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Monopsonist and the New Microeconomics  
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THE WAGE CHOICE OF THE DYNAMIC MONOPSONIST

AND THE NEW MICROECONOMICS OF INFLATION:

A COMMENT

In the late sixties a new view on the generation of inflation and unemployment emerged which sought to provide a choice-theoretic rationalization for inflation and employment dynamics. Much of this work was collected in a book edited by Phelps, "The New Microeconomics of Inflation and Unemployment." One of the most widely quoted contributions was Mortensen's "A Theory of Wage and Employment Dynamics," the first elaboration of a rigorous theory of optimal wage setting with incomplete information in the labor market. His model of wage setting in which the firm has dynamic monopsony power provided the conceptual framework for other studies into the process of inflation (Salop, 1973; Iwai, 1981; Pissarides, 1976; Baily, 1975, and others). Many of the references to Mortensen's work apply to his search model,<sup>1)</sup> but others to the dynamics of the labor market, and Mortensen's paper (and that of others in the Phelps volume) is credited with having provided a rationale for the short-run Phillips curve and the independence of the "natural rate of unemployment" from the rate of inflation.<sup>2)</sup>

Mortensen's model has been criticized by several writers for ignoring the phenomenon of layoffs and specifying quits as the only source of turnover and unemployment.<sup>3)</sup> This comment does

not, however, take issue with Mortensen's search model. His conclusion that the monopsonist faces an upward sloping labor supply curve seems general enough for an analysis of the firm's optimal behavior (Mortensen, 1970A, p.181). The present paper first examines Mortensen's article<sup>4</sup>) and then takes up some of the extensions provided by Baily (1975).

An analysis of the firm leads Mortensen to conclude that the optimal response of a dynamic monopsonist to a rise in the unemployment rate is likely to be positive; this result is contrary to intuition and to the results obtained by Phelps in the same volume. This, together with another result derived from the investigation of the firm's wage setting process, seems to imply in the market model that a reduction in the unemployment rate reduces wage inflation: "An increase, *ceteris paribus*, in employment decreases algebraically the difference between the target and current level of employment. Those firms affected attempt to reduce their wages relative to the average. The result is that the market average wage falls relative to its previous level" (p.206, emphasis in original).<sup>5</sup>)

This counterintuitive result seems to have caused some confusion in the literature. Brinner (1977, pp.389-418) claims that "Mortensen's work perhaps represents the most complete exposition of the neoclassical framework," and concludes that "Mortensen has developed an internally consistent voluntary unemployment-inflation model, whereas Phelps' model appears to be

inconsistent."6)

I. Brief Outline of Mortensen's Model

The firm is a dynamic monopsonist. The rate of change of the labor flow to the firm is derived from a model of search in the labor market and depends on the wage paid by the firm relative to the average wage ( $w$ ) and on the (expected) unemployment rate  $u^e$ . Specifically,

$$(1) \quad \frac{\dot{L}}{L} = g\left(\frac{w}{\bar{w}}, u^e\right)$$

where  $g_{11} < 0$ ,  $g_{12} > 0$ .

Since  $g$  is monotonic, it can be inverted to yield

$$(2) \quad \frac{w}{\bar{w}} = h\left(\frac{\dot{L}}{L}, u^e\right)$$

$$\text{and } h_1 = \frac{1}{g_1} > 0, \quad h_2 = -\frac{g_2}{g_1} < 0, \quad h_{11} = -\frac{g_{11}}{g_1^3} > 0,$$

$$h_{12} = \frac{g_2 g_{11} - g_1 g_{12}}{g_1^3} < 0.$$

The firm's value can be written as

$$(3) \quad V = \bar{w}(0) \int_0^{\infty} \left[ F(L(t)) - \frac{w(t)}{\bar{w}(t)} L(t) \right] e^{-rt} dt,$$

where  $F(t)$  is the production function, and

$$v = \frac{p(0)}{\bar{w}(0)}$$

$v$  is the initial ratio of the firm's expected product price  $p(t)$  to the expected average wage rate.<sup>7)</sup>  $r$  is the real interest rate. In the following equations the time subscripts are omitted.

The firm chooses  $L(t)$ ,  $t > 0$  so as to maximize

$$(4) \int_0^{\infty} [vF(L) - h(\frac{\dot{L}}{L}, u^e)L] e^{-rt} dt.$$

Applying the method of Calculus of Variations and defining

$$(5) \lambda = h_1(\frac{\dot{L}}{L}, u^e),$$

the Euler-Lagrange condition is

$$(6) \dot{\lambda} = (r - \frac{\dot{L}}{L})\lambda + h(\frac{\dot{L}}{L}, u^e) - vF'(L).$$

The Legendre condition is

$$(7) \frac{\partial \lambda}{\partial \dot{L}} = h_{11}(\frac{\dot{L}}{L}, u^e) \gg 0.$$

The transversality condition is

$$(8) \lim_{t \rightarrow \infty} \lambda e^{-rt} = 0.$$

The singular point  $(\lambda^+, L^+)$  is a saddle point. Along the optimal trajectory

(9)  $\frac{\partial \lambda}{\partial L} < 0$ . The optimal solution is unique, if  $F_{LL} < 0$ .

## II. Property of the Optimal Wage Path

Let us first turn to the relationship between the firm's optimal wage choice and a change in the unemployment rate. Denote by  $h^0$  the value of  $h$  and of  $\lambda$  along the optimal trajectory. By noting that  $g_1(h, u^e) = 1$  from definition (5) and differentiating with respect to  $u^e$ , Mortensen derives the response of the optimal wage path to a change in the unemployment rate.

$$(10) \frac{\partial h^0}{\partial u^e} = \frac{-1}{g_{11}(h^0, u^e)} \left[ \frac{\partial \lambda^0}{\partial u^e} + g_{12}(h^0, u^e) \right]$$

Mortensen argues that, with  $g_{12}$  positive and  $\frac{\partial \lambda^0}{\partial u^e}$  likely to be small in absolute value, the likely response to a decrease in  $u^e$  is a decrease and not an increase in the relative wage (p.191).

By expressing  $\frac{\partial h^0}{\partial u^e}$  directly in terms of the  $h$  function, however, it is possible to determine the sign of  $\frac{\partial h^0}{\partial u^e}$  unambiguously near the equilibrium point. Using the definitions



$$g_{11} = -\frac{h_{11}}{h_1^3}, \lambda^0 = h_1, \text{ and } g_{12} = \frac{h_2 h_{11} - h_1 h_{12}}{h_1^3}$$

we arrive at the following expression:

$$(11) \frac{\partial h^0}{\partial u^e} = \frac{h_1}{h_{11}} \left( \frac{\partial \lambda^0}{\partial u^e} - h_{12} \right) + h_2.$$

At the equilibrium point  $h^0 = h^0(0, u^e)$  and  $\frac{\partial h^0}{\partial u^e} = h_2 < 0$ .

Therefore, as  $L$  approaches  $L^+$ , the steady state value of  $L$ , the term  $\frac{\partial \lambda^0}{\partial u^e} - h_{12}$  approaches zero, and from a certain point onward  $\frac{\partial h^0}{\partial u^e}$  is negative.<sup>8)</sup>

Another important property of the optimal wage path concerns the time pattern: if the initial value of employment is below (above) the equilibrium value,  $L$  increases (decreases) along the optimal path and  $h^0$ , the optimal wage rate, declines (increases). This can be seen as follows.

$$(12) \frac{dh^0}{dt} = h_1 \left( \frac{\dot{L}}{L}, u^e \right) \frac{d}{dt} \left( \frac{\dot{L}}{L} \right).$$

Note that  $\lambda^0 = h_1 \left( \frac{\dot{L}}{L}, u^e \right)$ . Therefore,

$$(13) \dot{\lambda}^0 = \frac{d}{dt} h_1 \left( \frac{\dot{L}}{L}, u^e \right) = h_{11} \left( \frac{\dot{L}}{L}, u^e \right) \frac{d}{dt} \left( \frac{\dot{L}}{L} \right)$$

and

$$(14) \frac{d}{dt} \left( \frac{\dot{L}}{L} \right) = \frac{\dot{\lambda}^0}{h_{11} \left( \frac{\dot{L}}{L}, u^e \right)}.$$

Therefore,

$$(15) \frac{dh^0}{dt} = \frac{\lambda^0 \dot{\lambda}^0}{h_{11} \left( \frac{\dot{L}}{L}, u^e \right)} \geq 0 \text{ as } \dot{\lambda}^0 \geq 0.$$

Since  $\frac{\partial \lambda^0}{\partial L} < 0$  (equation 9),

$$\frac{dh^0}{dt} < 0 \text{ for } L(0) < L^+ \text{ and } \frac{dh^0}{dt} > 0 \text{ for } L(0) > L^+.$$

The relationship between the change in  $w$  and  $L$  can also be established in a slightly different way.

$$(16) \frac{\partial \lambda}{\partial L} = \frac{\partial}{\partial L} h_1 \left( \frac{\dot{L}}{L}, u^e \right) = h_{11} \frac{\partial \left( \frac{\dot{L}}{L} \right)}{\partial L} < 0$$

by equation (9).

$$(17) \frac{\partial \left( \frac{\dot{L}}{L} \right)}{\partial L} < 0, \text{ since } h_{11} > 0.$$

If the initial value of employment is below (above) the equilibrium value,  $L$  increases (decreases) and  $\frac{\dot{L}}{L}$  decreases (increases) along the optimal path, and by equation (2) the optimal relative wage falls. With  $u^e$  unchanged, the relative wage prevailing at  $t_0$  is re-established when  $\frac{\dot{L}}{L} = 0$ .

It seems paradoxical that the wage is adjusted downward when the

employment level is raised to a higher equilibrium level. This puzzle is solved, however, once it is realized that the relative wage is a free variable that can be changed costlessly. The relative wage does not change smoothly from one equilibrium position to the next, but there is a discontinuity at  $t_0$ . If, say, the initial employment level is below the equilibrium employment level, the optimal wage policy consists in raising the wage rate instantaneously and then letting it fall back to the level prevailing at  $t_0$ .

Thus, the wage adjustment of the monopsonistic firm in response to a change in output prices, for example, is contrary to the often evoked notion of gradual adjustment. In particular, the argument of Holt and David (1966), that the wage path is related in a simple way to the gap between the initial and the desired labor force so that the proportional rate of increase in the wage rate is a positive function of the divergence between the actual and the target level of employment, does not hold, first, because the wage rate is not continuous at the initial point, second, because beyond this point the change in the wage is negatively related to the employment gap.

The wage adjustment pattern of the monopsonistic firm does not even provide a rationale for a permanent wage increase. An increase in the product price faced by the firm results not in a permanent, but only in a temporary increase in the firm's wage rate, a point already noted by Hansen (1970, p.18). This

conclusion contradicts (see below) a result derived by Baily (1975, p.342, property 2). A high product price does not imply a high wage; only a rising product price does. The derivation of the process of inflation and employment changes in response to changes in exogenous parameters (other than the unemployment rate) must then rest on the aggregation of the dynamic response of all firms in the economy.

The optimal wage path is different when there is a once and for all change in the unemployment rate. Suppose the rate of unemployment falls. At the new equilibrium point the wage rate is higher and the firm's optimal employment level is lower.<sup>9)</sup> With  $u^e$  fixed at the new lower level, the wage will rise at  $t_0$ , but, according to equation (1), not enough to make  $\frac{L}{L}$  equal to zero. Beyond  $t_0$ , the relative wage rises to the new equilibrium wage.<sup>10)</sup>

### III. The Wage Response of the Monopsonistic Firm in Baily

Mortensen's wage model is further analyzed in Baily's paper, with one modification. While Mortensen treats labor as the only factor of production, Baily considers a production function with labor and capital. Capital (K) is rented at a constant rate ( $\rho$ ) and is freely adjustable. The production function is characterized by constant returns to scale, a seemingly innocuous assumption, but which will prove to be of importance for some of the results

derived.

The maximization problem is now as follows:<sup>11)</sup>

$$(18) \text{ Max } \int_0^{\infty} (pF(L,K) - wL - \rho K) e^{-rt} dt.$$

Since there are no adjustment costs for capital, the addition of capital as a factor of production does not change the dynamic equation governing the optimal trajectory. As in the static case, only the following condition is added:

$$(19) pF_K = \rho.$$

By noting that  $\lambda$  (in Mortensen's as well as in Baily's notation) is defined as

$$(20) \lambda = h_1\left(\frac{\dot{L}}{L}, u\right) = \frac{1}{g_1(w, u)}$$

$$\text{and that } \dot{\lambda} = \frac{d\lambda}{dt} = \frac{-g_{11} \frac{dw}{dt}}{g_1}$$

we can represent the Euler equation as a differential equation in  $w$ , instead of an equation in  $L$  as in (6).<sup>12)</sup>

$$(22) pF_L = h(p) = w + \left(\frac{r-g}{g_1}\right) + \frac{g_{11}}{2g_1} \frac{dw}{dt}$$

This is equation (9) in Baily.<sup>13)</sup>

We now turn to Properties 1 to 3 in Baily. These are derived under the assumption that the production function is homogenous of degree one. This specification raises the problem of indeterminacy familiar from the static analysis: the scale of the firm is not determinate. Just as the firm in the static case is unable to fulfill the second-order conditions for profit maximization (Henderson and Quandt, 1981, p.109), the sufficiency condition for the maximization problem is not satisfied. A sufficient condition for the maximum is the concavity of the integrand of (18) in  $L$ ,  $\dot{L}$ , and  $K$ . If the integrand is strictly concave, the optimal solution is unique (Hadley and Kemp, 1971, p.102).

Let  $H$  be the Hessian of the integrand

$$(26) \quad H = \begin{vmatrix} -h_{11} \frac{1}{L} & h_{11} \frac{\dot{L}}{L^2} & 0 \\ h_{11} \frac{\dot{L}}{L^2} & pF_{LL} - h_{11} \frac{\dot{L}^2}{L^3} & pF_{KL} \\ 0 & pF_{KL} & pF_{KK} \end{vmatrix}$$

$$(27) \quad /H/ = h_{11} \frac{1}{L} p^2 (F_{KL}^2 - F_{LL} F_{KK})$$

For a production function homogenous of degree one  $F_{KL}^2 - F_{LL} F_{KK} = 0$ ,<sup>14)</sup> and the integrand is not strictly concave. Thus, there may be many optimal paths.

The indeterminacy problem arises also in the steady-state solution. In the steady state, equation (22) reduces to

$$(23) \quad pF_L = w + \frac{r}{g_1(w)} .$$

This equation says that the labor force will be expanded to the point where the marginal product of labor is equal to the wage rate plus the interest charge on the cost of attracting labor. Similarly, capital input is increased to the point where

$$(19) \quad pF_K = \rho .$$

Furthermore, the relative wage is determined by the requirement that the change in the firm's employment is equal to zero, according to equation (1).<sup>15)</sup>

$$\dot{L}/L = g(w,u) = 0$$

Under constant returns to scale, the marginal product of labor and of capital are functions only of  $k=K/L$ , the capital-labor ratio (Henderson and Quandt, 1981, p.106). Thus, with  $w$  determined according to equation (1), equations (19) and (23) can be viewed as equations involving the product price  $p$  and  $k$ . A solution of these equations yields unique values for  $p$  and  $k$ . This result can now be contrasted with property 1 in Baily, which states: "There is a unique price  $p$  that is consistent with

stationary  $w, L,$  and  $K$  on the optimal path." This is not correct. Only the capital labor ratio is determined, the scale of the firm, i.e., the values of  $K$  and  $L$  remain indeterminate.

In a more general formulation of the production function, the price  $p$  is considered exogenous and has no special significance, as long as the firm does not make a loss at this product price. Equation (19) and (23) determine instead the value of  $L$  and  $K$ .

Properties 2 and 3 concern the dynamic path of wages and employment. Property 2: "for any price  $p$  such that  $p_{\min} < p < p_{\max}$  there is a unique optimal path. This path is to set a constant wage  $w(p)$  where  $w$  and  $p$  are related by:  $h(p) = w + [r - g(w)]/g_1$ ". (equation 14 in Baily)

Property 3: "(a) If the price  $p(t)$  is always above (below)  $p$  then the optimal wage path  $w(t)$  is always above (below)  $w$  from (14). (b) If  $p(t)$  converges from above (below) to  $p$  where  $p_{\min} < p < p_{\max}$  then the optimal path converges from above (below) to  $w$ ."

As is apparent from the discussion in Mortensen and in this paper, the firm does not set a constant wage along the optimal trajectory, but the optimal wage path is characterized by a discontinuity at  $t_0$  and is then described by equations (22) and (1). Likewise, employment along the optimal path does not, as implied by Baily's results,<sup>16)</sup> expand (contract) at a constant



growth rate, but follows a path given by a second order differential equation. According to Property 2, however, if the price is within a certain profit-making range the firm will expand (contract) its labor force (and capital) indefinitely, another manifestation of the indeterminacy problem.

#### IV. The Industry Model in Baily

The transition from firm equilibrium to industry equilibrium is simple given Baily's assumptions. All firms whose number is fixed are identical, and even more crucial, the industry is so small relative to the rest of the economy that wage and employment changes do not affect the aggregate economy. This means that both the economy-wide average wage rate and the unemployment rate are unaffected by developments in the industry. The industry can then be viewed as a monopsonist in the labor market, just as the individual firms were, and all previous results hold.

Industry demand is now a function of output price, and it is assumed that there is always equilibrium in the product market. The addition of the extra equation for the product market makes the product price also endogenous and dependent on the exogenous shift variable in the demand for output equation. But the equilibrium price has no special significance, because the steady state wage rate in the industry is determined outside the

industry by the economy-wide average wage and unemployment. As long as the equilibrium output price is not below the price where the firms will shut down and not above the point where the integral (18) does not converge, pure profits will be made by the firms in the industry, and Baily's conclusion for the industry is incorrect, being based on Property 1 - 3, i.e., that the firms operate where price just covers average costs and do not reap a pure profit.<sup>17)</sup> Naturally, once entry into the industry is allowed, such an entry of new firms would tend to lower the output price and thus profits.

The comparative static analysis of industry equilibrium is unsatisfactory. It shows the limitations of the model of dynamic monopoly at the aggregate level. While doing away with the auctioneer in the labor market, the model clings to this notion in the product market and stands in the way of a dynamic analysis. Such an analysis would have to model the adjustment path of the industry's output in response to a demand shift, perhaps in the vein of a cobweb cycle, and would require abandonment of the point input-point output production function.

Still, such an analysis neglects the interactions between the change in employment following an increase in aggregate demand, its corresponding change in unemployment, and the aggregate wage path. Let us now return to Mortensen where in fact an attempt has been made to make the unemployment rate endogenous to the model.

V. The Macroeconomic Model in Mortensen

The function describing the optimal wage choice of the individual firm in Mortensen's model is given by the following equation:

$$(28) \quad w_i/\bar{w} = h^0 = P(L_i, v_i, r, u^e),$$

(equation (55) in Mortensen).

By aggregation over all firms the following relationship is derived.

$$(29) \quad g = p^e + \frac{1}{b} \ln(\theta(1-u, v, r, u^e))$$

where  $\theta_1 < 0, \theta_2 > 0, \theta_3 < 0, \theta_4 = ?$

The signs of the partial derivatives of  $\theta$  are the same as those derived for  $P$ .

$p^e$  is the rate of product price inflation.  $g$  is the rate of wage inflation.  $\dot{v}/v = p - g$  holds approximately.  $\sum L_i = L$ ,  $1 - u = L/N$ ,  $N$  is the total labor force.  $b$  is the length of the expectations lag in adjusting the expected wage to the actual wage.

$h^0$  is the optimal wage path, with a discontinuity at time  $t_0$ ,

as described earlier. To understand the market model, remember that  $L_i$  in equation (28) is the initial employment level, which is one of the determinants of the optimal employment and wage path. The derivative of  $h^0$  with respect to  $L_i$  reflects one property of the optimal wage path established earlier (equation (15)): after the initial discrete change the wage rate decreases (increases) as employment in the individual firm increases (decreases), i.e., approaches the equilibrium employment level from below (above). After aggregating over all firms, this derivative reappears in equation (29) as the derivative of  $g$  with respect to  $(1-u)$ .

Thus, equation (29) describes only the path of the optimal wage after the discontinuity at  $t_0$ . This results in the conclusion quoted at the beginning of this comment, namely that

$$(30) \quad \left. \frac{dg}{du} \right|_{v=\text{const.}} > 0$$

(Mortensen, 1970A, equation (83a), p.199).

"thus ... the rate of wage inflation increases as the unemployment ratio increases if the price/wage ratio is held constant." (Mortensen, 1970A, p.199).

While the inverse relation between wage changes and changes in employment describes the firm's optimal wage and employment path after time  $t_0$ , it cannot describe the wage behavior of the aggregate economy: as each firm in the economy attempts to raise its wage above the economy-wide level in response, say, to a rise

in the output price, none will succeed in attracting additional labor. Therefore, the second phase of the wage path after  $t_0$  in which the wage is lowered as the firm's employment increases, will never begin. Instead, the wage-wage spiral will go on, if left unchecked. It is here that attention to the interaction between the firms in the wage setting process is important in aggregating over the whole economy. A process of staggered wage setting such as the one analyzed by Phelps (1970) or Akerlof (1969) seems to capture the essence of the wage-wage spiral very well. In such an analysis, wage changes and the unemployment rate are negatively related.<sup>18)</sup>

### Summary

Mortensen's "A Theory of Wage and Employment Dynamics" was one of the first attempts to provide a choice-theoretic rationalization for inflation and employment dynamics. His study of a dynamic monopsonist was later extended by Baily. This comment takes up Mortensen's conclusion that a reduction in the unemployment rate reduces wage inflation, a conclusion contrary to the Phillips curve analysis. This result has two roots. First, Mortensen claims that the optimal response of a dynamic monopsonist to a rise in the unemployment rate is ambiguous, but likely to be positive. In fact, this response can be shown to be negative near the equilibrium point. The second root presents a more serious problem. The adjustment path of the firm's wage is characterized

by a discontinuity at the initial time and then by a differential equation. If, say, after a rise in the product price the initial employment level is below the equilibrium employment level, the optimal wage policy consists in raising the wage rate in a discrete step and then reducing it to the original level as the firm's employment level rises. Failure to recognize the special characteristics of the wage adjustment path in the market model, where both unemployment and wage inflation are endogenous, leads to the erroneous conclusion cited above. Baily's analysis of the dynamic monopsonist is invalidated by the assumption of constant returns to scale, an assumption which makes the solution to the optimization problem indeterminate.

Footnotes

1) Barro-Grossman (1976, p.240); Okun (1981, p.78); Santomero-Seater (1978, p.518); Negishi (1979, p.40); Hall (1974, p.356); Frisch (1977, p.1297).

2) See Hey (1979, p.191); Gordon (1977, pp.58-59); Warren (1983, p.390); Siven (1979, *passim*) who credits Mortensen with having combined the demand side with the supply side in the labor market in a lengthy exposition of Mortensen's model; see Hines (1976, p.79) for some critical remarks on the implication of the search model for the working of the aggregate economy. Brinner's comments are taken up later.

3) See, for example, Gordon (1977, 58ff) and Barro-Grossman (1976, p.249) and Tobin (1972, p.6).

4) In a related paper, Mortensen (1970B) presents a search model which allows for differences in both wage offers and worker quality. The macroeconomic part (dealing with the interaction between inflation and unemployment) is, however, identical to that presented in the Phelps volume. This comment therefore refers explicitly only to the first paper where the results pertaining to the optimal firm behaviors and the aggregate economy are derived.

5) This statement is repeated in Mortensen (1970B, p.859) as:  
"Conversely, the rate of wage inflation falls relative to that expected as employment approaches equilibrium." and is formalized as equation (28).

6) According to Brinner an increase in unemployment in Mortensen's model can be interpreted as "a leftward shift in the labor supply curve creating excess demand and necessitating an increase in wage offers by 'dynamic monopsonist employers'", while in Phelps' model "unemployment is properly interpreted in the traditional way as an excess supply of labor following a leftward shift of the labor demand curve at the current wage and lower wages will suffice to attract the required work force." (p.394, emphasis in original) This view of Mortensen's model then prompts Brinner to conclude that the "accelerationist denial of a durable trade-off between inflation and unemployment rests primarily on a voluntary employment model" (p.395).

7) Later on, it will prove convenient to standardize all nominal variables on  $w$  and to set  $w$  equal to 1.

8) It is important to be clear about the meaning of equation (11). At time zero the firm's labor force is not equal to the optimal labor force, say, because, of an upward movement in the product price. The firm now adjusts its labor force along the optimal path, given by equation (6), to the new equilibrium point, with all parameters of the model ( $r, v, u$ ) being held



constant. We can now ask the question, how does the optimal path of employment and of the relative wage change, if one of the other parameters is changed. Equation (11) answers this question when  $u$  is changed. It does not answer the question what the response of a firm in equilibrium will be to a change in unemployment. See below.

9) Differentiating  $vF_L = h + rh_1$  with respect to  $u$  yields

$$\frac{\partial L}{\partial u} = \frac{h_2 + rh_{12}}{vF_{LL}} > 0$$

10) The possibility that  $w$  rises at  $t_0$  above the new equilibrium value and then declines to the equilibrium value can be excluded because along the optimal path the optimal wage increases as  $L$  decreases (see equation (9)).

11) For convenience and in agreement with Baily's notation we standardize all nominal variables on  $\bar{w}$  and set  $\bar{w}$  equal to 1.

12) At this point it is perhaps worth emphasizing that the Euler equation, which is obtained by substituting (5) into (6), is a second order differential equation in  $L$ , which like any second order differential equation can be written as a system of two first order equations. Likewise, equation (22) together with  $g(w) = L/w$  is a system of two first order equations in  $L$  and  $w$ .

13) In Baily, this equation is derived directly by application of the Maximum Principle.

14)  $F(L,K) = Lf(k)$ , where  $k = \frac{K}{L}$ .

$$F_L = f - f'k,$$

$$F_K = f',$$

$$F_{LL} = f''k^2 \frac{1}{L},$$

$$F_{KK} = f'' \frac{1}{L},$$

$$F_{LK} = -f''k \frac{1}{L},$$

$$\text{and } F_{KL}^2 - F_{LL}F_{KK} = (f'')^2 k^2 \frac{1}{L^2} - (f'')^2 k^2 \frac{1}{L^2} = 0.$$

15) It is useful to recall at this point that  $w$  is the wage relative to the economy's average wage. Furthermore, the change in employment depends not only on the wage rate but also on other factors, most importantly on the state of the labor market, characterized, say, by the unemployment rate. This is explicit in Mortensen, implicit in Baily (see the discussion on pp.338-39).

16) At a constant wage rate, employment changes at a constant proportionate rate according to equation (1).

17) If the industry is viewed as one big firm, i.e., as a monopolist, the scale of production may also be determined, even if the production function is not strictly concave (Henderson and Quandt, p.181).

18) This change also affects the characterization of the singular point in the stability analysis of Mortensen's market model. The singular point might not be a node, but a spiral point.

Another point concerns the expected unemployment rate. Mortensen drops the term  $u^e$  from the market model (explicitly in Appendix A, footnote 28, p.208) because the term is ambiguous, while in fact it is negative near the equilibrium point.

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