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Assessing the Dynamical Behavior of Austrian and German Industrial Production

1. Introduction

The purpose of this paper is to search for evidence of genuine nonlinear structure in the monthly growth rates of Austrian and West-German industrial production. As for industrial production data, so far evidence of nonlinearity has only been reported for the U.S. (Brock and Sayers 1988, Scheinkman and LeBaron 1989B). Besides U.S. industrial production, evidence for nonlinear dynamics has been detected for a variety of monetary aggregates (DeCoster and Mitchell 1991); for various exchange rates (Hsieh 1989, Kugler and Lenz 1990), and variables such as U.S. unemployment, U.S. employment (Scheinkman and LeBaron 1989B); for rates of return on gold and silver (Frank, Gencay, and Stengos 1989); and for stock returns (Scheinkman and LeBaron 1989A, Hsieh 1991).

From an econometrician's point of view, looking for evidence of nonlinear structure in time series data is important for at least one reason: to obtain good predictions. As commonly known, nonlinear systems can exhibit complex behavioral dynamics which cannot be captured by linear stochastic models. Thus, if the data generating process of interest is nonlinear, the chances to make good predictions on the basis of linear stochastic models are rather low.

Recently new algorithms have been proposed which seem to have some power in distinguishing between data generated by a deterministic 'chaotic' system and data generated by a random system (e.g. Grassberger and Procaccia 1983). These algorithms and related test procedures also proved to be useful in searching for nonlinear structure in time series. Among the algorithms on this line of reasoning the so-called BDS approach (Brock, Dechert and Scheinkman 1987) turned out to be quite successful in detecting nonlinear dynamics in economic time series. In addition, this test meets most of the standards required by statistical inference theory (Brock and Baek 1991). The BDS procedure tests the null hypothesis that the variable of interest is independent and identically distributed (IID).

The paper is organized as follows: We begin by describing the BDS approach. Then the data and the proposed testing procedure are discussed. Finally the empirical results are presented.

2. Statistical Test: The BDS Statistic

Brock, Dechert, and Scheinkman (1987) devised a statistical test on the basis of the correlation integral concept introduced by Grassberger and Procaccia (1983). The correlation integral is defined as follows

(1) $$C_n(\varepsilon) = \lim_{T \to \infty} \# \left\{ (t,s),\ 0<t,s<T: \| x_t^n - x_s^n \| < \varepsilon \right\} / T^2$$

where x_t^n is the n-dimensional history representation $(x_{t-n+1},...,x_t)$ of the time series $(x_t:t=1,...,T)$, and $|| \; ||$ is the sup- or max-norm. The correlation integral $C_n(\varepsilon)$ can be interpreted as an estimate of the likelihood that x_s^n and x_t^n are within a distance ε.

Following Hsieh (1991), if $(x_t:t=1,...,T)$ is IID, then:

(2) $$C_n(\varepsilon) = C_1(\varepsilon)^n$$

and

(3) $$W_{n,T}(\varepsilon) = \sqrt{T}\,[C_{n,T}(\varepsilon) - C_{1,T}(\varepsilon)^n]/\sigma_{n,T}(\varepsilon)$$

has a limited standard normal distribution, $\sigma_{n,T}$ an estimate of the asymptotic standard error of $[C_{n,T}(\varepsilon) - C_{1,T}(\varepsilon)^n]$. W-statistic (3) is usually referred to as BDS statistic. (Note that (2) does not imply IID).

Monte Carlo simulations by Hsieh (1991) show that the BDS test has good power against the alternative of at least four types of non-IID behavior: linear dependence, nonstationarity, chaos, and stochastic nonlinearity.

3. Description of Data and Procedure

As for industrial production on a monthly basis, we have to face the very fact that these data are usually contaminated by 'artifical dependencies' which will be picked up by any good test of nonlinear dynamics. The most relevant artifical dependencies are caused by seasonal and calendar effects. In order to find out as to what extent dependencies in the data are caused by calendar or seasonal effects we will apply our test procedures to the raw data as well as to the data adjusted for both effects.

In addition, serial correlation also affects tests concerned with nonlinear dependence. We will remove linear dependence based upon serial correlation by filtering the data using AR- and ARIMA-models, respectively. The lag length of these processes is chosen according to the Akaike (1974) and/or Schwarz (1978) information criterion.

The BDS statistics will be calculated with the BDS STATS program by W. D. Dechert.

3.1 The Data

Our data base consist of monthly production data for Austria and Germany, covering the period 1962:1 to 1991:8. Graphs of the original series and of the autocorrelation functions show that the monthly data used are heavily contaminated by calendar effects (Figure 1).

The original series of industrial production for both countries are clearly nonstationary and their variances increase over time. Therefore, before calculating the autocorrelations, the raw data are transformed to natural logarithms, and differences of order 1 and 12 are taken. The logarithmic transformation should stabilize the variance of the raw data, and the difference operators are applied to obtain stationary series. The autocorrelation functions exhibit a confused pattern as it is typical for

Figure 1a

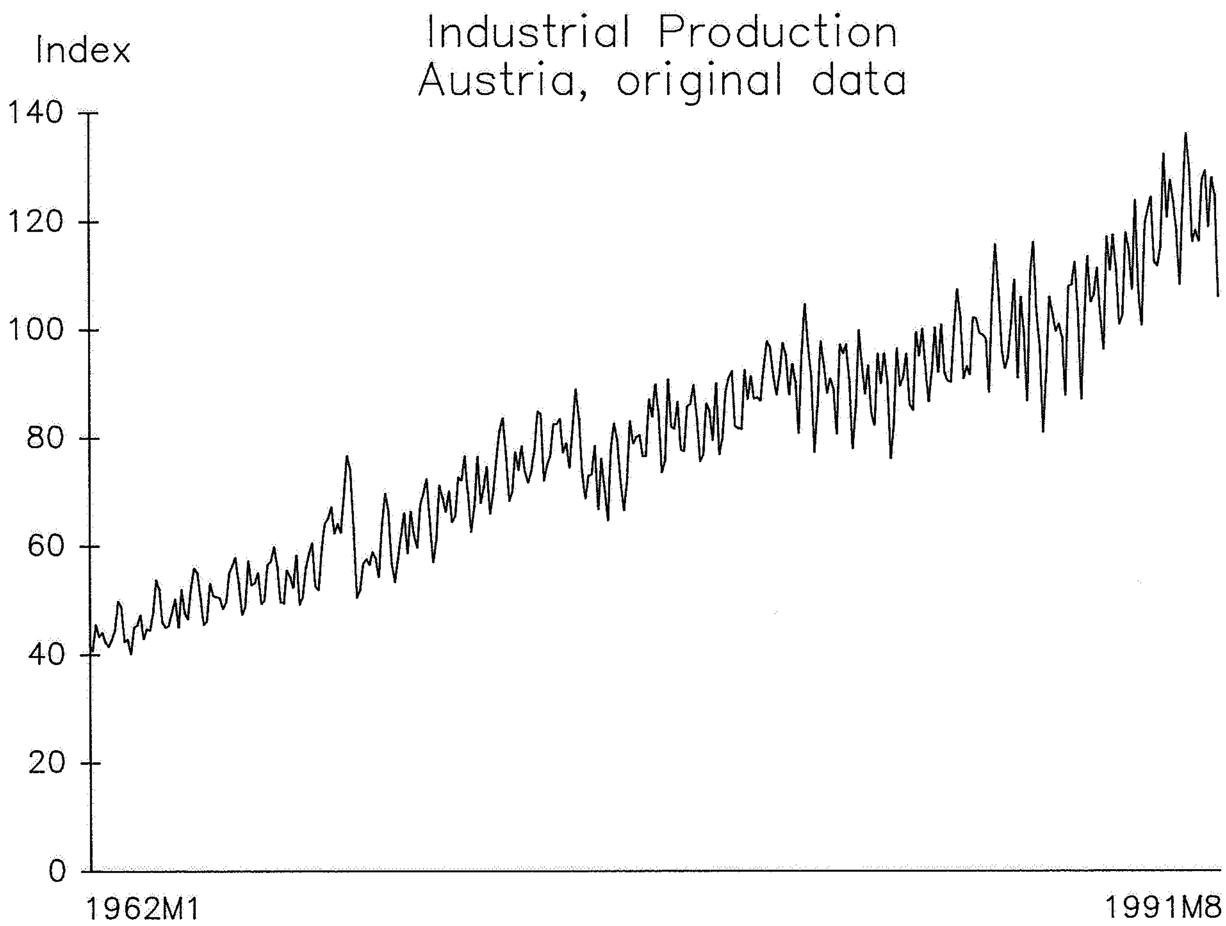

Industrial Production
Austria, original data
Index
140
120
100
80
60
40
20
0
1962M1
1991M8

Figure 1b

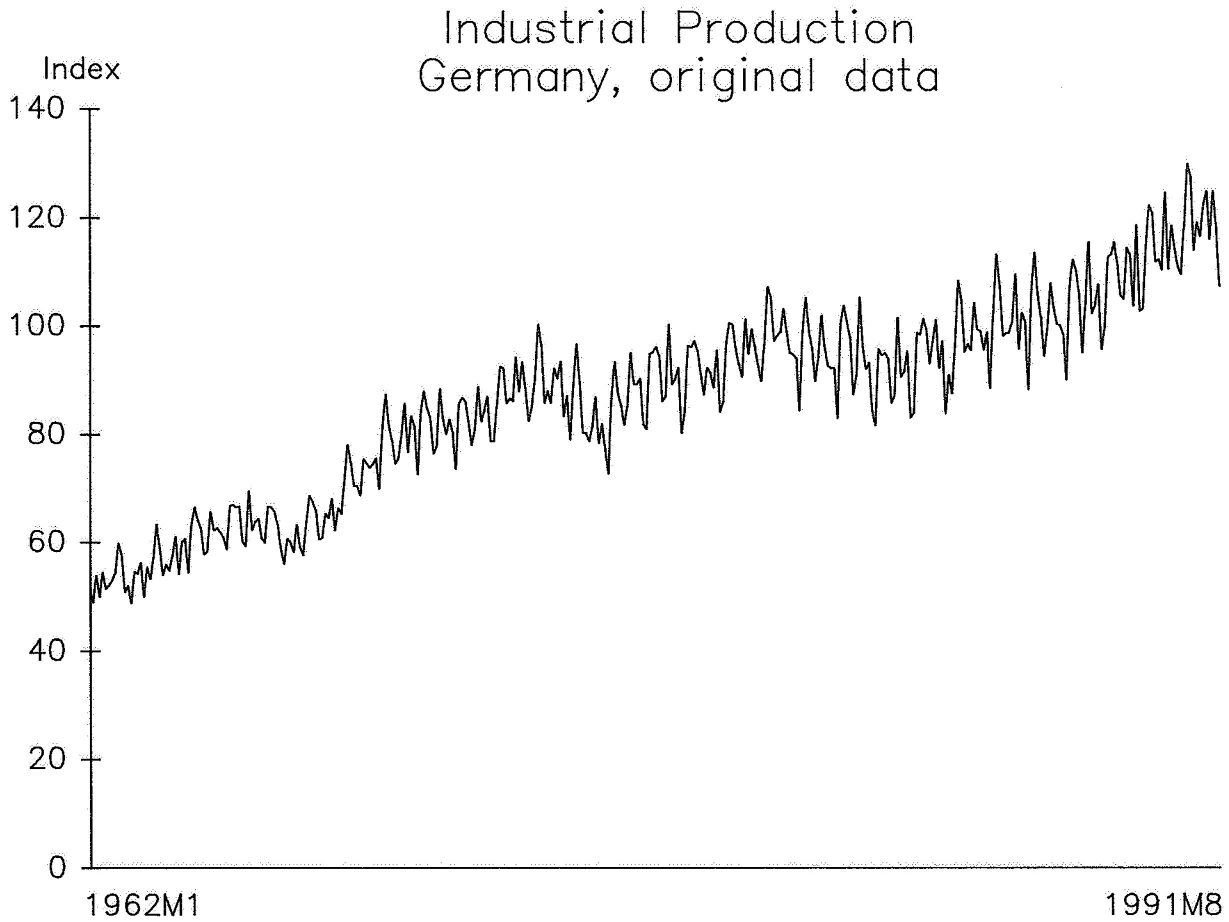

Industrial Production
Germany, original data
Index
140
120
100
80
60
40
20
0
1962M1
1991M8

Figure 2a

Autocorrelation Function
Austria, original data

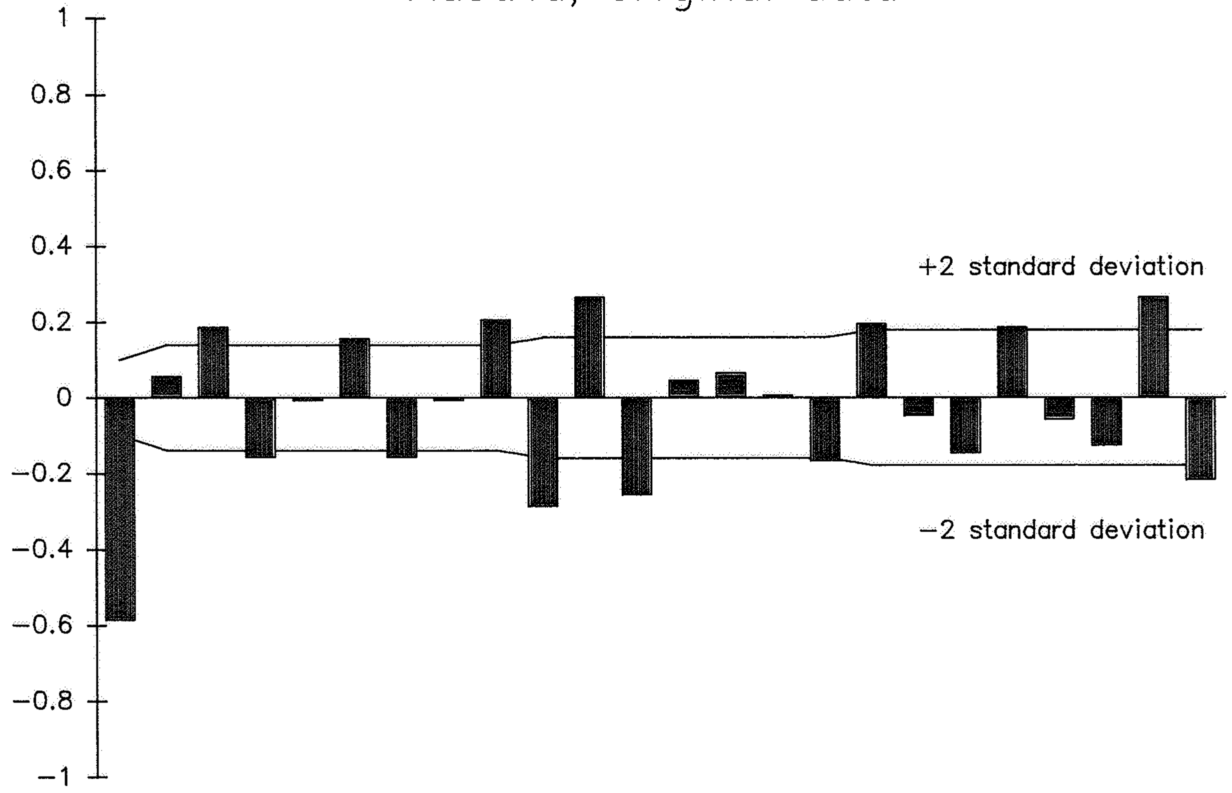
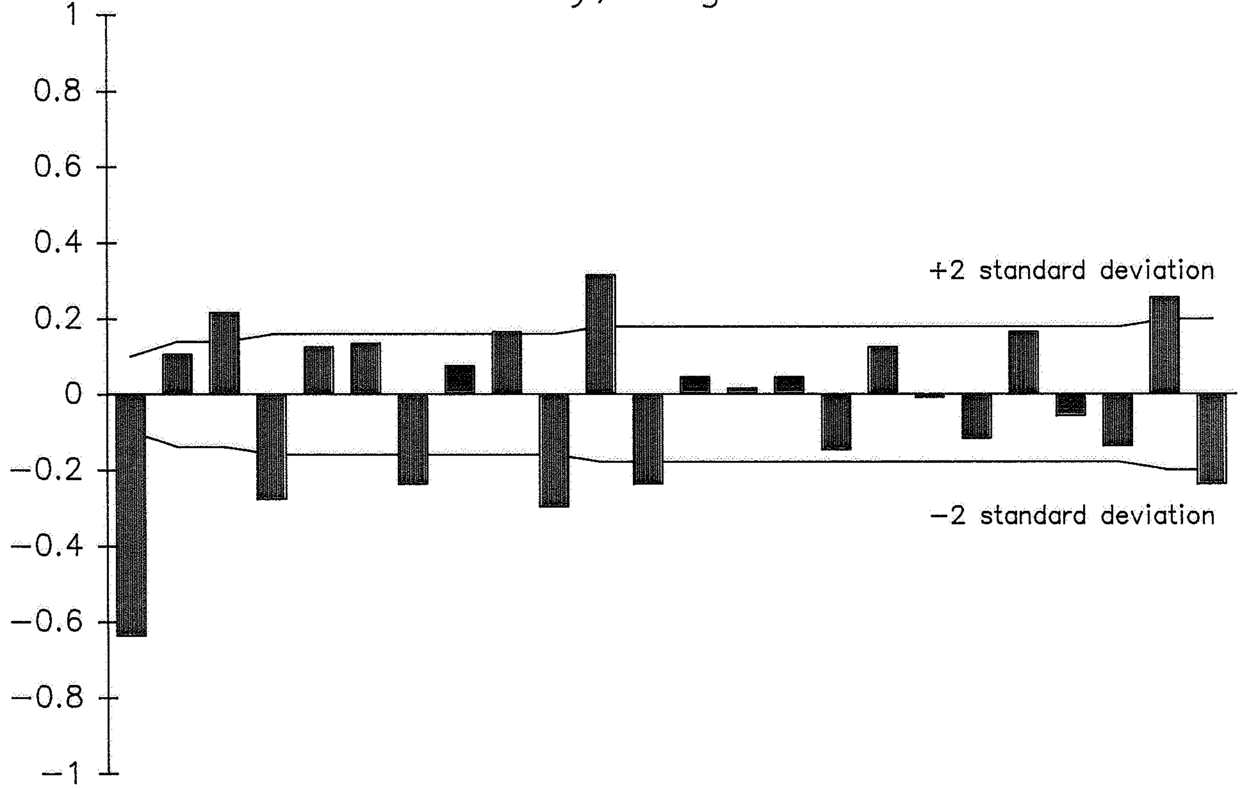


Figure 2b

Autocorrelation Function
Germany, original data



series, which are dominated by strong calendar effects (Figure 2). Before adequate ARIMA models can be identified and estimated for such series, it is obligatory to remove these calendar effects.

3.2 Removing Calendar and Seasonal Effects

Calendar effects constitute a rather complex problem. The number of working days per month varies from year to year as a consequence of moving festival days. But it is not only the existence of moving festival days that might cause disturbances in industrial production. An additional problem lies in the changing weekday composition of the different months of a year, because we observe substantial differences in productivity for different days of the week. Since calendar effects are caused primarily by institutional factors, their removal seems desirable. We try to accomplish this by adopting a deterministic approach. We assume that the calendar effect in month t, denoted by CEt, is a function of the number of different weekdays in that month, namely

$$(4) \qquad CE_t = \sum_{i=1}^{7} \beta_i T_{it}$$

Here, T_{it}, $i = 1, 2, \ldots, 6$, is the number of Mondays, Tuedays, ..., Saturdays minus the number of Sundays, respectively, in month t. T_{7t} denotes the total number of days in month t. It is included to capture leep-year effects.

A time series model for an observed series of industrial production could be of the following form:

Exhibit 1

Estimates for Parameters β_i

Model:

$$(I-B)(I-B^{12}) \ln IP_t = \sum_{i=1}^{7} \beta_i (I-B)(I-B^{12}) T_{it} + N_t$$

Parameter Estimates For

	Austria			Germany	
β_1 =	.0070	(5.87)	β_1 =	−.0006	(−0.55)
β_2 =	.0078	(6.44)	β_2 =	.0074	(6.57)
β_3 =	.0072	(6.24)	β_3 =	.0096	(9.31)
β_4 =	.0109	(9.02)	β_4 =	.0059	(5.83)
β_5 =	.0009	(0.71)	β_5 =	.0031	(3.14)
β_6 =	−.0069	(−8.25)	β_6 =	−.0055	(−8.46)
β_7 =	.0467	(8.81)	β_7 =	.0408	(8.34)

t – values in parentheses

(5) $\ln IP_t = CE_t + N_t,$

where IP_t denotes industrial production and N_t follows an ARIMA model which will be identified and estimated at a later stage.

Before estimates for the parameters β_i can be derived, the calendar variables T_{it} must be constructed. The PC-version of the SCA System produces immediately the weekday composition of each month for the desired number of years under analysis. But, this composition does not reflect the special calendar situations of Austria and Germany with their relatively large number of moving festival days concentrated in certain months of the year (May or June). To capture these effects, we first change these festival days into Sundays. Additionally, the festival days in May or June, which always fall on Thursdays, may induce employees to take off the intermediate Fridays also. Taking this fact into account and experimenting with different alternatives, we finally end up for both countries with a weekday composition, which gives rise to plausible looking estimates of the parameters β_i (Exhibit 1).

These parameter estimates can now be used to extract the calendar effects from the industrial production series of Austria and Germany. The resulting adjusted series together with their corresponding autocorrelation functions are displayed in Figure 3 and 4.

As expected, the adjusted series for both countries are much smoother. This visual impression is fully corroborated by the shape of the autocorrelation functions. These functions exhibit a clearly interpretable pattern. In both countries, industrial production seems to follow a simple multiplicative seasonal ARIMA model, known as airline model in the literature (Thury 1989). Parameter estimates and

Figure 3a

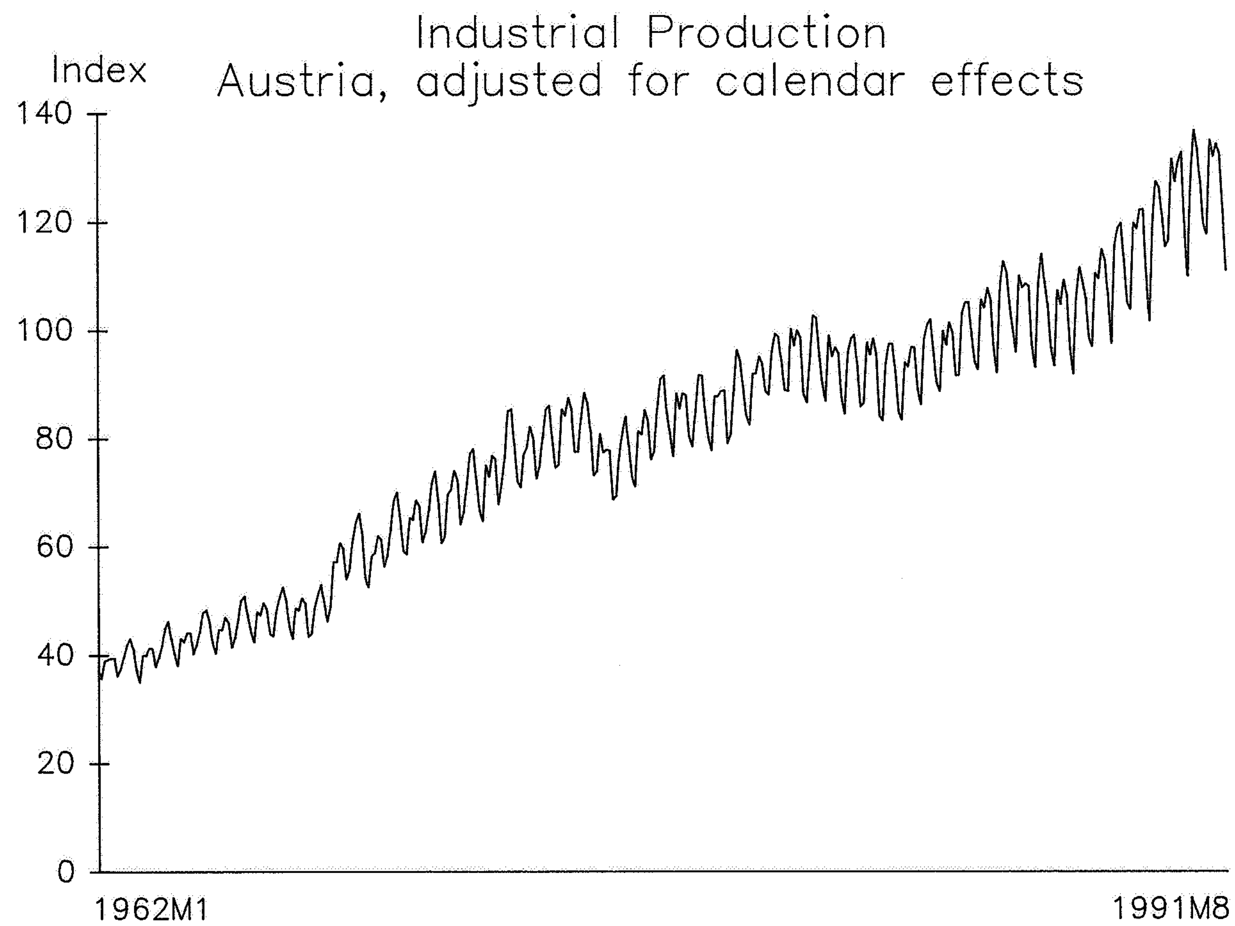

Industrial Production
Austria, adjusted for calendar effects
Index
140
120
100
80
60
40
20
0
1962M1
1991M8

Figure 3b

Industrial Production
Germany, adjusted for calendar effects

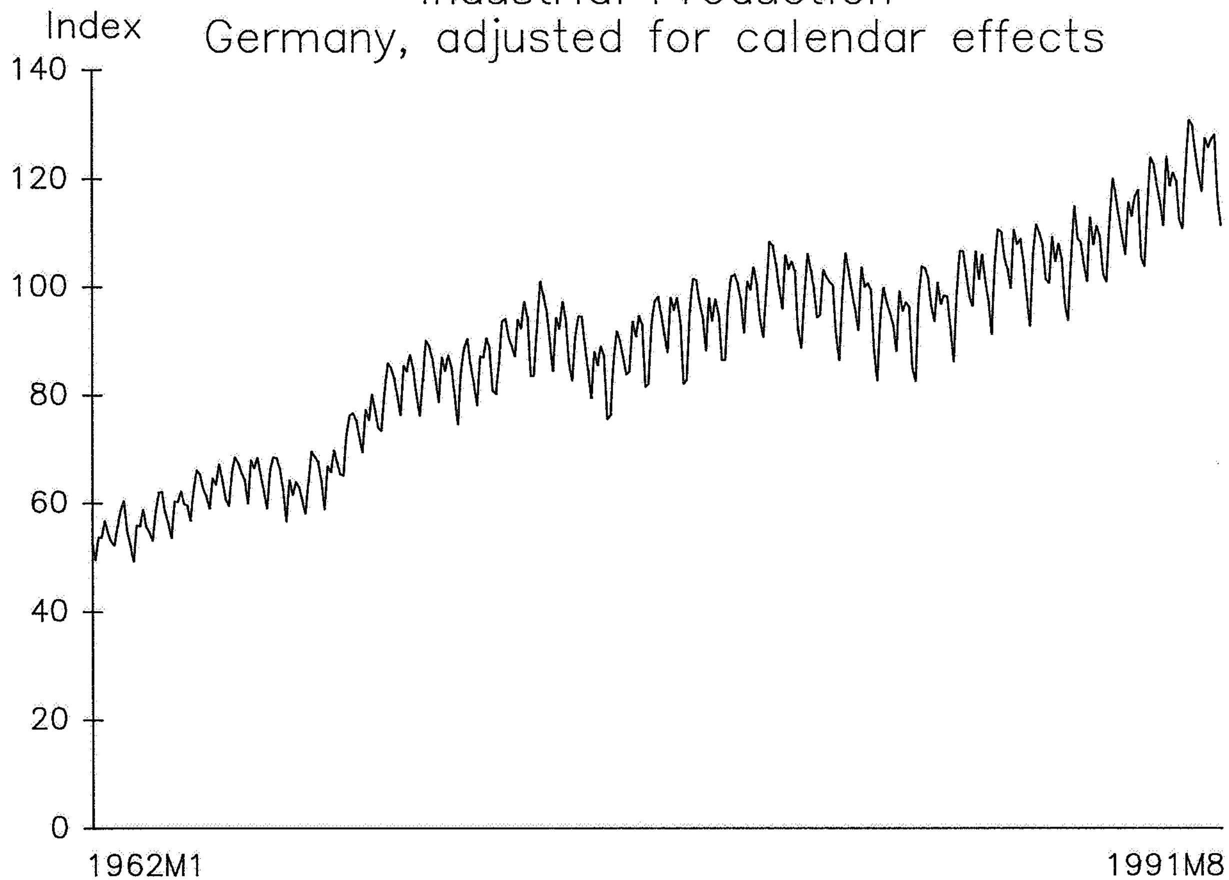


Figure 4a

Autocorrelation Function
Austria, adjusted for calendar effects

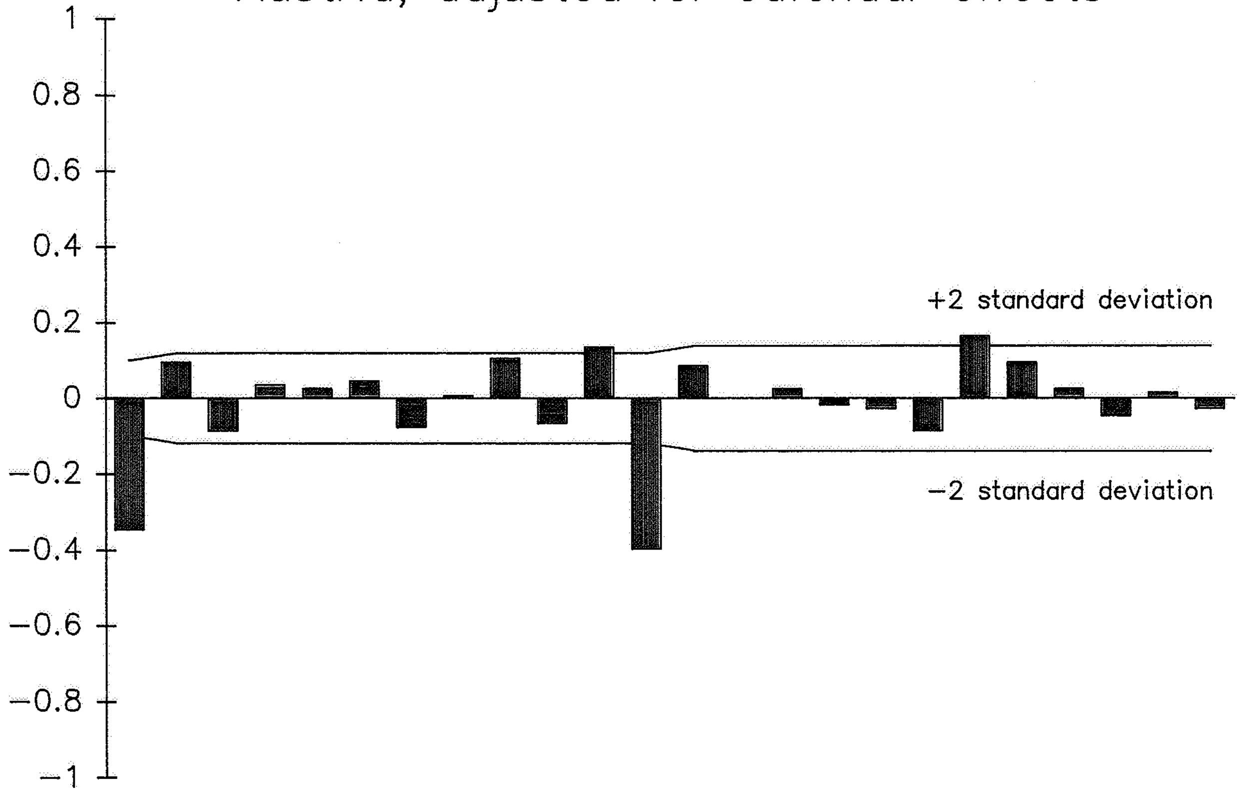


Figure 4b

Autocorrelation Function
Germany, adjusted for calendar effects

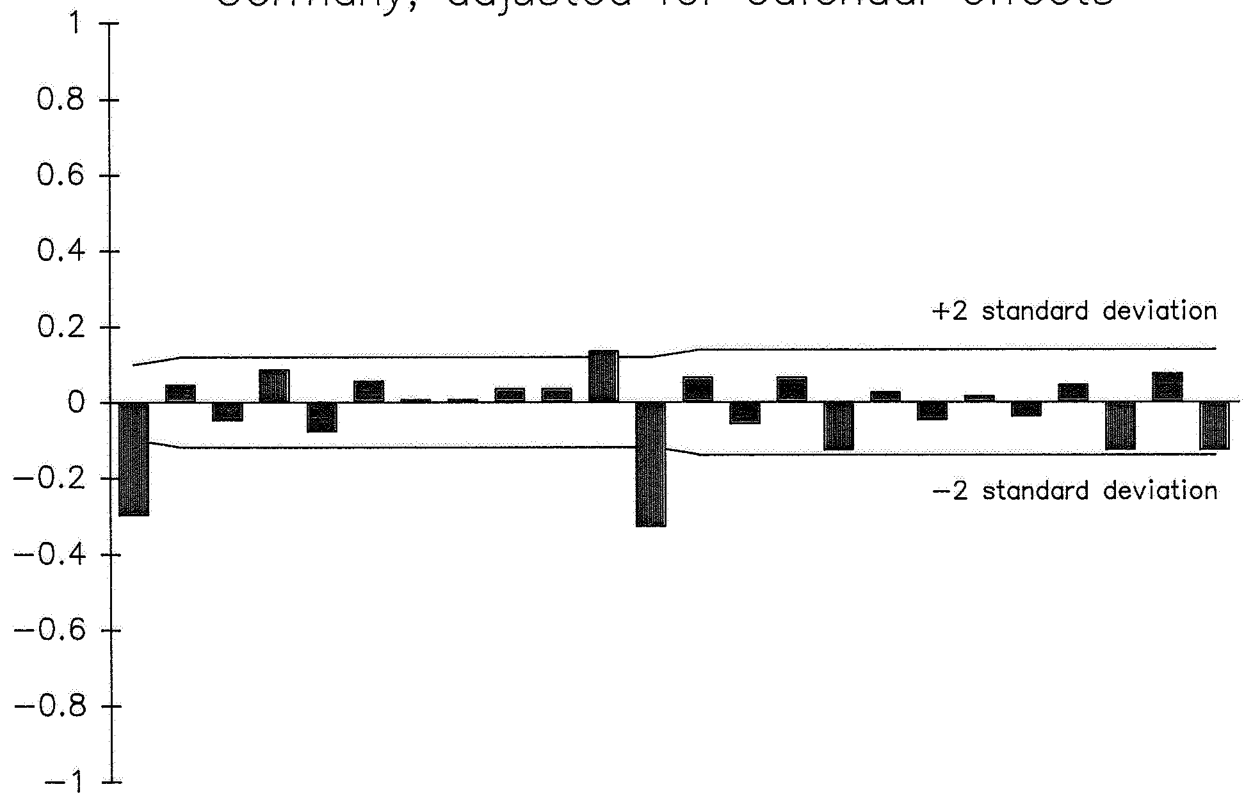


Exhibit 2

Arima Models for Calendar Adjusted Industrial Production

Arima Model:
$(I-B)\ (I-B^{12})\ \ln IPadj_t = (I-\Theta_1 B)\ (I-\Theta_{12} B^{12})\ a_t$

Parameter Estimates For

Austria	Germany
Θ_1 = .1996 (3.75)	Θ_1 = .4064 (8.20)
Θ_{12} = .6440 (15.62)	Θ_{12} = .7423 (21.71)
S.E. = .0188	S.E. = .0153
Q (24) = 24.4	Q (24) = 34.4

X^2 .05 (24) = 36.4

test statistics for these models are given in Exhibit 2. We will see in the following section that this interpretation is supported by the test procedure proposed.

Finally, we remove the seasonal effects from our data base. To this end, we apply the ARIMA model-based approach of Hillmer and Tiao (1982).

4. Empirical Results

To begin with, all presented BDS statistics are calculated on the basis of dimensions 2 through 5 and of ε equaling 0.5, 1, and 1.5 standard deviations of the data.

The results of the BDS test for the raw data (logarithmic differences of the monthly production index, unadjusted for seasonal and calendar effects) are given in Table I. The test strongly rejects the hypothesis that the monthly growth rates of the Austrian and German industrial production are IID.

In order to exclude calendar effects as a reason for the rejection of IID we apply our test procedure to the industrial production adjusted for these effects. Table II shows that, for both countries, the thus-adjusted industrial production data still remain non-IID.

Next we calculate the BDS statistic for the industrial production adjusted for calendar and seasonal effects. Again, the results show that independence cannot be achieved by these corrections, either (Table III).

Having removed any relevant artificial dependencies from our data base we can now proceed with testing whether the remaining non-IID bevavior of the monthly growth rates of the Austrian and German industrial production is in fact caused by 'genuine' linear or nonlinear stochastic dependence.

Before doing so, we have to explain why we do not test for deterministic chaos explicitely as a possible source of non-IID. Besides the fact that deterministic chaos is as aesthetically appealing as practically implausible a reason for non-IID of Austrian and German industrial production, we do not have the stastistical means yet, as far as small data sets are concerned, to discriminate between low dimensional deterministic chaos, stochastic nonlinearity, and linear dependence. The available techniques in 'statistical chaos' (or 'chaotic statistics') have been devised by scientists who use typically 100,000 or more data points to detect low dimensional chaotic systems. Economic data sets consist at best of 2,000 observations (i.e. financial data sets; Remember, our data set has only 356 data points).

The major deficiency of the available techniques in 'statistical chaos' is that, applied to small data sets, they show a strong tendency to bias the test results in favor of detecting chaos, even if there is none (Ramsey and Yuan 1989).

Now, testing for linear versus nonlinear dependence requires that we first pre-filter the data using linear AR- and/or ARIMA-models and then apply the BDS-test to the fitted residuals. According to the residual test theorem by Brock (1986), applying the BDS stastistic to the residuals of linear models indicates reliably if there is genuine nonlinear dependence in the time series or not. As mentioned above, the linear filters used are fitted to the monthly growth rates of the Austrian and German industrial production (both adjusted for seasonal and/or calendar effects) by using the Akaike and/or the Schwarz information criterion. Since we have a relatively large number of degrees of freedom, we use the longer lag lengths.

Table IV shows the test results based upon the residuals generated by AR-processes of order 1 (adjusted for calendar and seasonal effects) and of order 24 (adjusted for calendar effects only). In addition, we also subjected the residuals of the ARIMA 'airline' models (Exhibit 2) to our test procedure.

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Table I

BDS – Statistics for Raw Data

	Dimension	ε/σ 0.5	1.0	1.5
Austria	2	1.37	−0.55	−1.19
	3	10.01[1]	2.71	1.21
	4	17.86[1]	4.11[1]	1.95
	5	33.82[1]	7.29[1]	3.09[1]
Germany	2	4.41[1]	1.47	0.34
	3	6.76[1]	2.37	0.78
	4	12.65[1]	2.84	0.21
	5	25.20[1]	5.25[1]	0.75

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[1]) Indicates significance at the 1 percent level.

Table II

BDS-Statistics for Calendar Effect Adjusted Data

	Dimension	ε/σ 0.5	1.0	1.5
Austria	2	9.97[1])	−2.27	−4.99[1])
	3	44.63[1])	9.50[1])	−0.04
	4	95.81[1])	19.14[1])	2.84
	5	201.62[1])	30.97[1])	7.43[1])
Germany	2	0.83	−1.96	−0.60
	3	7.05[1])	2.25	0.04
	4	14.46[1])	7.32[1])	1.36
	5	24.31[1])	11.38[1])	2.52

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[1]) Indicates significance at the 1 percent level.

Table III

BDS-Statistics for Seasonally and Calendar Effect Adjusted Data

	Dimension	ε/σ 0.5	1.0	1.5
Austria	2	2.63	4.14[1)]	3.90[1)]
	3	2.89	4.21[1)]	4.29[1)]
	4	1.85	3.69[1)]	3.93[1)]
	5	1.85	3.21[1)]	3.60[1)]
Germany	2	2.99	4.21[1)]	3.42[1)]
	3	2.42	3.90[1)]	2.83
	4	0.54	3.08[1)]	2.08
	5	−0.87	2.46	1.44

[1)] Indicates significance of the 1 percent level.

Table IV

BDS-Statistics for Filtered Data

		Dimension	ε/σ 0.5	1.0	1.5
Austria	AR1	2	1.05	1.96	2.63
		3	1.26	2.21	2.75
		4	0.54	1.69	2.38
		5	0.67	1.38	1.94
	AR24	2	1.56	0.94	1.44
		3	0.91	0.32	0.71
		4	0.97	0.28	0.80
		5	1.02	0.08	0.79
	'Airline'	2	1.35	2.32	3.02[1])
		3	1.18	2.49	2.82
		4	0.50	2.00	2.36
		5	− 0.92	1.58	1.94
Germany	AR1	2	− 0.39	0.83	0.18
		3	− 0.52	0.66	0.03
		4	− 1.53	0.37	− 0.26
		5	− 1.23	0.04	− 0.58
	AR24	2	− 0.22	0.19	0.42
		3	0.26	−0.10	0.12
		4	− 0.37	−0.28	− 0.22
		5	− 2.23	−0.58	− 0.49
	'Airline'	2	0.49	0.58	− 0.00
		3	0.31	−0.09	− 0.32
		4	0.41	−0.32	− 0.52
		5	− 0.52	−0.64	− 0.93

[1]) Indicates significance at the 1 percent level.

Basically, for all estimated residuals, the independence hypothesis can no longer be rejected. The residuals of almost all linear filters pass the BDS test at conventional significance levels. That is to say, the genuine non-IID behavior of the time series considered is very likely to be caused by linear rather than nonlinear dependence, and the linear ARIMA model of type 'airline' seems to be a good representation of the monthly dynamics of the Austrian and German industrial production.

5. Conclusion

This paper tests for dependence of monthly growth rates of the Austrian and German industrial production using a test procedure proposed by Brock, Dechert, and Scheinkman (1987). Having removed artificial dependencies caused by seasonal and calendar effects we find strong evidence that the real source of non-IID behavior of both time series is linear dependence. For both countries, the data indicate that this linear dependence is conveniently represented by a simple seasonal multiplicative ARIMA model known as the 'airline model'. Though multiplicative seasonal models of type 'airline' have considerable practical appeal, the class as a whole is not a particularly natural one (for a detailed discussion of this point, see Harvey 1989). The more natural approach is structural time series modeling which allows for explicitely representing the main features of a time series such as trends, cycles, and seasonals. Selecting structural time series models for the Austrian and German industrial production with a similar fit as the presented ARIMA 'airline' models will be the subject of a forthcoming paper by the authors.

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