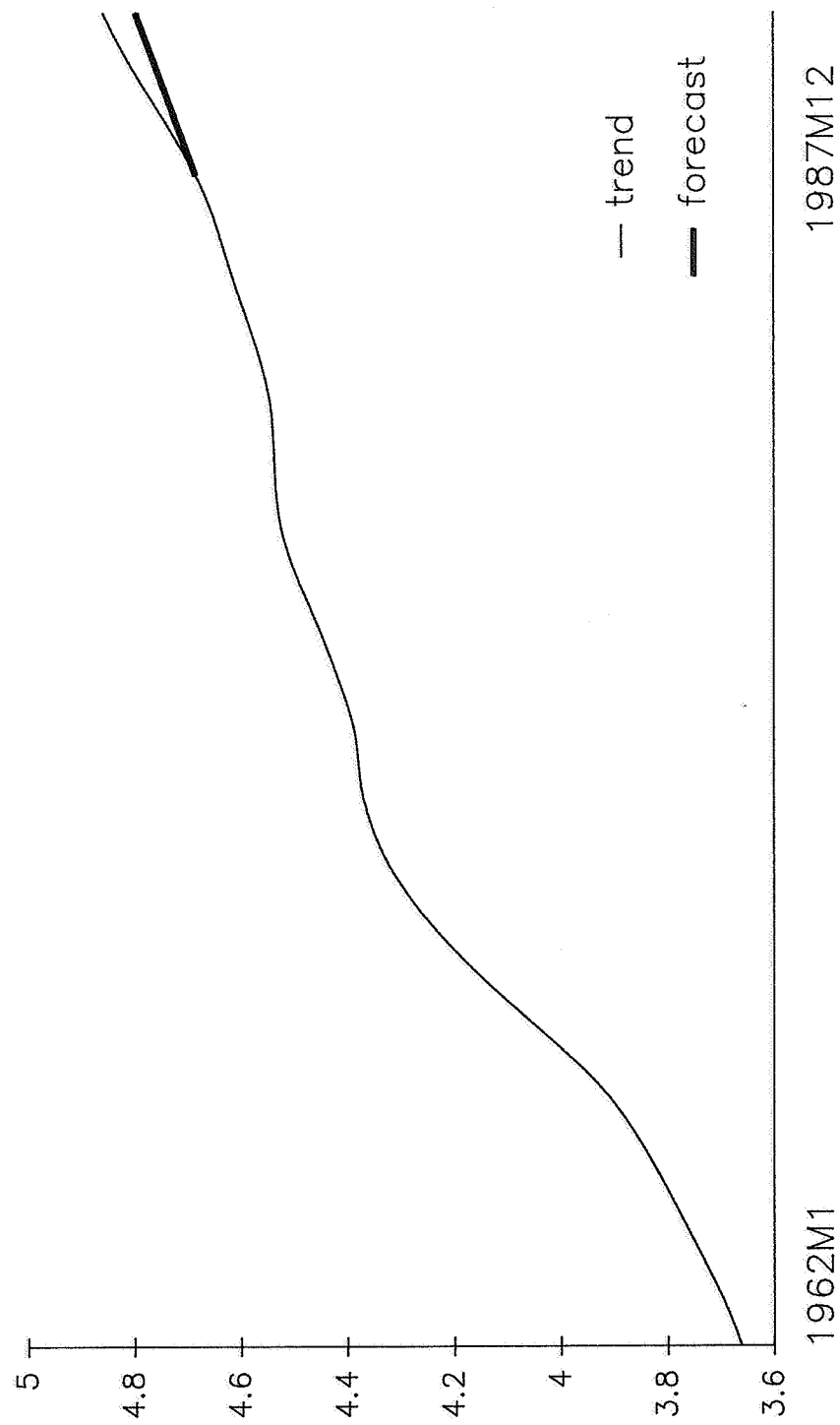


Fig.5b

Forecast for trend, Germany

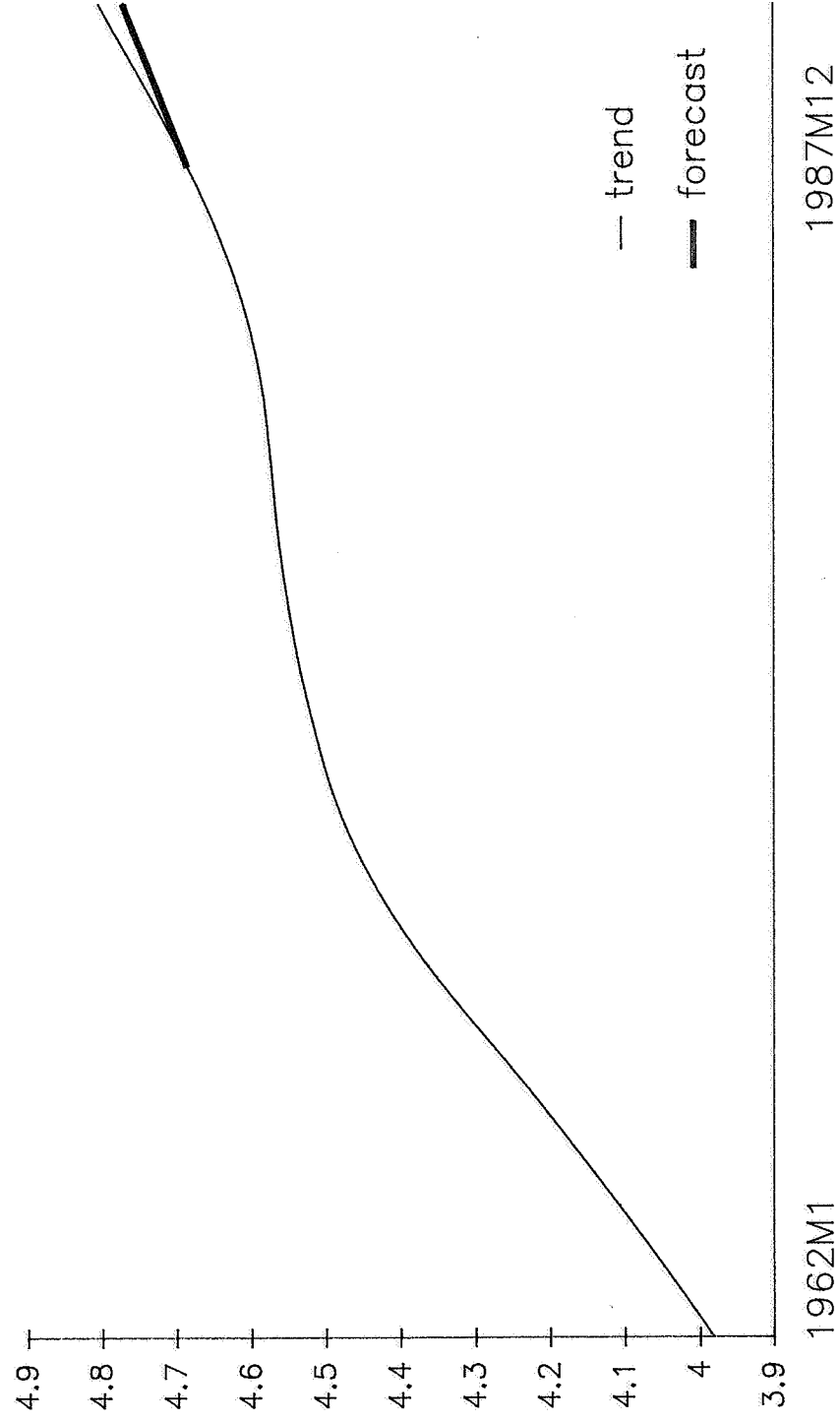


Fig.5c

Forecast for cyclical component, Austria

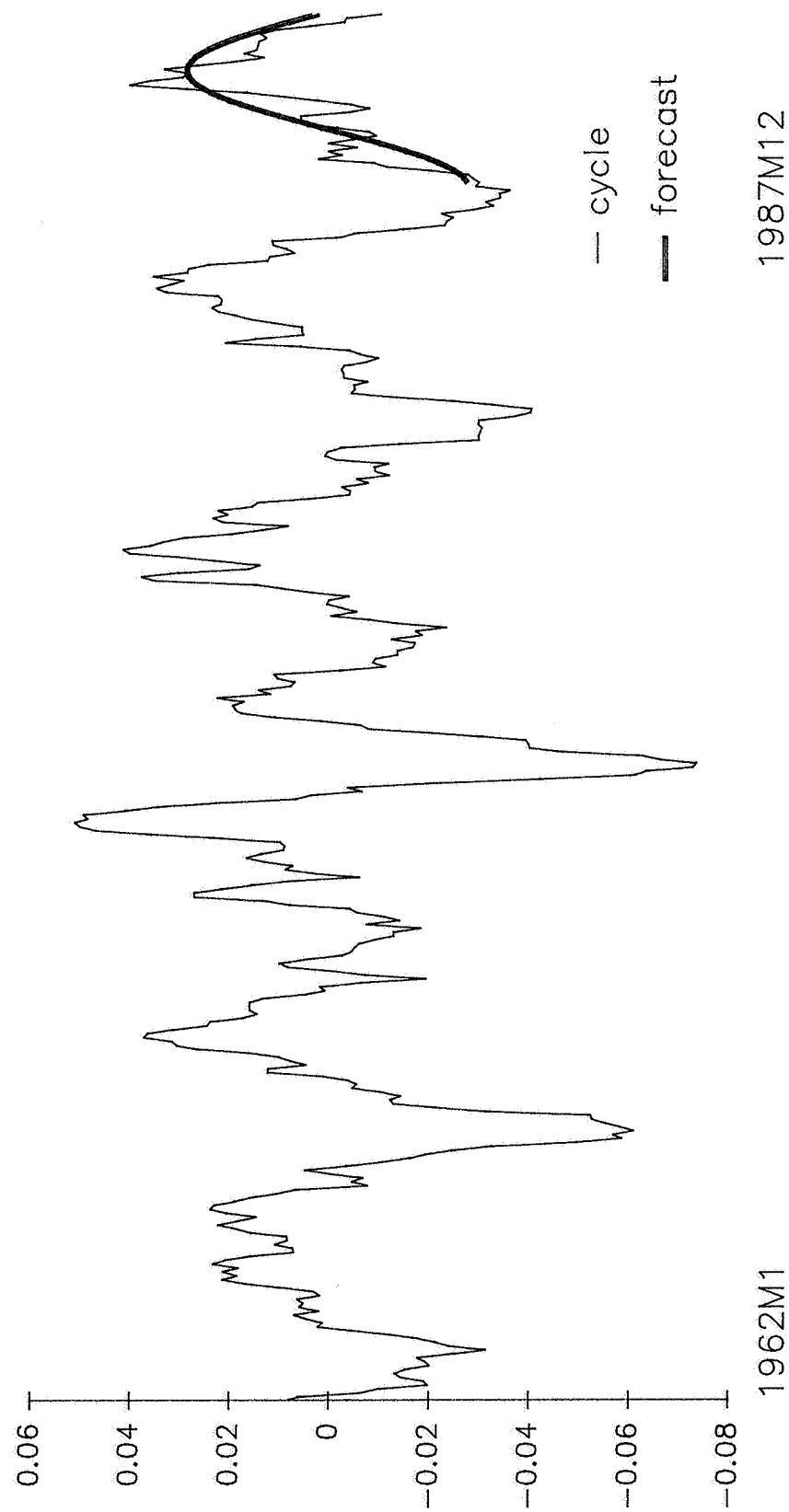
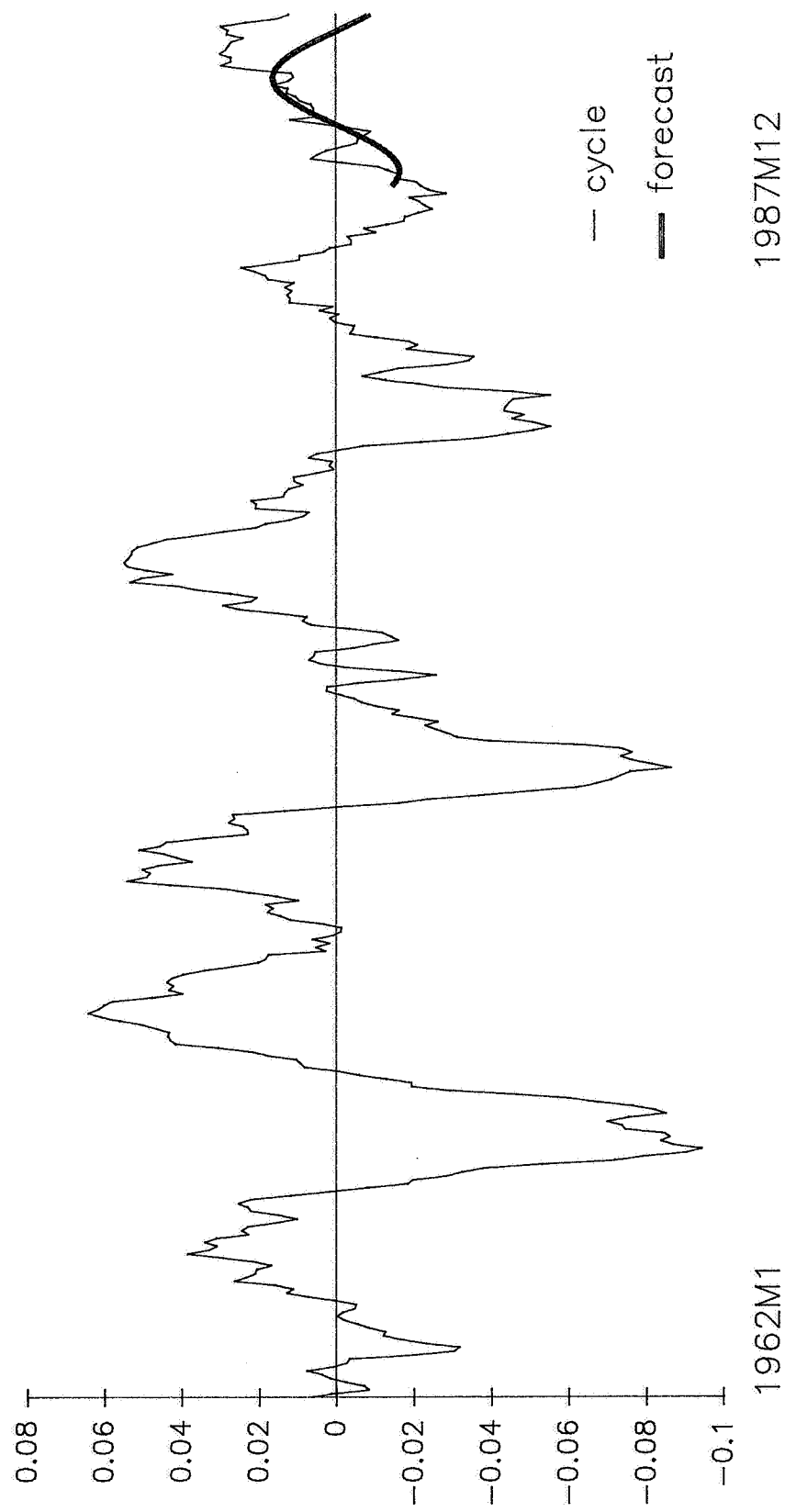


Fig.5d

Forecast for cyclical component, Germany



time series models. The regression coefficient of 1 in the case of the Austrian industrial production, indicating perfect forecasts, seems to be in conflict with the outcome of the above CUSUM test, which signals systematic underprediction. These two results are not fully comparable, however. We analyze levels for the CUSUM test and use changes of these levels in the regression.

More interesting than one-step-ahead forecasts of the original series, which could be obtained from other sources also, are extrapolations of trends and cyclical components for longer forecast horizons. Forecasts of these component series can only be derived from structural time series models what makes up one of the major advantages of this model type. Again, we estimate the two models until 1987:12 and then derive forecasts for the trend and cyclical components until 1991:8. These forecasts are confronted with the outcome from an estimation of the models until 1991:8. The results are shown in Figure 5.

We see that, for both countries, the slope of the trend lines is underpredicted with the underprediction being more severe for Austrian industrial production. Really astonishing, however, are the extrapolations of the cyclical components in two countries. The extrapolations for the cyclical component of the Austrian industrial production seem to be extremely accurate while, for Germany, we observe bigger deviations between extrapolations and realizations. We do not believe that these observed discrepancies point to an inadequacy of the estimated model. They are obviously caused by the political change which took place in that country. They might be considered as measure for the effects of the German reunion on industrial production.

#### **4. Conclusions**

In this paper we present structural models for the manufacturing sector of Austria and Germany which allow for the explicite decomposition of the monthly production data into a trend, a cyclical, a seasonal, and a irregular component. The approach chosen is a suitable vehicle to deepen our understanding of the dynamical structure of industrial production. In particular, it sheds new light on the relationship between the long and the short run dynamics of Austrian and German industrial production. It is the qualitative pattern of the trend and the cyclical components, provided by these models, which underlines the high degree of congruence between the Austrian and the German industrial sector. But it is also the pattern of these very component series which brings out the differences between the Austrian and German industry over the short and long run. It remains to be seen if plausible explanations can be found for the similarities as well as for the discrepancies observed. Only future research can tell. However, what we can tell at this very stage of research is that the estimated models perform outstandingly well in terms of forecasting accuracy over the short and medium run.

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Forecasting Performance of Structural  
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# **Forecasting Performance of Structural Time Series Models**

## **A Case Study for Austrian and German Industrial Production**

In a recent paper, Hahn and Thury (1992) present estimates of structural time series models for the Austrian and German industrial production. A comparison of the forecasting performance of this model type with that of traditional univariate time series models might be informative. As measure of reference, we use the multiplicative seasonal *ARIMA* model, which is widely used in applied economic forecasting. We have 372 monthly observations on the indices of industrial production in Austria and Germany. Since calendar variations strongly influence production data, the two series are first adjusted for these effects before the different time series models are estimated. Details about this calendar adjustment can be found in Thury (1989). Since the number of observations at our disposal is relatively large, we can retain a substantial portion of these observations in order to test the forecasting performance of the estimated time series models. Thus, for the different forecasting methods and forecasting horizons, we generate 120 genuine ex-ante predictions covering the period 1983:1 to 1992:12 which, then, form the basis for an evaluation of the forecasting accuracy of the methods under test. We are convinced that 120 observations should be sufficient to derive reliable estimates for various test statistics of forecasting performance.

The organisation of the paper is as follows. We start out with a short description of the measures of forecasting accuracy which we shall employ in this paper. The main part of the paper consists of a presentation and interpretation of our empirical results. In a short concluding section, finally, we summarize our main findings.

### **Theoretical considerations**

In assessing the forecasting accuracy of the time series models under consideration, we closely follow the path proposed by Witt and Witt (1992). We begin with a detailed analysis of the committed forecast errors. Since it is often claimed, however, that directional accuracy is, at least, as important as the magnitude of the forecast error, we investigate the performance in this respect also very carefully.



### *Measures of numerical accuracy and statistical tests*

In order to evaluate the accuracy of a forecasting method, it is necessary to have a yardstick. There exist various measures of forecasting accuracy but, unfortunately, none of them is universally accepted. Following Witt and Witt, we shall concentrate on two relative measures of forecasting accuracy, namely the mean absolute percentage error and the root mean square percentage error.

The mean absolute percentage error (*MAPE*) is given by

$$(1) \quad MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|e_t|}{IP_t} \cdot 100,$$

where  $|e_t|$  denotes the absolute value of the forecast error and  $n$  is the number of forecasts. The forecast error is given by

$$(2) \quad e_t = \hat{IP}_t - IP_t,$$

where  $\hat{IP}_t$  and  $IP_t$  symbolize predicted and actual values of the index of industrial production, respectively. *MAPE* is a measure of overall accuracy which offers an indication of the degree of spread between predicted and observed values. All forecast errors are assigned equal weights. Table 1 contains typical *MAPE* values for industrial data and their interpretation, which were published originally by Lewis (1982).

Table 1

| Interpretation of typical <i>MAPE</i> values |                             |
|--|-----------------------------|
| <i>MAPE</i>                                  | Interpretation              |
| < 10 percent                                 | Highly accurate forecasting |
| 10 – 20 percent                              | Good forecasting            |
| 20 – 50 percent                              | Reasonable forecasting      |
| > 50 percent                                 | Inaccurate forecasting      |

Reproduced from Witt and Witt(1992), p. 86

The root mean square percentage error (*RMSPE*) is given by

$$(3) \quad RMSPE = \sqrt{\frac{1}{n} \sum_{t=1}^n \left[ \frac{e_t}{IP_t} \right]^2} \cdot 100.$$

The *RMSPE* is also a measure of overall accuracy which provides an indication of the degree of spread. But, contrary to *MAPE*, large errors are penalized by additional weight.

In studies of forecasting performance it has been common practice for a long time to simply present accuracy measures in tabular form. Supplementing these presentations by statistical tests will provide additional insight and might allow firmer conclusions. An approach, which seems to be especially adequate for the purposes of this paper, is the ANOVA technique because it allows for varying numbers of factors to be tested simultaneously. Thus, we can test whether there exist significant differences between forecasting methods, forecasting horizons, and production countries. A certain drawback of the ANOVA approach lies in the fact that it may indicate significant differences among factor levels, but not between which levels if there are more than two. Multiple comparison tests, as for example Scheffe's test, or pairwise *t*-tests can provide answers to open questions of this type.

### *Measures of directional accuracy*

Numerical accuracy is one of the desirable features of a forecast, directional accuracy is another, perhaps even more important, property. With directional accuracy, we must distinguish between direction of change errors (sometimes also called tracking errors) and trend change errors. A direction of change error occurs if the forecast misses the actual direction of change. There are several possibilities. The predicted change is positive and the actual change is negative or vice versa. Additionally, an observed change in direction can be missed by the forecast or a change in direction can be predicted which, then, does not realize. We compress these different possibilities into a single measure of direction of change error by calculating the percentage of correctly predicted changes of direction.

Instead of just looking generally at directional accuracy, it may be informative to analyze the situation more closely by examining trend change accuracy. A trend change error is observed when either a forecasting method fails to predict a realized change in the trend (a missed trend change) or incorrectly predicts a trend change (a false signal). Trend changes may be divided into downturns and upturns, and varying numbers of observations can be employed in their definition. Following Witt and Witt, we define them as follows:

$$(4) \quad y_{n-2} < y_{n-1} < y_n \quad \text{and} \quad \begin{cases} Z < y_n & = \text{Downturn (DT)}, \\ Z \geq y_n & = \text{No downturn (NDT)}, \end{cases}$$

and

$$y_{n-2} > y_{n-1} > y_n \quad \text{and} \quad \begin{cases} Z > y_n & = \text{Upturn (UT)}, \\ Z \leq y_n & = \text{No upturn (NUT)}, \end{cases}$$

where  $y_1, y_2, \dots, y_n$  denote given past realizations of a time series, and  $Z \equiv y_{n+1}$  is the first future value of this series. Four consecutive observations are used to define downturns and upturns. A downturn is observed when an increasing trend has been established by the two observations preceeding the current one and the following observation is smaller than the current one. Similarly, an upturn occurs when a decreasing trend has been established, and the following observation is greater than the current one.

## Empirical results

In the following, we analyze the forecasting performance of four univariate time series models:

basic structural model (*BSM*);

structural model with additive cycle (*SMAC*);

structural model with additive cycle and damping factor 1.00 (*SMACX*);

Box-Jenkins airline model (*ARIMA*).

These four model versions are used to generate predictions for the Austrian and German industrial production with forecasting horizons of 1, 6, 12, 18, and 24 months covering the period 1983:1 to 1992:12. In order to obtain these forecasts, the models are always reestimated for the relevant sample periods. Only information, which would have been available already at the date of the forecast origin, is utilized. The predictions, which are analyzed in this paper, are thus genuine ex-ante forecasts<sup>1)</sup>.

### *Numerical measures of forecasting accuracy*

Tables 2 and 3 contain measures of forecasting accuracy for the different sets of predictions. Table 2 summarizes the results for *RMSPE*, Table 3 those for *MAPE*. Generally, both *RMSPE* and *MAPE* are subject to distortion caused by outlying observations, in that one or two poor forecasts will affect these average error measures. However, given the large number of forecast errors under analysis, this is unlikely to be a serious problem here.

Before entering into a detailed discussion of the different accuracy measures, some form of general assessment might be useful. Comparing the *MAPE*'s in Table 3 with the typical values given in Table 1, we see that, according to this standard, even the 24-months-ahead predictions figure as highly

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<sup>1)</sup> In the computations for this paper the PC versions of the following programs are applied: SCA, SPSS, STAMP, and TSP. I wish to thank Sonja Patsios for her short, but very informative introductory course to SPSS.

accurate. Our judgement is less euphoric. We would say that forecasts with a horizon of up to 12 months might provide valuable information.

Table 2

**Forecasting performance by forecasting horizon, forecasting method, and production country: *RMSPE***

| Forecasting horizon<br>(months) | Forecasting method | Production country |          |
|---------------------------------|--------------------|--------------------|----------|
|                                 |                    | Austria            | Germany  |
| 1                               | <i>BSM</i>         | 1.644(1)           | 1.734(1) |
|                                 | <i>SMAC</i>        | 1.657(2)           | 1.741(2) |
|                                 | <i>SMACX</i>       | 1.674(3)           | 1.765(4) |
|                                 | <i>ARIMA</i>       | 1.695(4)           | 1.744(3) |
| 6                               | <i>BSM</i>         | 2.675(1)           | 2.605(1) |
|                                 | <i>SMAC</i>        | 2.757(2)           | 2.870(2) |
|                                 | <i>SMACX</i>       | 2.856(4)           | 3.157(4) |
|                                 | <i>ARIMA</i>       | 2.761(3)           | 2.933(3) |
| 12                              | <i>BSM</i>         | 3.626(1)           | 3.002(1) |
|                                 | <i>SMAC</i>        | 3.658(2)           | 3.678(2) |
|                                 | <i>SMACX</i>       | 3.901(3)           | 4.279(4) |
|                                 | <i>ARIMA</i>       | 4.039(4)           | 3.750(3) |
| 18                              | <i>BSM</i>         | 4.793(1)           | 3.934(1) |
|                                 | <i>SMAC</i>        | 4.891(2)           | 4.943(2) |
|                                 | <i>SMACX</i>       | 5.157(3)           | 6.006(4) |
|                                 | <i>ARIMA</i>       | 5.613(4)           | 5.090(3) |
| 24                              | <i>BSM</i>         | 5.593(2)           | 4.377(1) |
|                                 | <i>SMAC</i>        | 5.589(1)           | 5.943(3) |
|                                 | <i>SMACX</i>       | 5.818(3)           | 7.421(4) |
|                                 | <i>ARIMA</i>       | 6.714(4)           | 5.888(2) |

Now, we shall turn to a detailed analysis of these accuracy measures. As expected has the length of the forecasting horizon the biggest effect for accuracy. Both measures, *RMSPE* and *MAPE*, give identical results in this respect. The longer the forecasting horizons the larger are the errors, although the differences for longer horizons (18 and 24 months) are less pronounced. The different forecasting methods also give rise to variations in the size of the forecast errors. The observed differences are however by far less significant than in the case of forecasting horizons. Additionally, we observe here slight discrepancies between the results for Austria and Germany. For Germany both measures, *RMSPE* and *MAPE*, yield identical results. Here, the ranking of forecasting methods is unequivocal. *BSM* is best, followed by *SMAC*. It is perhaps a little surprising that *ARIMA* outperforms *SMACX*. For Austria, the outcome is slightly more controversial, as we observe certain differences in the ranking according to *RMSPE* and *MAPE*. Relying on *RMSPE*'s, the ranking is identical to the German one apart

from the fact that, in Austria, the *ARIMA* model gives the worst forecasts. The *MAPE* based results also show that the structural time series models clearly outperform the traditional *ARIMA* model. Only the ranking within the group of structural models varies somewhat between the *RMSPE* and the *MAPE* results.

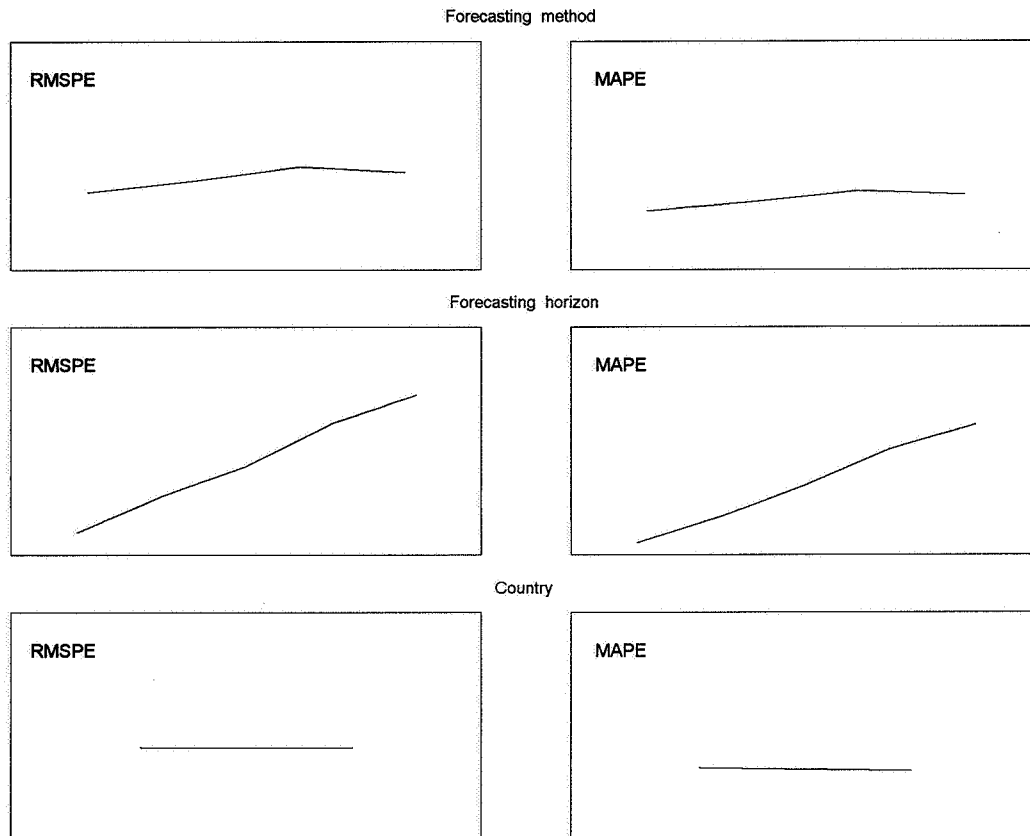
Table 3

| Forecasting performance by forecasting horizon, forecasting method, and production country: <i>MAPE</i> |                    |                    |          |
|---|--------------------|--------------------|----------|
| Forecasting horizon<br>(months)   | Forecasting method | Production country |          |
|   |                    | Austria            | Germany  |
| 1   | <i>BSM</i>         | 1.355(1)           | 1.382(2) |
|   | <i>SMAC</i>        | 1.373(3)           | 1.378(1) |
|   | <i>SMACX</i>       | 1.372(2)           | 1.389(4) |
|   | <i>ARIMA</i>       | 1.393(4)           | 1.388(3) |
| 6   | <i>BSM</i>         | 2.125(3)           | 2.063(1) |
|   | <i>SMAC</i>        | 2.053(1)           | 2.226(2) |
|   | <i>SMACX</i>       | 2.119(2)           | 2.511(4) |
|   | <i>ARIMA</i>       | 2.221(4)           | 2.228(3) |
| 12  | <i>BSM</i>         | 3.192(3)           | 2.498(1) |
|   | <i>SMAC</i>        | 3.026(1)           | 3.069(2) |
|   | <i>SMACX</i>       | 3.138(2)           | 3.556(4) |
|   | <i>ARIMA</i>       | 3.592(4)           | 3.176(3) |
| 18  | <i>BSM</i>         | 4.127(1)           | 3.245(1) |
|   | <i>SMAC</i>        | 4.179(2)           | 4.143(2) |
|   | <i>SMACX</i>       | 4.286(3)           | 4.919(4) |
|   | <i>ARIMA</i>       | 4.905(4)           | 4.306(3) |
| 24  | <i>BSM</i>         | 4.794(1)           | 3.583(1) |
|   | <i>SMAC</i>        | 4.853(2)           | 4.926(3) |
|   | <i>SMACX</i>       | 4.996(3)           | 6.217(4) |
|   | <i>ARIMA</i>       | 5.782(4)           | 4.835(2) |

Summarizing the outcome of this ranking exercise, one might say that, in both countries, a basic structural time series model gives the predictions of industrial production with the smallest forecast error. This result is somewhat surprising. Intuitively, one would expect that the explicit introduction of a cyclical component should improve the tracking performance of a model. But, although estimates of such cyclical components may provide valuable qualitative information on the current state of the business cycle, their extrapolation into the future seems to be too unreliable in order to give rise to improved quantitative forecasts. This problem becomes particularly visible, if the cyclical component enters a model in undamped form, as it is the case in *SMACX*.

Figure 1

**Main effects for different factors**



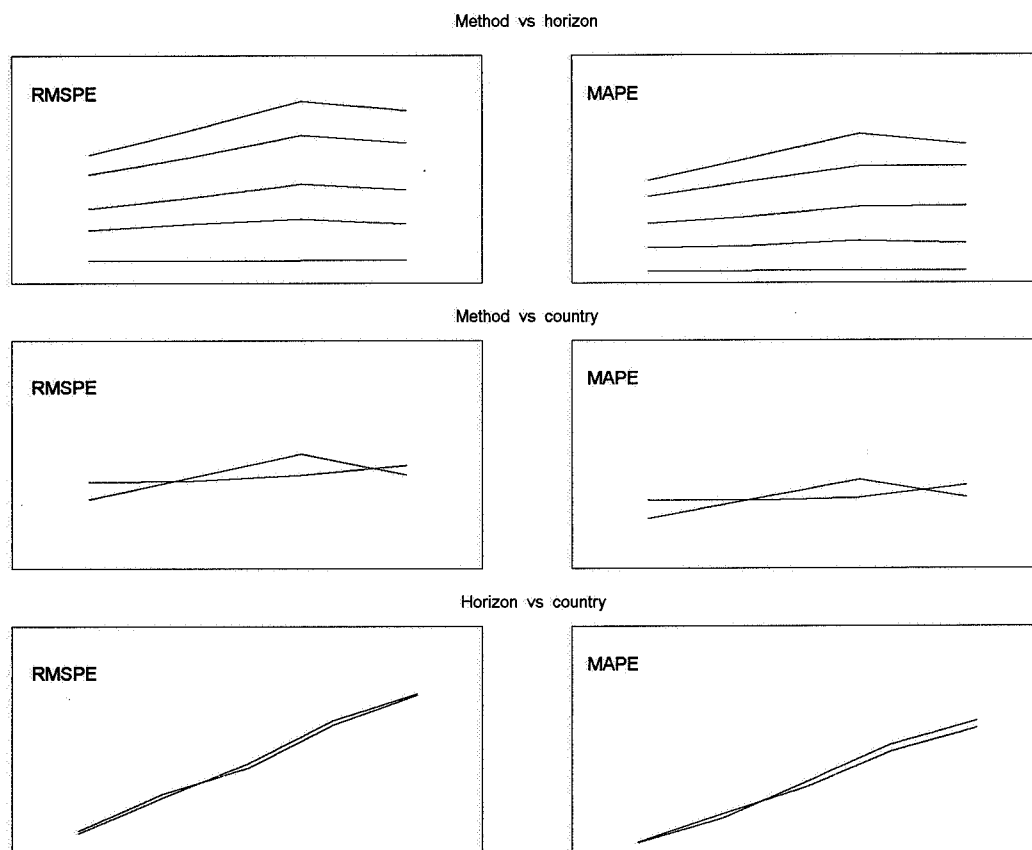
*ANOVA results and statistical testing*

Many empirical studies of forecasting performance present the results in terms of the accuracy measures considered. Often however additional analysis, such as an application of the ANOVA technique, might provide valuable further insight. There exist simple graphical techniques, which allow to decide quickly whether such further analysis is worthwhile or not. Figures 1 and 2 contain such simple graphs. In Figure 1 we depict the main effects for different factors individually. Thus, we have only one line in a graph. If this line slopes, this is an indication for the existence of a significant effect of a particular factor. The steeper the slope, the more significant the effect will be. If the line is horizontal, no significant effect is present. In Figure 2 we depict interaction effects for pairs of factors. Consequently, we have two lines in a graph. If these lines are parallel, no interaction between the

factors occurs. If these lines intersect, significant interaction between two factors is present. cursory inspection of these graphs provides some evidence for the existence of both main and interaction effects. Thus, further analysis seems to be worthwhile.

Figure 2

Interaction effects for different factors



Tables 4 and 5 present the outcome of an application of the ANOVA technique to *RMSPE*'s and *MAPE*'s. In each ANOVA table, we investigate if any of the three factors (forecasting method *FM*, forecasting horizon *H*, production country *C*) or interactions of these three factors have a significant effect on forecasting accuracy, as measured either by *RMSPE* or *MAPE*. *FM* has four levels, *H* five, and *C* two. Contrary to the above ranking example, the ANOVA results for *RMSPE*'s and *MAPE*'s are fully identical. In both cases, we find statistically significant effects for two of the three factors, namely for forecasting methods (*FM*) and forecasting horizon (*H*). The production country has no effect on the

accuracy of the forecasts. There does not exist a statistically significant difference in the size of forecast errors for Austria and Germany. Of the three possible two-way interactions only one is significant, namely that between forecasting methods and production country. This implies that there exists a significant difference in the performance of the different forecasting methods in the two countries under study. We did not have enough data to test for the existence of a three-way interaction.

Table 4

ANOVA results for pooled data of Austria and Germany: *RMSPE*

| Source of variation  | Sum of squares | Degrees of freedom | Mean square | <i>F</i> | Significance of <i>F</i> |
|----------------------|----------------|--------------------|-------------|----------|--------------------------|
| Main effects         | 94.661         | 8                  | 11.833      | 89.704   | 0.000                    |
| Forecasting methods  | 3.647          | 3                  | 1.216       | 9.217    | 0.002                    |
| Forecasting horizon  | 91.012         | 4                  | 22.753      | 172.493  | 0.000                    |
| Production country   | 0.002          | 1                  | 0.002       | 0.012    | 0.915                    |
| Two-way interactions | 4.046          | 19                 | 0.213       | 1.614    | 0.199                    |
| <i>FM – H</i>        | 1.945          | 12                 | 0.162       | 1.229    | 0.363                    |
| <i>FM – C</i>        | 1.994          | 3                  | 0.665       | 5.039    | 0.017                    |
| <i>H – C</i>         | 0.107          | 4                  | 0.027       | 0.203    | 0.932                    |
| Explained            | 98.707         | 27                 | 3.656       | 27.715   | 0.000                    |
| Residual             | 1.593          | 12                 | 0.132       |          |                          |
| Total                | 100.290        | 39                 | 2.572       |          |                          |

Table 5

ANOVA results for pooled data of Austria and Germany: *MAPE*

| Source of variation  | Sum of squares | Degrees of freedom | Mean square | <i>F</i> | Significance of <i>F</i> |
|----------------------|----------------|--------------------|-------------|----------|--------------------------|
| Main effects         | 72.008         | 8                  | 9.001       | 97.171   | 0.000                    |
| Forecasting method   | 2.342          | 3                  | 0.781       | 8.427    | 0.003                    |
| Forecasting horizon  | 69.582         | 4                  | 17.395      | 187.792  | 0.000                    |
| Production country   | 0.085          | 1                  | 0.085       | 0.917    | 0.357                    |
| Two-way interactions | 3.542          | 19                 | 0.186       | 2.012    | 0.108                    |
| <i>FM – H</i>        | 1.529          | 12                 | 0.127       | 1.376    | 0.295                    |
| <i>FM – C</i>        | 1.821          | 3                  | 0.607       | 6.553    | 0.007                    |
| <i>H – C</i>         | 0.191          | 4                  | 0.048       | 0.517    | 0.725                    |
| Explained            | 75.550         | 27                 | 2.798       | 30.207   | 0.000                    |
| Residual             | 1.112          | 12                 | 0.093       |          |                          |
| Total                | 76.661         | 39                 | 1.966       |          |                          |



The above ANOVA results tell us only that there exist significant differences in forecasting accuracy for different forecasting horizons and different forecasting methods. However, they provide no ranking for different levels of a particular factor. Thus, if a significant factor has more than two levels, further statistical tests become necessary in order to find out which of these levels are different from each other.

Table 6

Modified pairwise *t*-tests for differences in accuracy between forecasting methods: pooled sample

| <i>RMSPE</i> | <i>BSM</i> | <i>SMAC</i> | <i>SMACX</i> | <i>ARIMA</i> |
|--------------|------------|-------------|--------------|--------------|
| <i>BSM</i>   |            |             |              |              |
| <i>SMAC</i>  | 3.744**    |             |              |              |
| <i>SMACX</i> | 8.051**    | 4.307**     |              |              |
| <i>ARIMA</i> | 6.244**    | 2.500       | 1.807        |              |
| <i>MAPE</i>  |            |             |              |              |
| <i>BSM</i>   |            |             |              |              |
| <i>SMAC</i>  | 2.862      |             |              |              |
| <i>SMACX</i> | 6.139**    | 3.277*      |              |              |
| <i>ARIMA</i> | 5.462**    | 2.600       | 0.677        |              |

\* indicates 5% level of significance,

\*\* indicates 1% level of significance

Of our two significant factors, forecasting horizon has five and forecasting method four levels. In order to determine a hierarchy among these levels, we apply various alternatives of multiple range tests (among them Scheffe's test). Only for forecasting horizons significant differences are revealed by these tests. *MAPE*'s and *RMSPE*'s are different for each particular forecasting horizon. That forecasts with shorter horizons have smaller errors, is selfevident. That all five horizons are classified as significantly different by multiple range tests, is somewhat surprising. For forecasting methods on the other side, no significant differences could be found by these tests. The reason for this failure may be twofold. Multiple range tests are, in general, rather conservative and the observed differences in the *MAPE*'s and *RMSPE*'s are numerically small. We use a modified version of a paired *t*-test in order to overcome these problems. The null of nonsignificant differences between forecasting methods is rejected if

$$(5) \quad |x_i. - x_j.| > \sqrt{2 n S_R F_{n-a}}.$$

In this expression,  $x_{i.}$  and  $x_{j.}$  denote sample sums of *MAPE*'s and *RMSPE*'s, respectively, for method *i* and method *j*, *n* is the sample size, *a* the number of levels, and *SR* the residual mean square of the

ANOVA table with  $(n-a)$  degrees of freedom. The outcome of this testing procedure is given in Table 6. We see that for both accuracy measures, *MAPE* and *RMSPE*, *BSM* is definitely superior to *SMACX* and *ARIMA*. There exists some evidence that it also dominates *SMAC*. The poor performance of *SMACX* is an obvious consequence of setting the damping factor equal to 1.00. This operation is far from optimal in the context of forecasting. It might, however, provide valuable qualitative information on the future cyclical development of industrial production.

Of the three possible interaction effects only that between forecasting methods (*FM*) and production country (*C*) is statistically significant. There exist apparently some differences between Austria and Germany in the accuracy of the tested forecasting method. The situation can be looked at from two perspectives:

- (i) the factor *FM* is specified and the data, on which the analysis is performed, are restricted to a given country;
- (ii) the factor *C* is specified, and the data are restricted to a given method.

The first approach shows where any statistical differences lie among forecasting methods for a given country. The second approach informs us about differences between the two countries for a given forecasting method.

Table 7

Modified pairwise *t*-tests for interaction effects between forecasting methods and production country: *RMSPE*

Case (i)

|              | Austria    |             |              |              | Germany    |             |              |              |
|--------------|------------|-------------|--------------|--------------|------------|-------------|--------------|--------------|
|              | <i>BSM</i> | <i>SMAC</i> | <i>SMACX</i> | <i>ARIMA</i> | <i>BSM</i> | <i>SMAC</i> | <i>SMACX</i> | <i>ARIMA</i> |
| <i>BSM</i>   |            |             |              |              |            |             |              |              |
| <i>SMAC</i>  | 0.221      |             |              |              | 3.523*     |             |              |              |
| <i>SMACX</i> | 1.075      | 0.854       |              |              | 6.976**    | 3.453*      |              |              |
| <i>ARIMA</i> | 2.551**    | 2.330*      | 1.476        |              | 3.753*     | 0.230       | 3.223*       |              |

Case (ii)

|         |              | Austria    |             |              |              |
|---------|--------------|------------|-------------|--------------|--------------|
|         |              | <i>BSM</i> | <i>SMAC</i> | <i>SMACX</i> | <i>ARIMA</i> |
|         | <i>BSM</i>   | 2.679*     |             |              |              |
| Germany | <i>SMAC</i>  |            | 0.623       |              |              |
|         | <i>SMACX</i> |            |             | 3.222*       |              |
|         | <i>ARIMA</i> |            |             |              | 1.417        |

\* indicates 5% level of significance,

\*\* indicates 1% level of significance

Table 8

Modified pairwise *t*-tests for interaction effects between forecasting methods and production country: *MAPE*

Case (i)

|              | Austria    |             |              |              | Germany    |             |              |              |
|--------------|------------|-------------|--------------|--------------|------------|-------------|--------------|--------------|
|              | <i>BSM</i> | <i>SMAC</i> | <i>SMACX</i> | <i>ARIMA</i> | <i>BSM</i> | <i>SMAC</i> | <i>SMACX</i> | <i>ARIMA</i> |
| <i>BSM</i>   |            |             |              |              |            |             |              |              |
| <i>SMAC</i>  | 0.109      |             |              |              | 2.971*     |             |              |              |
| <i>SMACX</i> | 1.318      | 0.427       |              |              | 5.821**    | 2.850       |              |              |
| <i>ARIMA</i> | 2.300*     | 2.409*      | 1.982        |              | 3.162*     | 0.191       | 2.659        |              |

Case (ii)

|         |              | Austria    |             |              |              |
|---------|--------------|------------|-------------|--------------|--------------|
|         |              | <i>BSM</i> | <i>SMAC</i> | <i>SMACX</i> | <i>ARIMA</i> |
|         | <i>BSM</i>   | 2.822*     |             |              |              |
| Germany | <i>SMAC</i>  |            | 0.258       |              |              |
|         | <i>SMACX</i> |            |             | 2.681*       |              |
|         | <i>ARIMA</i> |            |             |              | 1.960        |

\* indicates 5% level of significance,

\*\* indicates 1% level of significance

Since multiple range tests yield no significant results, we apply the above mentioned modified *t*-tests. The outcome of these tests is presented in Tables 7 and 8. For both measures of accuracy, *RMSPE* and *MAPE*, we observe differences in the performance of the forecasting methods for a given country. These observed differences are for Germany much more significant than for Austria. For Germany, we find that *BSM* definitely outperforms the other methods. *SMAC* and *ARIMA* yield predictions of similar absolute accuracy. *SMACX* gives by far the worst results. For Austria, we find only weak evidence that *BSM* and *SMAC* do somewhat better than the other two methods, at least, as far as absolute forecasting accuracy is concerned. When looking for differences between the two countries for a given method, we find that *BSM* does significantly better in Germany than in Austria while, for *SMACX*, the exact opposite is the case. The performance of the remaining two methods is similar in both countries.

### *Directional forecasting accuracy*

It is often argued by practitioners that, in situations where both goals cannot be achieved simultaneously, a forecasting method with a high degree of directional accuracy is preferable to a method with better absolute accuracy.

### *Directional change accuracy*

Naive no-change extrapolations are neither correct nor incorrect predictions of the direction of change in a time series. A forecasting method outperforms such a naive model, if it can predict over 50% of the directions of change which occur. With monthly, seasonally unadjusted data, we have two possibilities to measure the directions of change: first differences (against the previous month) and annual differences (against the corresponding month of the previous year). The month-to-month differences of industrial production, for which strong seasonal movements are characteristic, exhibit wild fluctuations but are stationary in general. Annual differences, on the other side, are far less volatile but, often, a significant time trend is present. Moreover, by forming annual differences much information is lost. Since the results for month-to-month and annual differences diverge substantially for the two production series under analysis, we decided to present both. They are given in Tables 9 and 10. What we present in these tables are the percentages of directions of change predicted correctly by a particular forecasting method.

Table 9

| Forecasting performance: Direction of change error in month-to-month differences |  |      |      |      |      |
|--|--|------|------|------|------|
| Forecasting methods  | Percentage of direction changes forecast correctly |      |      |      |      |
|  | Forecasting horizon in months                      |      |      |      |      |
|  | 1  | 6    | 12   | 18   | 24   |
| Austria  |  |      |      |      |      |
| <i>BSN</i>   | 88.2   | 92.4 | 91.6 | 89.9 | 91.6 |
| <i>SMAC</i>  | 88.2   | 89.9 | 91.6 | 89.1 | 91.6 |
| <i>SMACX</i>   | 89.9   | 87.4 | 86.6 | 88.2 | 89.9 |
| <i>ARIMA</i>   | 89.1   | 91.6 | 90.8 | 89.1 | 89.9 |
| Germany  |  |      |      |      |      |
| <i>BSM</i>   | 95.8   | 95.0 | 95.8 | 94.1 | 96.6 |
| <i>SMAC</i>  | 95.8   | 92.4 | 93.3 | 90.8 | 91.6 |
| <i>SMACX</i>   | 95.0   | 90.8 | 87.4 | 83.2 | 79.0 |
| <i>ARIMA</i>   | 95.8   | 95.0 | 93.3 | 89.9 | 95.0 |

We begin with a discussion of the results for first differences. We are somewhat surprised that here the relative forecasting performance is practically independent of the length of the forecasting horizon. The percentage of directions of change, which are predicted correctly by a particular method, hardly decreases for longer horizons (*SMACX* in Germany is the only exception). The explanation lies in the fact that we have eliminated the trend by taking first differences. Erratic fluctuations in the slope of a trend line seem to be the main cause of big forecast errors. The rest, that remains after removing the trend, is dominated strongly by seasonal fluctuations and, obviously, can be predicted very accurately. A substantial difference in the forecasting performance between Austria and

Germany is also worth noting. Apart from *SMACX*, all other forecasting methods exhibit a better relative performance in Germany, whereby the high percentages of correctly forecast month-to-month changes for *BSM* and, most surprisingly, *ARIMA* are especially remarkable. One possible explanation might lie in more stable climatic conditions in Germany.

Table 10

| Forecasting performance: Direction of change error in annual differences |  |      |      |      |      |
|--|--|------|------|------|------|
| Forecasting methods  | Percentage of direction changes forecast correctly |      |      |      |      |
|  | Forecasting horizon in months                      |      |      |      |      |
|  | 1  | 6    | 12   | 18   | 24   |
| Austria  |  |      |      |      |      |
| <i>BSN</i>   | 91.7   | 81.5 | 75.0 | 65.7 | 62.0 |
| <i>SMAC</i>  | 91.7   | 81.5 | 75.0 | 66.7 | 64.8 |
| <i>SMACX</i>   | 91.7   | 82.4 | 69.4 | 64.5 | 68.5 |
| <i>ARIMA</i>   | 90.7   | 81.5 | 71.3 | 61.1 | 54.6 |
| Germany  |  |      |      |      |      |
| <i>BSM</i>   | 82.4   | 81.5 | 72.2 | 63.0 | 58.3 |
| <i>SMAC</i>  | 83.3   | 83.3 | 72.2 | 58.3 | 54.6 |
| <i>SMACX</i>   | 85.2   | 82.4 | 67.6 | 52.8 | 49.1 |
| <i>ARIMA</i>   | 83.3   | 79.6 | 69.4 | 58.3 | 52.8 |

Removing the seasonality by annual differencing and leaving the trend untouched yields results, which are more conform to expectations. In both countries, the relative performance deteriorates continuously with the increasing length of the forecasting horizon. This substantial decrease in the percentage of correctly predicted directional changes is caused by errors in the trend extrapolations.

The completely different relative forecasting performance of the analysed methods for month-to-month changes and for annual differences is in open conflict with the opinion of many experts. They often believe that month-to-month changes are unpredictable because of erratic cyclical and seasonal fluctuations, while the evolvement of the trend is thought to be stable and, therefore, easier to predict. For the industrial production of Austria and Germany just the opposite seems to be true however. Since correct predictions of the trend play such a prominent role for the forecasting performance, we turn to a closer inspection of this problem before concluding this paper.

### *Trend change accuracy*

Making use of the definitions given in the relations (4) of the theoretical section, we can classify forecasts into downturns / no downturns / upturns / no upturns. Comparing these values with the

actual trend movements, we can calculate the percentage of trend changes forecast correctly. These percentages for different forecasting horizons, different forecasting methods and different production countries are given in Table 11. The method, by which these figures are derived is demonstrated exemplarily in Table 12 for one month ahead forecasts with *BSM*.

Table 11

| Forecasting performance: trend change error    |                    |                    |         |
|--|--------------------|--------------------|---------|
| Percentage of trend changes forecast correctly |                    |                    |         |
| Forecasting horizon<br>(months)                | Forecasting method | Production country |         |
|  |                    | Austria            | Germany |
| 1  | <i>BSM</i>         | 94(4)              | 93(4)   |
|  | <i>SMAC</i>        | 95(3)              | 97(1)   |
|  | <i>SMACX</i>       | 97(1)              | 96(2)   |
|  | <i>ARIMA</i>       | 96(2)              | 95(3)   |
| 6  | <i>BSM</i>         | 81(3)              | 71(3)   |
|  | <i>SMAC</i>        | 83(2)              | 77(2)   |
|  | <i>SMACX</i>       | 86(1)              | 79(1)   |
|  | <i>ARIMA</i>       | 80(4)              | 74(3)   |
| 12   | <i>BSM</i>         | 64(3)              | 59(3)   |
|  | <i>SMAC</i>        | 64(3)              | 62(2)   |
|  | <i>SMACX</i>       | 83(1)              | 72(1)   |
|  | <i>ARIMA</i>       | 68(2)              | 58(2)   |
| 18   | <i>BSM</i>         | 53(3)              | 44(3)   |
|  | <i>SMAC</i>        | 54(2)              | 48(2)   |
|  | <i>SMACX</i>       | 79(1)              | 66(1)   |
|  | <i>ARIMA</i>       | 52(4)              | 43(4)   |
| 24   | <i>BSM</i>         | 36(4)              | 36(3)   |
|  | <i>SMAC</i>        | 42(3)              | 44(2)   |
|  | <i>SMACX</i>       | 84(1)              | 63(1)   |
|  | <i>ARIMA</i>       | 44(2)              | 33(4)   |

From Table 11, we note immediately that the percentage of trend changes forecast correctly drops significantly with the increasing length of the forecasting horizon. This result is no surprise and corroborates our above hypothesis that trend change errors are a more serious problem than inaccuracies in the prediction of short term components. We detect some differences in the trend change accuracy between Austria and Germany. Apart from the predictions with a forecasting horizon of 1 month, the trend change accuracy for all tested forecasting methods is in Austria substantially better. The most interesting information of this table is the excellent performance of *SMACX* in this context. Splitting up the long run component of a time series into a slowly changing trend component and a repetitive cyclical component leads to a substantial improvement in trend change accuracy. For

Austria, the percentage of trend changes forecast correctly by *SMACX* is, even for a forecasting horizon of 24 months, with more than 80% extremely high. Thus, we observe here a substantial trade-off between absolute and relative forecasting performance, and as a source of qualitative information *SMACX* should not be ignored. This result has a very interesting implication. Apparently, statistical techniques can provide valuable information, if only the right questions are asked. To do this, may not be easy for analysts who are no trained statisticians.

Table 12

| Trend change accuracy: one month ahead forecasts with <i>BSM</i> |         |                      |       |         |                      |       |
|--|---------|----------------------|-------|---------|----------------------|-------|
|  | Correct | Austria<br>Incorrect | Total | Correct | Germany<br>Incorrect | Total |
| 1 month ahead  |         |                      |       |         |                      |       |
| Downturn   | 2       | 0                    | 2     | 2       | 0                    | 2     |
| No downturn  | 93      | 2                    | 95    | 86      | 6                    | 92    |
| Total  | 95      | 2                    | 97    | 88      | 6                    | 94    |
| Percent  | 97.9    | 2.1                  | 100.0 | 93.6    | 6.4                  | 100.0 |
| Upturn   | 0       | 1                    | 1     | 1       | 0                    | 1     |
| No upturn  | 15      | 4                    | 19    | 20      | 2                    | 22    |
| Total  | 15      | 5                    | 20    | 21      | 2                    | 23    |
| Percent  | 75.0    | 25.0                 | 100.0 | 91.3    | 8.7                  | 100.0 |
| Overall total  | 110     | 7                    | 117   | 109     | 8                    | 117   |
| Overall percent  | 94.0    | 6.0                  | 100.0 | 93.2    | 6.8                  | 100.0 |

## Conclusions

In the present paper, we compare the forecasting performance of structural time series models with that of a traditional *ARIMA* model. The forecasting performance of model can be evaluated from two perspectives. One can use the absolute magnitude of the committed forecast errors as yardstick or one can rely on a model's ability to predict turning points as evaluation criterion. The computed statistics are often referred to as measures of absolute and relative (or directional) accuracy, respectively.

As far as absolute accuracy is concerned, even 24-months-ahead forecasts might be classified as highly accurate according to international standards. Our own standards are more stringent, and we would say that forecasts with a horizon of up to 12 months can be considered as sufficiently reliable. Next, we investigate whether the length of the forecasting horizon, the forecasting method, and the production country have significant effects on the absolute accuracy. The forecasting horizon turns out to be the most influential factor. The longer the horizon, the larger the committed errors. This

result is more or less trivial. The forecasting method also has a significant effect. A ranking of the tested methods can be determined. Structural models outperform the traditional *ARIMA* model clearly. Somewhat surprising is however, that the basic structural model does better than more sophisticated model versions with an additive cyclical component. The third tested factor, namely the production country, is of no relevance.

Basing the judgement on relative measures of accuracy changes the ranking of the tested forecasting methods completely. The structural models remain superior, but now the sophisticated model versions with additive cyclical component dominate. Here, we also come upon one of the most surprising results of the whole paper. We detect that problems with the prediction of the trend are mainly responsible for a poor directional forecasting performance. Once the trend is removed, the remaining rest can be forecast perfectly, even for a horizon of 24 months.

It follows as final conclusion from this study, that structural time series models are superior to a traditional *ARIMA* model for modelling industrial production in Austria and Germany. Besides a forecasting performance, which is definitely at least as good as that of a *ARIMA* model, they offer additionally valuable information about trend, seasonal and, possibly, cyclical components.

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