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A GENERALIZATION OF THE PHILLIPS CURVE

Fritz BREUSS

Austrian Institute of Economic Research (Wifo), Vienna, Austria*)

Abstract: The Phillips curve has many interpretations. Either it offers a stable enduring trade-off (this was the belief of the earlier proponents) or it offers no trade-off at all (this is the folklore of the "New Classical" macroeconomics).
The "stylized facts" of modern business cycle analysis state that in "normal" business cycles inflation moves procyclically and unemployment countercyclically. This constellation results in a "normal" negatively sloped short-run Phillips curve. In a period of stagflation, when prices and unemployment move countercyclically, one gets a positively sloped Phillips curve. Using the "wave-analytic" approach, one gets a generalized Phillips curve, whichs is able to map all possible patterns of the empirical curve and, hence, is also open to all theoretical interpretations. Therefore one can speak of a true "Phillips possibility curve".
As an interesting side-result the so-called Lucas supply function is derived as a special case of the generalized Phillips curve.

*) An earlier version of this paper was written while the author was a visiting scholar at the University of California (Department of Economics), Berkeley in the Spring Semester 1985.

I. Introduction

More than a quarter of a century has passed since Professor A. W. PHILLIPS (1958) first brought the attention of the ecocomics profession to the relationship between inflation and unemployment and hence gave economics a new paradigm. The vast theoretical and empirical literature about the inflation-unemployment trade-off, which has accumulated in the meantime, has been critically reviewed repeatedly (see FRISCH, 1977, 1980; SANTOMERO-SEATER, 1978; HUMPHREY, 1985).

At the core of modern macroeconomics is some version or another of the Phillips curve. The Phillips curve, both in its original and reformulated expectations-augmented versions, has two main uses. In theoretical models of inflation, it provides the so-called "missing equation" that explains how changes in nominal income are divided into price and quantity components. On the policy front, it specifies conditions contributing to the effectiveness of expansionary and disinflationary policies.

Since 1958 Phillips curve analysis has evolved under the pressure of events and the progress of economic theorizing, incorporating at each stage such new elements as the natural rate hypothesis (PHELPS, 1967; FRIEDMAN, 1968), the adaptive expectations mechanism (GORDON, 1976; WACHTER, 1976), and most recently, the rational expectations hypothesis (LUCAS, 1972, 1973). Each radically altered its policy implications. As a result, whereas the Phillips curve was once seen as offering a stable enduring trade-off for the policymakers to exploit (a sin committed by SAMUELSON-SOLOW, 1960), it is now widely viewed as offering no trade-off at all. In short, the original Phillips curve notion of the potency of activist fine tuning has given way to the revised Phillips curve notion of policy ineffectiveness.

The purpose of this article is not to trace the sequence of steps that led to this change (this is done by HUMPHREY, 1985), but to present a generalized model of the Phillips curve which is able to incorporate all possible theoretical interpretations known hitherto and hence pushes the Phillips curve analysis to the extreme.

The paper is organized in five sections including this introduction. In Section II some "stylized facts" of the business cycle in four countries are discussed and used as a starting point for the reconstruction of the Phillips curve. Section III outlines the mathematical model of the generalized Phillips curve, showing that this curve is a mere statistical artifact. In Section IV the generalized Phillips curve is incorporated into a rational expectations model which shows that the LUCAS supply function is the same construct as the Phillips curve. In Section V some conclusions are drawn. The appendix deals with the specific solutions of difference and differential equations and possible links between both concepts.

II. Inflation and Unemployment as "Stylized Facts" of the Business Cycle

It is common sense that in capitalist economices, aggregate variables undergo repeated (but not regular) fluctuations about trend, a phenomenon called business cycles. Robert E. LUCAS (1977, p. 217), with reference to Arthur F. BURNS and Wesley C. MITCHELL (1946) mentions some "stylized facts" about the main qualitative features of economic time series which one calls "business cycle". Technically, movements about trends in reference series (e. g. gross national product) in any country can be described by a stochastically disturbed difference (or differential) equation of low (at least second) order.

These movements do not exhibit uniformity of either period or amplitude, which is to say, they do not resemble the deterministic wave motions which sometimes arise in the natural sciences. The regularities which are observed are in the movements among different aggregate time series.

It is not the purpose of this article to evaluate the completeness of standard lists of "stylized facts" (for such an exercise, see SCHEBECK-TICHY, 1984). Rather the behaviour of price and wage inflation and unemployment in the business cycle is of interest here. LUCAS and his followers argue that money wages and prices generally are procyclical. No such clear cut statement is given about the behaviour of labour market series. Yet the National Bureau of Economic Research (NBER) in its traditional business cycle analysis includes the series for employment and unemployment rate (inverted= proxy for excess demand for labour) in the list of "Roughly coinciding" indicators, which are seen to be procyclical (see MOORE, 1980, p. 78). Whereas there is unanimity that money wages and prices move "normally" procyclically and unemployment countercyclically, there is no unequivocal view concerning the behaviour of real wages. Some (e. g. TOBIN, 1980) believe that they are procyclical others (e. g. LUCAS, 1977, p. 226) think that observed real wages are not constant over the cycle, but neither do they exhibit consistent pro- or countercyclical tendencies, they are more or less neutral. LUCAS claims that any attempt to assign systematic real wage movements a central role in an explanation of business cycle is doomed to failure. Accordingly, he assumes that the real wage is fixed, using the terms "wages" and "prices" interchangeably. Jeffrey D. SACHS (1983, p. 265) stresses the necessity to differentiate between countries. The United States, with which the literature has mainly dealt, are different. Real wages in the Great Depression moved countercyclically in Europe and procyclically in the United States. But even in Europe it is doubtful that real wages move countercyclically in all business cycles after World War II.

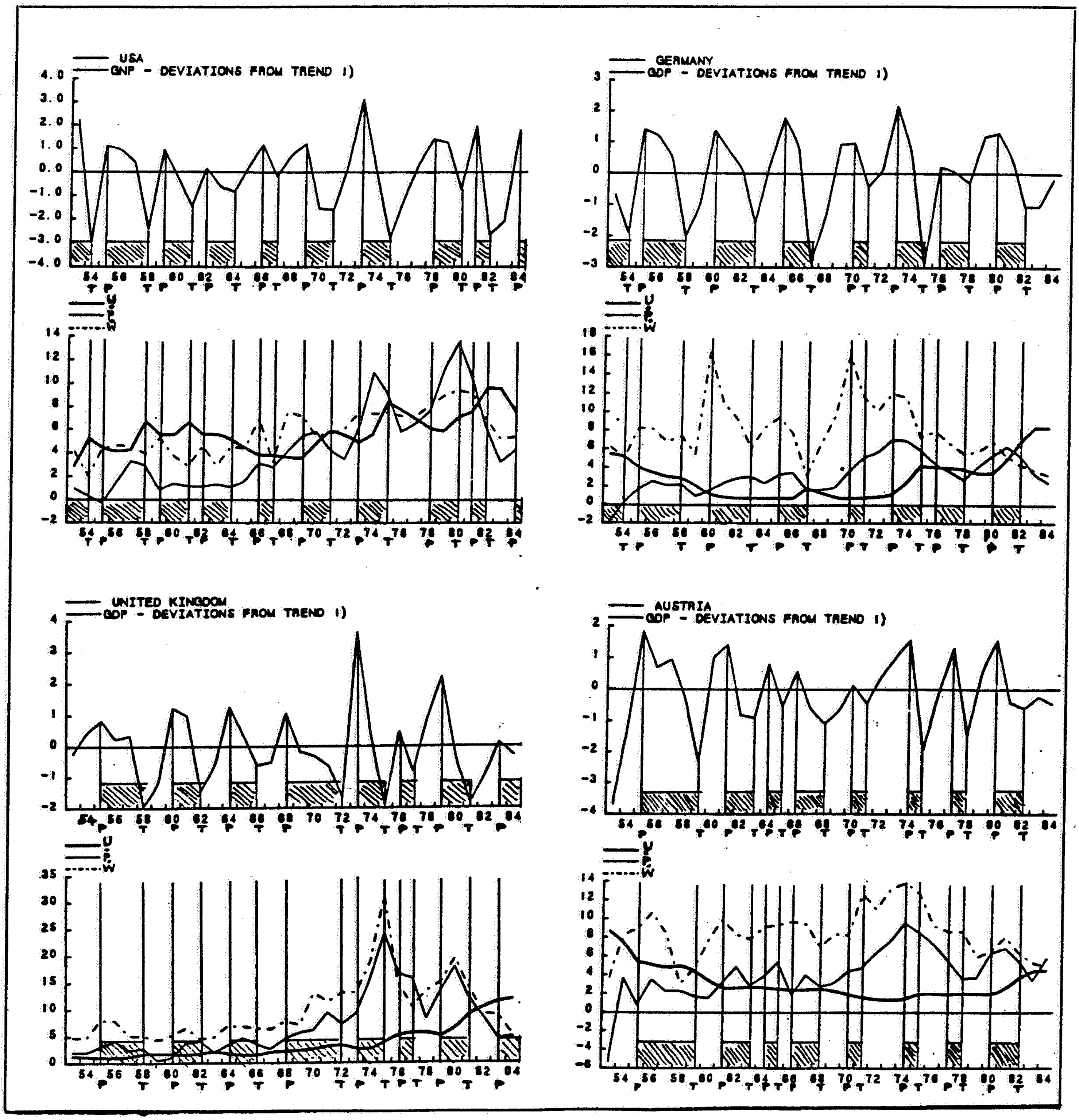
In order to exemplify the aforesaid the behaviour of money wages, prices and the unemployment rate over the business cycle for a small sample of four countries (United States, United Kingdom, Federal Republic of Germany and Austria) in the post World War II period is shown in Figure 1.
These countries were chosen for two reasons: On the one hand to see whether the clocks go differently in the United States and in Europe and on the other hand in order to differentiate between large and small countries.

The time series used here begin in 1953. The reason is that the period shortly after World War II was dominated more by institutional changes and shocks than by "normal" economic performance. At the end of the forties, an international race of currency depreciation against the Dollar took place in order to solve the Dollar shortage problem (currency reforms). The next price shocks were brought about by the Korean war (1950-52) with the highest price increases in 1951. Beyond that in some countries (e. g. Austria) the price-wage adjustment after these shocks were more or less administered by institutional arrangements (e. g. the Austrian Social Partnership was practically founded through the five price-wage agreements between 1947 and 1951; see BUTSCHEK, 1984).

Before reconstructing the Phillips curve from its statistical ingredients "inflation rate" and "unemployment rate" and their behaviour in the business cycle let us recapitulate how Phillips proceeded.
PHILLIPS (1958) tried to apply traditional economic propositions to the labour market in the United Kingdom. In particular, when there is excess commodity demand, prices are expected to rise. To find out, whether this also happens in the labour market, Phillips plotted the percentage changes of money wages ($\hat{w}$) against the unemployment rate (u), the inverse of it served as proxy for excess demand for labour. He found a rather stable

Figure 1

PRICES, WAGES AND UNEMPLOYMENT IN THE BUSINESS CYCLE



1) Trend = 4 years moving average

P = peak, T = trough ▨ = period of contraction

$\dot{p}$ = price inflation (percentage change of CPI)

$\dot{w}$ = money wage inflation (percentage change of compensation per employee)

u = unemployment rate in percent.

downward-sloping convex (nonlinear) long-term relationship for the United Kindom, 1861-1957. This statistical equation $\hat{w} = f(1/u)$ showed the response of money wages to the excess demand for labour, with causation running from unemployment to inflation.

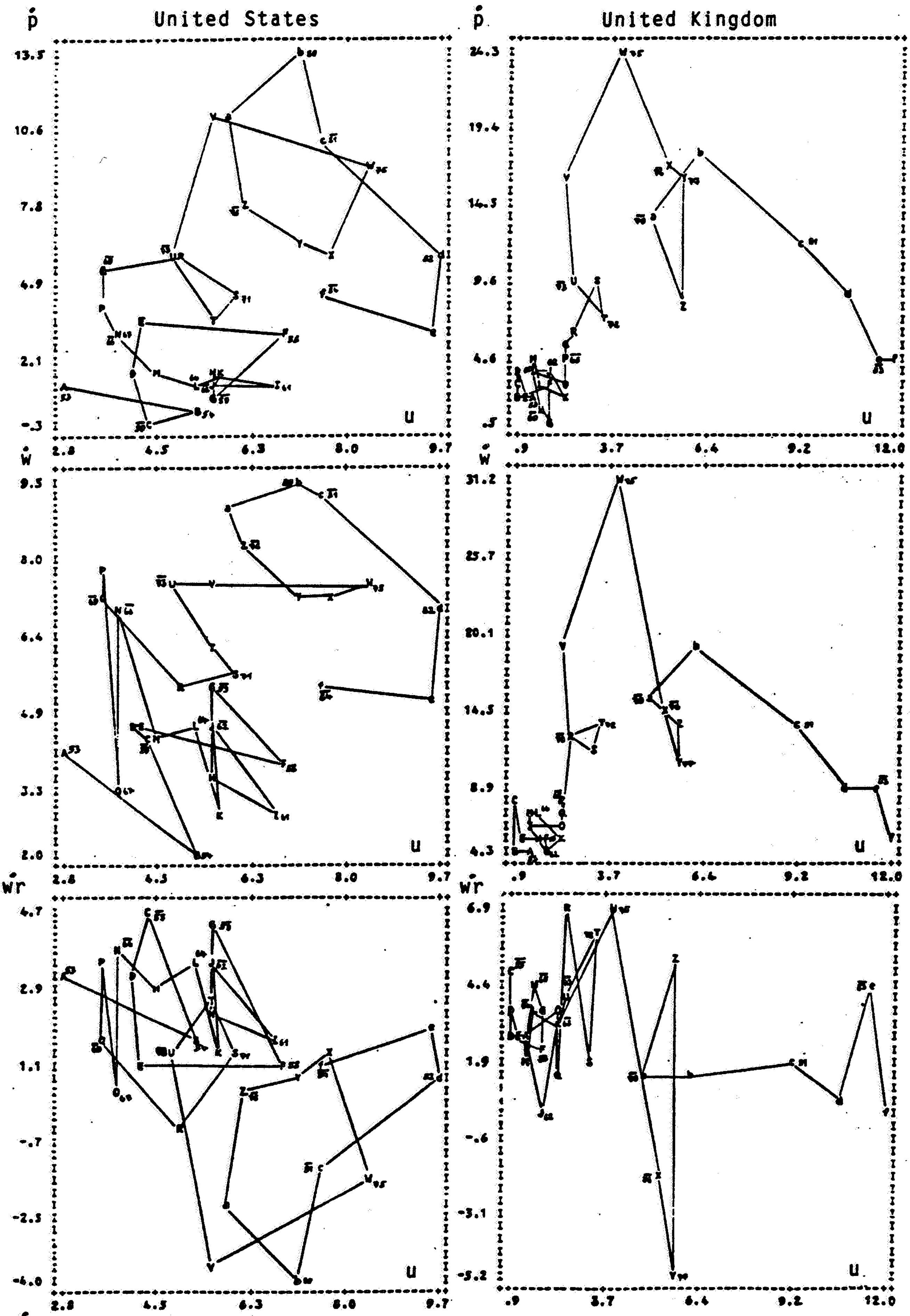
Neglecting at first the proposition that the Phillips curve is a causal relationship with economic meaning and assuming instead that the curve is a mere statistical construct, obtained by plotting the time series of two variables against each other, then we can use the wave analytic approach to study the Phillips curve. From the viewpoint of the "stylized facts" approach to the business cycle one can deduce the following tentative conclusions:

1. If the business cycle is "normal", wages and prices move procyclically and the unemployment rate countercyclically. Plotting the series for inflation against those for unemployment the result for a "normal" business cycle is a "normal" (negatively sloped) Phillips curve. Looking at the actual data (Figure 1) one can detect that these "normal times" ended with the first (1973/74) and second (1979/80) oil price shocks. As LUCAS (1977, p. 228, 229) points out, such shocks to supply which affect all, or many, sectors of the economy simultaneously lead to countercyclical wage/price movements. The succession and therefore permancency of supply shocks in the seventies (two oil price shocks) led to a phenomenon called "stagflation" (BRUNNER-CUCKIERMAN-MELTZER, 1980). In a "stagflation" inflation and unemployment move countercyclically whichs translates into a positively sloped Phillips curve.

2. Plotting the time series for the percentage change of money wages ($\dot{w}$), of prices ($\dot{p}$) and of real wages ($\dot{w}-\dot{p} = \dot{w}r$) against the unemployment rate (u), one gets the Phillips curves for the four selected countries (Figures 2 and 3).

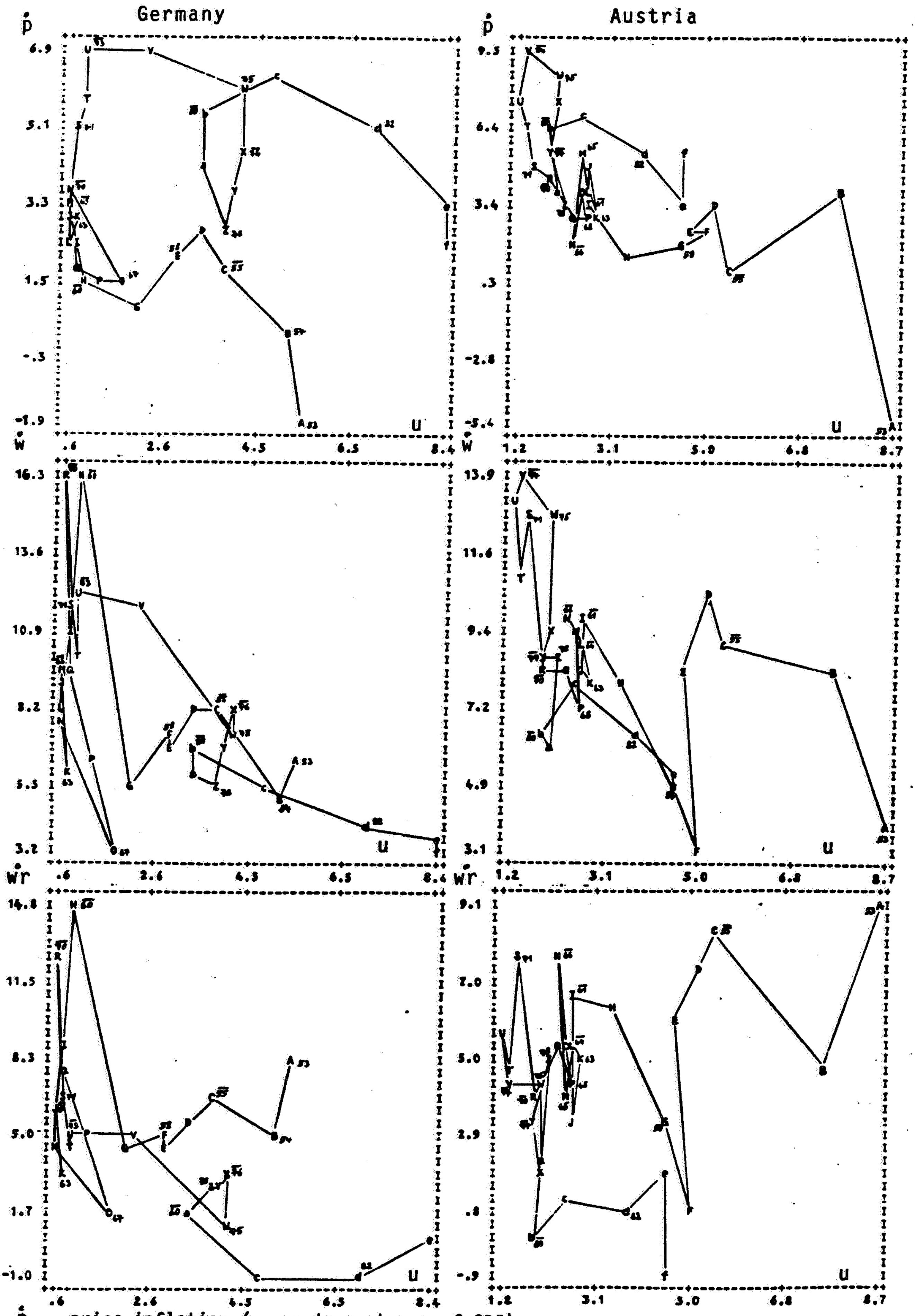
PHILLIPS CURVES

Figure 2



$\dot{p}$ = price inflation (percentage change of CPI)
$\dot{w}$ = wage inflation (percentage change of compensation per employee)
$\dot{w}r$ = real wage changes ($\dot{w}$ - $\dot{p}$)
u = unemployment rate in percent.

PHILLIPS CURVES Figure 3



ṗ = price inflation (percentage change of CPI)
ẇ = wage inflation (percentage change of compensation per employee)
ẇr = real wage changes (ẇ - ṗ)
u = unemployment rate in percent.

The initial Phillips curve depicted a relation between unemployment and money wage inflation. To make the Phillips curve more useful to policymakers (who usually specify inflation targets in terms of rates of change of prices rather than wages), it was transformed from a wage-change relationship to a price-change relationship by assuming that prices are set by applying a constant mark-up to unit labour cost and so move in step with wages - or, more precisely, move at a rate equal to the differential between the percentage rates of growth of wages and productivity.[1] The result of this transformation was the price-change Phillips relation.

Up until the mid-1960s numerous empirical studies doubted the temporal stability of the Phillips curve (see HUMPHREY, 1985, 9 ff.). BHATIA (1961) for example revealed for 1900-1958 U.S. data, that the menue for this country was hardly as stable as its original British counterpart and that the Phillips curve had a tendency to shift over time.[2]

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1) Let prices P be the product of a fixed markup K (including normal profit margin and provision for depreciation) applied to unit labour costs C,

(2.1) $P = KC$.

Unit Labour costs by definition are the ratio of hourly wages W to labour productivity or output per labour hour Q

(2.2) $C = W/Q$.

Substituting (1.2) into (1.1), taking logarithms of both sides of the resulting expression, and then differenctiating with respect to time yields

(2.3) $p = w - q$

where the lower case letters denote the percentage rates of change of the price, wage, and productivity variables. Assuming productivity growth q is zero and the rate of wage change w is an inverse function of the unemployment rate yields

(2.4) $p = ax(1/u)$

where x(1/u) is overall excess demand in labour and hence product markets and "a" is a price-reaction coefficient expressing the response of inflation to excess demand (see HUMPHREY, 1985, 5).

2) On the other hand ECKSTEIN-GIROLA (1978) found recently a strong and persistent long-run Phillips curve for the United States, 1891-1977. Accordingly, wages have been determined by the inverse of the unemployment rate and by consumer prices. The adjustment of wages to prices is nearly complete.

Accordingly, the trade-off equation was augmented with additional variables to account for such movements. The augmented Phillips curve can be written as $\dot{p} = ax(1/u) + z$, where z is a vector of shift variables (productivity, profits, trade union effects, unemployment dispersion etc.). Later, inflation expectations became the chief cause of the shifting short-run Phillips curve.[1)] The fact that the original Phillips curve required a price expectations term led to the development of the expectations-augmented Phillips curve (Natural rate hypothesis and accelerationist hypothesis). The driving force behind the discovery of the missing (shift) variable was the skepticism of some neoclassical thinkers (PHELPS, 1967; FRIEDMAN, 1968) about the validity of a stable Phillips curve, primarily on theoretical grounds. Since neoclassical economic theory teaches that real rather than nominal wages adjust to clear labour markets, it follows that the Phillips curves should have been stated in terms of real wage changes instead of nominal wage changes as in the original version.
In our Figures 2 and 3, therefore, the Phillips curve is also shown in terms of real wage changes. But instead of the theoretical correct specification in terms of expected real wage changes $\dot{w}-\dot{p}^e = f/1/u)$, the Phillips curves drawn here are in terms of realized (or actual) real wage changes $\dot{w}-\dot{p} = f(1/u)$.

Figures 2 and 3 illustrate that the first major shift from a (more or less) negatively sloped Phillips curve to a positively sloped one occured in the wake of the first oil price shock 1973/74, the second after the second oil price shock in 1979/80 in the United States, in Germany and (less pronounced) in Austria. In the United Kindom, however, the Phillips curve

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1) More recent candidates for shift variables are the real wage gap in the price-change Phillips curve (see SACHS, 1983, 268 ff.; BRUNO-SACHS, 1985, 198 ff.) and some proxy for wage moderation because of lost international competitiveness (e. g. current balance in % of GDP, lagged by one year) in the wage-change Phillips curve in small countries (see ROSNER-TINTNER-WÖRGÖTTER-WÖRGÖTTER, 1984; for a successful test for Austria, see POLLAN, 1985).

was positively sloped from the beginning up to 1975. In all the four countries this pattern is more pronounced in the price-change Phillips curve than in the wage-change curve. The curve for real wage changes is (perhaps with the exception of Germany) more or less "chaotic", a "statistical" result of the unstable cyclical pattern of real wages mentioned above.

The worsening of the trade-off in the seventies, i. e. the "stagflation" phenomenon, is now more frequently explained by the so-called "Friedman effect" (FRIEDMAN, 1977; at first proposed by LUCAS, 1973, 333-334). Accordingly, the increased variability of the inflation rate (induced by the oil price shocks) and hence the increase in uncertainty cause a reduction in the allocative efficiency of the price system, which in turn causes a reduction (increase) in the natural rate of real output (unemployment).[1)]

But as one can easily see from Figures 2 and 3, since 1975 (interrupted by the second oil price shock in 1979/80) a process of "disinflation" has taken place in all four countries.[2)] I.e., the transition from high inflation to low inflation traces a short-run Phillips curve. This process is brought about by lowering inflationary expectations via creating slack capacity or excess supply in the economy. Such slack

1) The "Friedman effect" appears to play a significant role in explaining the deterioration in the output-inflation trade-off in the United Kingdom over the period 1957-80. Other effects, jointly tested were the "Lucas effect" (aggregate demand variability) and the "supply side effect" (oil price shocks) (see FROYEN-WAUD, 1984). NEUMANN-HAGEN (1985) show for 1958-82 West German data that in a modified multimarkets equilibrium model, originally formulated by CUKIERMAN (1983), an increase in aggregate risk (higher price variability) simultaneously shifts the long-run Phillips curve to the right and steepens the short-run curves. AIGINGER (1985, Table 12) includes price uncercainty variables as shift variables into a (uncercainty-augmented) Phillips curve for Austria.

2) For a broader discussion about "disinflation", see the proceedings of the conference on "Disinflation - West European Experiences" in 1984 (Zeitschrift für Wirtschafts- und Sozialwissenschaften, 105. Jg., Heft 2/3, 1985; e. g. LÖWENTHAL, 1985).

raises unemployment above its natural level and thereby causes the actual rate of inflation to fall below the expected rate. This adjustment process can be well explained by the natural rate hypothesis and/or the accelerationist hypothesis respectively.

Whereas the disinflation path (especially concerning the price-change Phillips curve) is located well above former short-run Phillips curves in three countries, in Austria this adjustment takes place more or less on the "old" long-run Phillips curve. Generally, the Austrians make Keynesian stabilization policies mixed with an institutionalized incomes policy (via the "Social Partnership") cum "hard-currency" policy (a policy-mix coined "Austro-Keynesianism") responsible for the relatively favourable performance of the Austrian economy with respect to inflation and unemployment in the seventies and eighties (see HOLZMANN-WINCKLER, 1983; TICHY, 1984).[1] The main merits for the smoother adjustment to external price shocks and therefore the existence of a "nice" long-run Phillips curve in Austria is, however, attributable to the influence of the administered price-wage setting by the Austrian "Social Partnership" (see BREUSS, 1980; WÖRGÖTTER, 1983). By setting prices and wages jointly, this institution functions like a "super" auctioneer. The effect is a reduction of uncercainty and information costs and hence an overall increase of allocative efficiency.

3. In addition to the cyclical behaviour of inflation and unemployment in the post World War II period, discussed so far, another feature is worth mentioning: The time series have also different <u>trends</u> (see <u>Figure 1</u>). In all four countries price and wage inflation rates follow an upward trend from the mid-1950s up to the mid-1970s (OPEC I). Since 1974 inflation rates fluctuate around a downward trend, amplified by the OPEC II price shock. As far as unemployment is concerned the development

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1) In a study on the effects of disinflationary policies on unemployment and inflation in Austria, NECK (1985) found that a (simple) Keynesian econometric model does better in explaining the rate of unemployment and the rate of inflation than three Monetarist models.

differs from country to country. In the United States the unemployment rate declined steadily from 1958 to 1969. Since 1969 it followed an increasing trend (the "natural rate" of unemployment is now twice as hight as in the sixties). In the United Kindom unemployment increases steadily since the mid-1950s (with an acceleration in the seventies). In Germany and Austria unemployment rates followed a declining trend up to 1974. After 1974 unemployment increases in both countries, with a steeper upward trend in Germany.

If one abstracts from cycles and only looks at trends the following long-run Phillips curve constellations are possible: The combination of a secular increase of inflation and a decrease of unemployment results in a negatively sloped Phillips curve (inflation process). The same holds for the combination of an inflation with a decreasing trend and an unemployment rate with an increasing trend (disinflation process). If, however, both series have an increasing trend, we get a positively sloped Phillips curve (stagflation). The rare combination of decreasing prices and decreasing unemployment would characterize a deflationary process along a positively sloped Phillips curve.

To sum up, the actual Phillips curve can be seen as the statistical result of the complex interaction of cyclical and long-run movements of two time series: inflation and unemployment.

III. The Mathematical Model of the Generalized Phillips Curve

On the basis of the "stylized facts", discussed in section II, a mathematical model able to generate all possible shapes of Phillips curves can be formulated.

At first, we deal with the static case (or stationary state), where the consequences of the different cyclical behaviour of inflation and unemployment for the shape of the Phillips curve are studied (= "the cyclical model"). After having analysed the periodic (repetitive) case, it is also shown that the generalized Phillips curve can easily end in chaos. The model is then made dynamic by considering not only cycles (short-run Phillips curve) but also the trends in inflation and unemployment (short- and long-run Phillips curve combined in the "complete model").

A. The Static Case or the Pure Cyclical Model

A.1. Periodic Cycles

When modelling the "stylized facts" story one must assume that, technically speaking, the Phillips curve is the statistical result of the superimposition of two harmonic sinoidal waves (oscillations).[1)]

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1) A first step in this direction was made by HANSEN (1970). Although he was mainly interested in the relationship between vacancies and unemployment he derived a Phillips relation with a stationary (long term) part and cycles around it. But he concentrated only on the explanation of Phillips' historical counterclockwise cycles.
YOUNG-BARNETT (1978) also propose implicitly a wave-analytic approach to the Phillips curve. They deal with the dynamic properties of Phillips curves (interaction of the second differences of the inflation and the unemployment rates).

In a first approximation, business cycles can be represented by symmetric oscillations about a rising trend.[1)]

First, suppose there are two exogenous cycles, one in excess demand for labour (for which the unemployment rate is a proxy) and one in inflation.[2)]
Let the cycles (oscillations) in the unemployment rate ($u' = u-\bar{u}$) - where u' is detrended - be

$$(3.1) \qquad u' = r_1 \sin\omega t.$$

Or one can implicitly assume that u' oscillates around a constant trend or the "natural" value of u.

Similarly, the cycles (oscillations) in the (price)inflation ($\dot{p}' = \dot{p}-\bar{\dot{p}}$; $\dot{p}$ is percentage change in the CPI)[3)] shall be given by

$$(3.2) \qquad \dot{p}' = r_2 \sin(\omega t + \delta).$$

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1) BRADFORD DeLONG-SUMMERS (1984) found for six industrieal countries that GNP growth rates and industrial production growth rates do not provide significant evidence of cyclical asymmetry, i. e. that contractions are shorter and sharper than expansions.
2) This assumption is not unrealistic. It highlights the empirical fact that after World War II periods with a relatively stable Phillips curve were followed by stagflation periods. Therefore, as ROTHSCHILD (1982, 189) points out, firstly one should explain separately inflation (e.g. by monetary policy, incomes distribution conflicts, structural problems, terms of trade shocks etc.) and employment (e.g. by effective demand policy, profit expectations, international competitiveness, particitpation rates etc.). Only then one should examine the special conditions under which the not rare combinations of policies and economic behaviour occur which lead to Phillips curve-like relations.
3) As was mentioned in the preceding section, the terms "wages" and "prices" can be used interchangeably when analysing the Phillips curve.

As can be seen from the Appendix, the equations (3.1) and (3.2) are general solutions of homogeneous second-order difference and/or differential equations for the special case of regular oscillations.

The parameters in the equations (3.1) and (3.2) have the following interpretation: r_1 and r_2 are the (constant) amplitudes (or radius); ω is the angular frequency (or angular velocity; $\omega = 2\pi/T$; T is the length of the business cycle, measured e.g., in years); t is continuous time; δ is the angle of the phase (or phase difference between the two waves for u' and $\dot{p}'$).

Through the transformations $u' = u - \bar{u}$ and $\dot{p}' = \dot{p} - \bar{\dot{p}}$ the position of both variables is given in the $\dot{p}$-u diagram. In our example $\dot{p}'$ and u' are in the positive quadrant (Figure 4). But any other constellation is possible. E.g. the case of negative values of $\dot{p}'$ which results in a positive intercept on the u axis.

Second, both single exogenous cycles are combined in order to generate a nonlinear inflation-unemployment relationship, known as the empirical construct "Phillips curve". This procedure is called the addition of two superimposed regular sinoidal waves.

Rearranging equation (3.1) gives

$$\sin\omega t = u'/r_1 \quad \text{and} \quad \cos\omega t = \sqrt{1 - \sin^2\omega t} = \sqrt{1 - (u'/r_1)^2}\ .$$

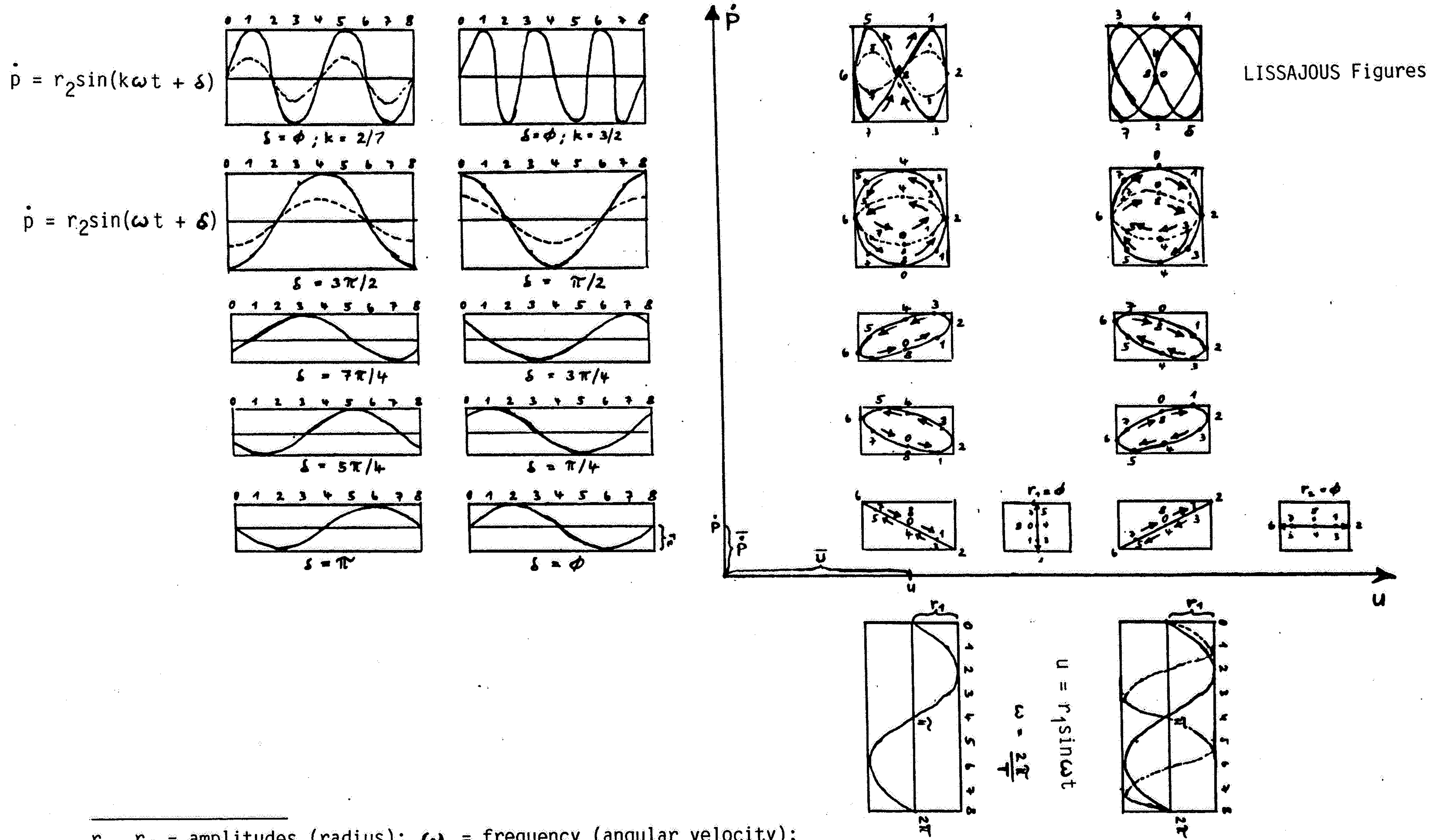
From equation (3.2) and the above rearrangements one gets

$$\dot{p}'/r_2 = \sin\omega t\cos\delta + \cos\omega t\sin\delta =$$

$$= (u'/r_1)\cos\delta \pm \sqrt{1 - (u'/r_1)^2}\,\sin\delta.$$

Figure 4

POSSIBLE SHAPES OF SHORT-RUN PHILLIPS CURVES



r_1, r_2 = amplitudes (radius); ω = frequency (angular velocity);
t = time; δ = angle of the phase (phase difference).

This can be rewritten such that

$$\left(\frac{\dot{p}'}{r_2} - \frac{u'}{r_1}\cos\delta\right)^2 = \left(1 - \frac{u'^2}{r_1^2}\right)\sin^2\delta ,$$

or

$$\frac{u'^2}{r_1^2} + \frac{\dot{p}'^2}{r_2^2} - \frac{2u'\dot{p}'}{r_1 r_2}\cos\delta = \sin^2\delta .$$

From this implicit elliptic function one gets the formula (model) for the generalized static Phillips curve, which turns out to be an ellipsis, namely

$$\text{(3.3)} \quad \dot{p}' = r_2\left[\frac{u'}{r_1}\cos\delta \pm \sqrt{1 - \left(\frac{u'}{r_1}\right)^2}\ \sin\delta\right]$$

The static Phillips ellipsis (curve) is the result of repeated regular (business) cycles and equation (3.3), hence, characterizes a stationary state because in the absence of time-trends the repetition of regular sinoidal waves over the given time length of a cycle is the only dynamic element.[1)] To indicate the absence of "historical" time the variables have no time subscript.

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1) The continuous time (t) involved here represents only the measure of an interval [T',T"] between "historical" time (T), defined as the length of a cycle. For these distinctions, see GEORGESCU-ROEGEN (1971), 135, 136.

Equation (3.3) depicts the short-run aspects of the Phillips curve.

Equation (3.3) can be interpreted as the "Phillips possibility curve"[1] because it allows us to generate different variants of Phillips curves. Varying the values of the parameters r_1, r_2 (amplitudes of the cycles) and δ (phase difference between the inflation and the unemployment cycles) one gets all possible shapes of short-run Phillips curves (see Figure 4):

1. We are now able to verify the heuristic propositions of the "stylized facts" story. In the case of "normal" business cycles one gets a "normal" or standard Phillips curve (a negative, linear[2] relationship between inflation and the unemployment rate). In the "normal" cycle inflation moves procyclically and unemployment countercyclically with respect to a common "reference cycle". If the cycle of the unemployment rate is taken as the "reference cycle" the inflation rate oscillates exactly countercyclically. Technically, this is the case if the phase difference parameter $\delta = \pi$.
Then (because $\delta = \pi$ implies $\sin\delta = 0$ and $\cos\delta = -1$) equation (3.3) yields

$$\dot{p}' = -\alpha u'$$

where $\alpha = (r_2/r_1)$. α is the relationship between the amplitudes for the inflation cycle (r_2) and those for the unemployment cycle (r_1).

1) This expression is borrowed from YOUNG-BARNETT (1978), 33.
2) Because of the static nature of the present analysis only a linear relationship results in this special case. In the dynamic case (next section) the Phillips curve for the "normal" cycle also becomes nonlinear.

The <u>steepness of the Phillips curve</u> is determined by the relationship of the amplitudes of both variables, namely by α . The greater (smaller) α the steeper (flatter) the Phillips curve. If there is no inflation cycle ($r_2 = 0$), then $\dot{p}'$ is zero or the Phillips curve becomes horizontal. If, in the other extreme, there is no unemployment cycle ($r_1 = 0$), then $\dot{p}'$ is not defined, i.e., the Phillips curve becomes vertical.[1)] The proposition that there is no systematic Phillips curve relationship belongs also to the folklore of the "New Classical" macroeconomics.
If there are no cycles in both variables ($r_1 = r_2 = 0$), then the Phillips curve, according to the static model of equation (3.3) would shrink to a single point.

2. In a period of "<u>stagflation</u>" inflation and unemployment move countercyclically in relation to a common "reference cycle". I.e. there is a comovement of both variables. Translated in the general model of equation (3.3) this means that the phase difference is zero ($\delta = 0$).
Then (because $\delta = 0$ implies $\sin\delta = 0$ and $\cos\delta = +1$) equation (3.3) yields

$$\dot{p}' = +\alpha u'$$

where $\alpha = (r_2/r_1)$ as before.
Thus the phenomenon of a "stagflation" results in a <u>positively sloped Phillips curve</u> (see <u>Figure 4</u>).

Within this static framework, we deal with the "pure" stagflation case, i.e., a situation where the "natural" rate of unemployment is constant. As mentioned in the previous section this can occur after shocks to supply (e.g. the oil price shocks) which lead to countercyclical wage/price movements.

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1) Extending this short-term model over many periods, this extreme case can also be interpreted as a situation in which the average value for excess demand for labour (the proxy is u) is equal to zero over successive cycles. This implies an average value of unemployment given by u* (constant "natural" rate). Thus, in the long run, the level of unemployment becomes independent of the rate of inflation (see BARRO-GROSSMAN, 1976, 207, 208). Hence, on average over all business cycles the long-run Phillips curve becomes vertical.

3. A further aspect of the Phillips curve analysis concerns the loops. A. W. PHILLIPS noted that the raw data points were distributed around his long run fitted curve in a systematic way: before World War II, the data described counterclockwise loops and thus tended to lie above the fitted curve when unemployment was falling (and inflation rising); after World War II, the loops became clockwise.

A look at the empirical Phillips curves in Figure 2 and 3 reveals that most of the loops were clockwise, although there are some counterclockwise loops (e.g. see wage-change Phillips curves for the United States, Germany and Austria).

A great variety of interpretations (for an overview, see SANTOMERO-SEATER, 1978, 503, 504) have been offered to explain both phenomena. According to BARRO-GROSSMAN (1976, 199-210)[1] both variants of loops have different theroetical explanations: Clockwise cycles are explained by a price expectations approach (expectation cycles), while counterclockwise cycles are produced by an adaptation mechanism for unemployment (adaptation cycles).

The generalized Phillips curve according to equation (3.3) is able to generate every possible looping pattern. The direction of the loops depends only on the value of the phase difference parameter δ . Clockwise cycles appear when the phase difference (δ) is for instance $\pi/4$, $3\pi/4$ or $\pi/2$. Counterclockwise cycles occur when δ is bigger, namely $5\pi/4$, $7\pi/4$ and $3\pi/2$ (in our example in Figure 4). Thus, a slight shift, say from $3\pi/4$ to $5\pi/4$ induces a change from a clockwise to a counterclockwise loop.

In addition, also the exact shape of the loops (cycles), whether they are circles, ellipses or whether they collapse to a straight line, as well as the position of the loops (vertical; horizontal; inclined to the right or to the left) can be determined by varying the parameters r_1, r_2 and .

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1) For an application of this traditional approach to explain the loops of the Austrian Phillips curve, see STIASSNY (1984).

A.2. From Order to Chaos

Up to now we have dealt with deterministic order and stability. We assumed that inflation and unemployment follow regular cycles, i.e., the frequencies (ω) of the two superimposed sinoidal waves stand in a one-to-one relation to each other. The most complicated geometrical shape of the Phillips curve was an ellipsis.

It can, however, be demonstrated that the "Phillips possibility curve", created by the superimposition of two harmonic sinoidal waves, can take on even more complicated geometrical loop forms than ellipses and circles. Moreover, it is demonstrated that a simple change in the initial constellation of parameters can lead to "chaos".

Only recently a scientific discovery keeps the scientific community in suspense. Since human records began, human beeings were trained to discover order in nature. Although in some fields (e.g. in astrophysics and in the quantum theory in physics) irregularities are well known, only recently one discovered in ordinary natural processes the "deterministic chaos" (see DEKER-THOMAS, 1983; BREUER, 1985; DEWDNEY, 1985). The conclusion is that in contrast to the traditional belief in science, not order is the rule in natural processes, but chaos.[1)]

1) Recent mathematical theories have been constructed by using the notion of the "bifurcation" of a dynamical system in order to explain the emergence of cycles and the transition to turbulent ("chaotic") behaviour in physical, biological, or ecological systems. Catastrophe Theory (CT) is one such theory. It is a branch of differential topology founded by R. THOM (1972, 1975). It deals with the problems of structural instability of dynamical systems. Simply spoken: It deals with the question under which conditions small changes in the system parameters can lead to qualitative changes (jumps) in the original system behaviour. In economics CT is applied in several special fields (for a mathematical and critical overview, see USPRUNG, 1982; for a more applied overview, see FISCHER, 1985). Recently, not only qualitative models are built but one concentrates more and more on empirical parameter estimations. One example in our context is the parameter estimation of a cusp catastroph of the Phillips curve for the United States (FISCHER, 1983; FISCHER-JAMMERNEGG, 1985). They specify the CT stagflation model by WOODCOCK-DAVIS (1978, 130-132). Formally, this is a nonlinear specification of the traditional expectations augmented Phillips curve.

In order to demonstrate that the Phillips curve can also end in chaos, we use the concept of the so-called "LISSAJOUS" figures, which are well known in physics.
One gets such Lissajous figures when the frequencies ω_1 and ω_2 of the sinoidal oscillations of the equations (3.1) and (3.2) stand in a specific proportion to each other.
In Figure 4 only two examples of much more complicated Lissajous figures are generated. In the first case, the relation of $k = \omega_1/\omega_2$ is 2:1 and in the second example k is 3:2. For the simplest case (k = 2/1) the Phillips curve has the general formula

$$(3.4) \qquad \dot{p}' = 2u' \pm \sqrt{1 - \left(\frac{u'}{r_1}\right)^2}$$

The Phillips loop has a so-called "point attractor" in the middle of the picture, i.e., the graphs of the trajectory intersect always in the middle. DEKER-THOMAS (1983) speak in this context of "strong causality" (i.e.: similar causes have similar effects)[1].
Translated in our example: Finite repetitions of the experiment always result in the same picture.
In principle the same is true for the case of k =3/2, except that one cannot derive the explicit formula for this already more complicated Phillips loop.

Now, the initial conditions of our simple nonlinear system are slightly changed. I.e. it is assumed that the frequency proportions k do not consist of integers but of dicimals.
The outcome of this experiment is shown in Figure 5. The first two pictures repeat the LISSAJOUS figures of Figure 4 for the cases k =2/1 and k =3/2 respectively.
Let the relationship of the frequencies in the first case be arbitrarily changed to k = 2.235/1.479 and in the second case to k = 3.557/2.213. These assumptions are not completely unrealistic because the first example just means that the sinoidal

---

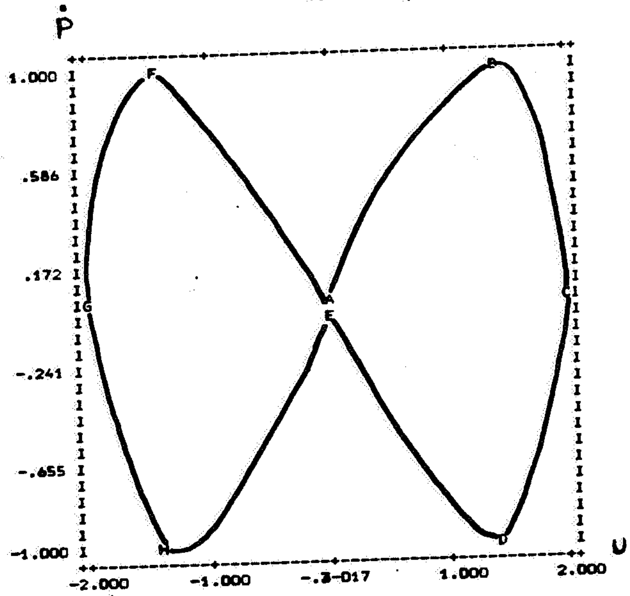
1) "Strong causality" implies "weak causality" (i.e. equal causes have equal effects). Traditional scientific measurement (with measurement errors included) is based on the "strong causality" principle.

Figure 5

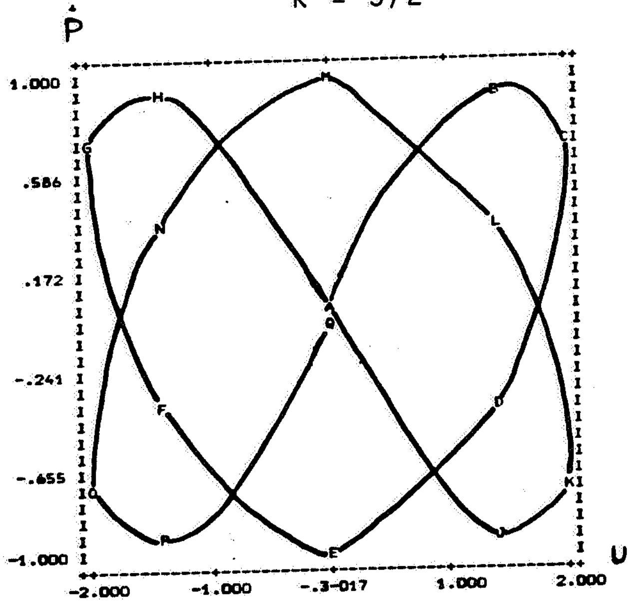
FROM ORDER TO CHAOS

1. ORDER: LISSAJOUS Figures

k = 2/1

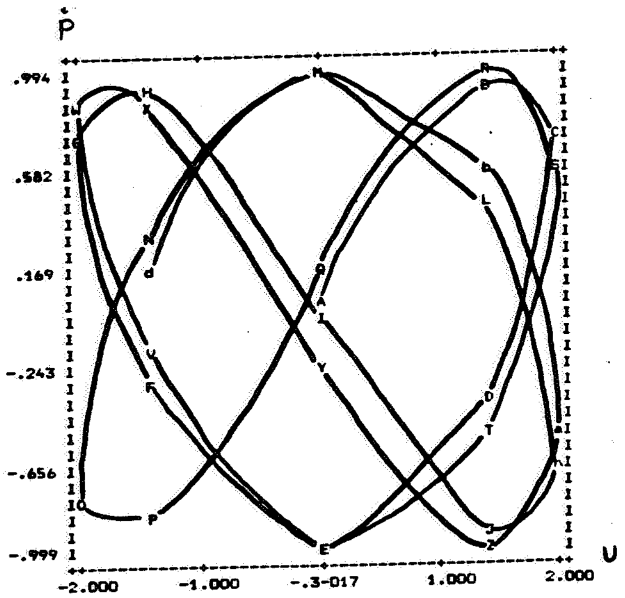


k = 3/2

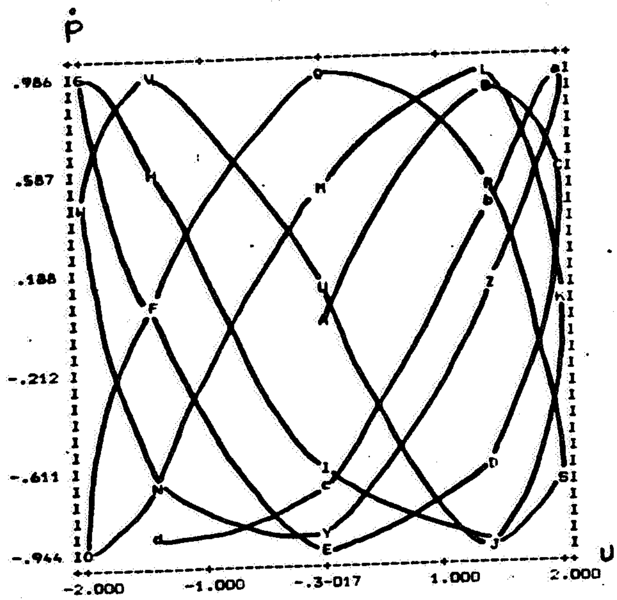


2. CHAOS: LISSAJOUS Figures

k = 2.235/1.479



k = 3.557/2.213



$\dot{p} = r_2 \sin(k\omega t + \delta)$; $u = r_1 \sin \omega t$; $r_2 = 1$; $r_1 = 2$

$\omega = 2\pi/T$; $T = 8$; $\delta = 0$.

wave of the inflation rate has a length of 2.235 times the given cycle lenght T , whereas the wave length of the unemployment rate is 1.4279 times T.

As a consequence, the parameter changes result in a "deterministic chaos", shown in the lower pictures of Figure 5. In both cases the trajectory does not coincide in a common attractor but diverges from former orbits in an unpredictable way. In this case one speakes of "chaotic attractors" (HAKEN, 1983, 42 ff.) or "crazy attractors" (DEKER-THOMAS, 1983, 74, 75).

That this experiment is not just l'art pour l'art can be verified by having a look at the actual empirical Phillips curves in Figure 2. In particular, the real wage-change Phillips curve for the United States is a good example for a "chaotic" pattern.[1)]

1) ROTHSCHILD (1982, 191) points to the fact that the inflation-unemployment trade-off is not symmetric. I.e. both, a negatively sloped and a positively sloped (stagflation) Phillips curve is possible. Assuming, that the possible combinations of inflation and unemployment rates are randomly distributed over the p-u diagram, he demonstrates that a weak negatively sloped Phillips curve emerges. Low unemployment rates are "on average" combined with higher inflation rates. He concludes that uncertain (not sharp) Phillips curves are not the rule, but they are nevertheless the more frequent case in "normal times".

B. The Dynamic Case or the Complete Model

Until now we have only dealt with the cyclical component of the variables inflation and unemployment. Having worked out the various consequences of the cyclical behaviour of the respective variables for the short-run Phillips curve "Gestalt" we are know ready to complete the model. For this purpose the model is dynamized, i.e., in addition to the cycles also the trends of the time series are taken into consideration.

In reality, the cycles are no longer static and selfrepeating, but their pattern changes over time. A look at Figure 1 reveals that there are trends (increasing and decreasing) in the "natural" rate of unemployment and also in the inflation rate. Hence, in order to model a realistic picture of a changing Phillips curve "Gestalt" over time, we assume that both series consist of a trend component and a cyclical term in the following way.

Let the development of the actual unemployment rate (u_t) be characterized by

$$(3.5) \qquad u_t = u_0 e^{g_u t} + e^{g_{r_1} t} r_1 \sin \omega_u t$$

where ($u_0 e^{g_u t}$) is the trend component and ($e^{g_{r_1} t} r_1 \sin \omega_u t$) is the business cycle term. g_u is the trend growth rate for the unemployment rate. u_0 is the starting value of the unemployment rate. g_{r_1} is the growth rate of the amplitude r_1. The cycle in "u" can be damped, if $g_{r_1} < 0$, uniform, if $g_{r_1} = 1$ and explosive, if $g_{r_1} > 0$.[1]

Similarly, let the development of the actual inflation rate ($\dot{p}_t$) be modelled by

1) As can be seen from the Appendix, the business cycle terms are special solutions of differential equations.

$$(3.6) \quad \dot{p}_t = \dot{p}_o e^{g_p t} + e^{g_{r2} t} r_2 \sin(\omega_p t + D\delta)$$

The parameters have the same interpretation as those of equation (3.5). g_u and p_p as well as g_{r1} and g_{r2} need not, of course, be indentical. $\omega_u (\omega_p)$ are the special angular frequencies for u_t and $\dot{p}_t$ respectively. Attached to the phase difference shift parameter δ is a Dummy variable D which may be used to distinguish different time periods with different phase differences.

The same derivation as in the static case yields the formula for the generalized dynamic Phillips curve, which is an exploding or imploding ellipsis, depending on positive or negative growth rates for the amplitude parameters (r_1, r_2).

$$(3.7) \quad \dot{p}_t = e^{g_{r2} t} r_2 \left[\left(\frac{u_t - u_o e^{g_u t}}{e^{g_{r1} t} r_1} \right) \cos D\delta \pm \pm \sqrt{1 - \left(\frac{u_t - u_o e^{g_u t}}{e^{g_{r1} t} r_1} \right)^2} \sin D\delta \right] + \dot{p}_o e^{g_p t}$$

or

$$(3.7') \quad \dot{p}_t - \dot{p}_o e^{g_p t} = e^{g_{r2} t} r_2 \left[\left(\frac{u_t - u_o e^{g_u t}}{e^{g_{r1} t} r_1} \right) \cos D\delta \pm \pm \sqrt{1 - \left(\frac{u_t - u_o e^{g_u t}}{e^{g_{r1} t} r_1} \right)^2} \sin D\delta \right]$$

This curve exists under the conditions that $r_1 > u_t - u_o$ and $\left(\frac{u_t - u_o e^{g_u t}}{e^{g_{r_1} t} r_1}\right) \leq 1.$

As one can easily recognize, the generalized Phillips curve of equation (3.7) is identical with the expectations-augmented standard version of the Phillips curve, except for the fact that our formula explicitly specifies the loopings, the shape and the shifts of the curve. In so far as it includes all possible shapes of Phillips curves, it is a true "Phillips possibility curve".

The similarity with the expectations-augmented version of the Phillips curve becomes clear, if one identifies $\dot{p}_o e^{g_p t}$ as the price expectations term ($\dot{p}^e_t$), the "shift variable", and if one interpretes $u_o{}^{g_u t}$ as the "natural" rate of unemployment (u^*_t), which in our case is not constant, but variable over time.

Equation (3.7') corresponds directly to the "natural rate hypothesis" interpretation of the Phillips curve, namely

$$\dot{p}_t - \dot{p}^e_t = f(u_t - u^*_t)$$

where "f" is an unspecified function. Hence, the trade-off is between unexpected inflation (the difference between actual and expected inflation $\dot{p}_t - \dot{p}^e_t$) and unemployment. That is, only "surprise" price increases could induce deviations of unemployment from its natural rate (u^*_t).[1)]

---

1) For the special case of $\delta = \pi$ (i.e. $\sin\delta = 0$ and $\cos\delta = -1$) one can derive the explicit form of an "expectations-augmented Phillips curve" from equation (3.7')

$$\dot{p}_t - \dot{p}^e_t = -\varepsilon(u_t - u^*_t)$$

where the expression $\varepsilon = (e^{g_{r_2} t} r_2 / e^{g_{r_1} t} r_1)$ determines the steepness of the curve.

In order to capture the main characteristics of the morphogenesis of the actual $\dot{p}$-u relationship in most industrial countries after World War II (a fairly stable Phillips curve in the sixties; the phenomenon of stagflation with a positive $\dot{p}$-u relationship after the oil shocks; and then a period of disinflation and hence, the return to a normal Phillips curve) a "stylized" generalized Phillips curve is designed in Figure 6 and interpreted by the help of the generalized Phillips curve model of equation (3.7).

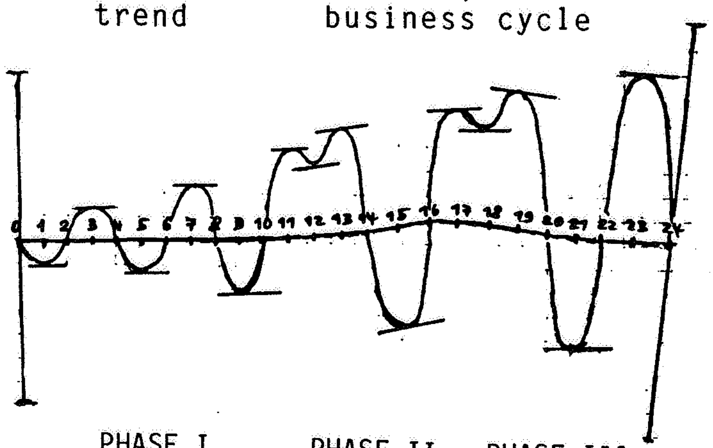
For this purpose three phases are distinguished:
Phase I is characterized by "normal" business cycles. In addition, due to a secular growth rate of GDP in the range of 4 to 5 % the trends in unemployment (downward) and in inflation (upward) point into the opposite direction. A stable international framework on the one hand concerning international trade (increasing liberalization under the OEEC and GATT), on the other hand with respect to international finance (Bretton-Woods fixed exchange rate system; liberalization of international capital transactions under the IMF), as well as more or less constant raw material prices dampen the business cycles. Mild fluctuations around an increasing trend in GDP provoke no drastic policy interventions. This scenario is not fictitious, but a historical sketch of the already glorified period of post World War II up to 1973, the "golden age" of growth in the industrialized world.

This period coincides with the "golden years" of a strong belief in the stability of the Phillips curve. Two factors strengthen the niceness of a negatively sloped Phillips curve: first, the general properties of a "normal" business cycle, which imply - due to the equality between $\delta = \pi$ - also a "normal" Phillips curve; second, the opposite rends in prices and unemployment. Theses combinations yield a nonlinear downward-sloping Phillips curve.

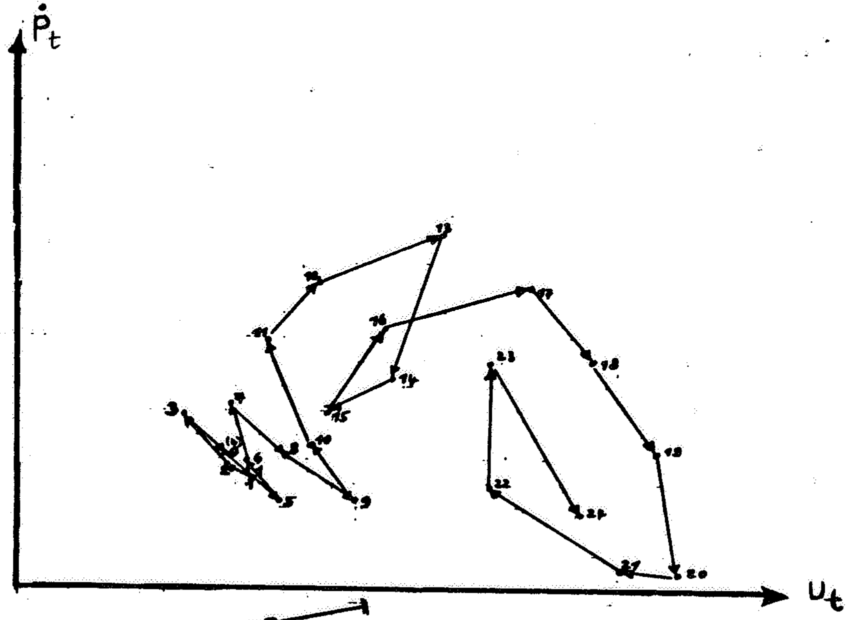
"STYLIZED" GENERALIZED PHILLIPS CURVE

Figure 6

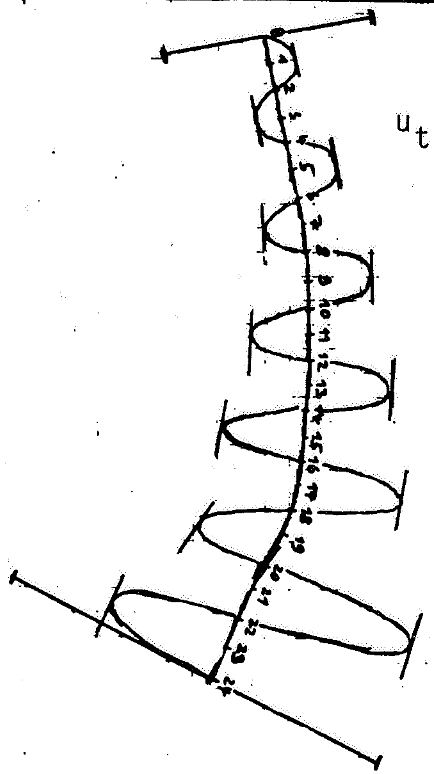
$$\dot{p}_t = \underbrace{\dot{p}_o e^{g_p t}}_{\text{trend}} + \underbrace{e^{g_{r_2} t} r_2 \sin(\omega_p t + D\delta)}_{\text{business cycle}}$$



PHASE I (t=0-11)	PHASE II (t=12-17)	PHASE III (t=18-24)
"normal" business cycle $\delta = \pi$	"stag-flation" (oil shocks) $\delta = \pi/4$	"disin-flation" $\delta = \pi$



$$u_t = \underbrace{u_o e^{g_u t}}_{\text{trend}} + \underbrace{e^{g_{r1} t} r_1 \sin\omega_u t}_{\text{business cycle}}$$



In Figure 6 this scenario is depicted by 6 cycles of equal length (T = 4 years). During the first three cycles we have modelled a "normal" business cycle, i.e. $\delta = \pi$. The unemployment rate has a downward trend, inflation rate follows a slight upward trend. Furthermore, the business cycle has mildly expanding amplitudes. This results in a nonlinear negatively sloped Phillips curve in the years t = 0 to 6.

Then, according to the mechanism of the "natural rate" interpretation of the expectations-augmented version of the Phillips curve the short-run Phillips curve shifts to the right (in the years 6 to 11). The transition to Phase II follows.

Phase II is characterized by "stagflation". Two oil price shocks (1973/74 and 1979/80) lead - via a deterioration of the terms of trade in the industrialized countries - to an upsurge in inflation (to countercyclical wage/price movements), to a decline in productivity and GDP growth rates, to wider swings in the business cycles, and to a secular increase in unemployment. It is also the period of the transition from the Bretton-Woods system to a system of more or less mananged floating. In this period of real and financial instability, real interest rates reach unpredecent levels. It is a period of dangerous debt crisis in the developing world with the revival of protectionist tendencies (Neo-Protectionism) in world trade.

The period sees radical changes in the policy stance: from a generally accepted Keynesian fiscal policy stance to a more monetarist oriented policy in the largest countries.

Summing up, one could speak of a "policy regime change" in a more broadly defined sense than LUCAS (1976) had in mind. LUCAS warned us, that the coefficients of econometric models might change in the face of policy changes. This is probable true for the most important coefficient in our model, namely δ .

The oil price shocks led to a comovement of prices and unemployment, technically, to $\delta = \pi/4$. Furthermore the "natural" rate of unemployment increased in trend, a phenomenon which can be explained either by the so-called "Friedman effect" (increase in price uncertainty), mentioned earlier or by the persistence of supply shocks (BRUNNER-CUKIERMAN-MELTZER; 1980).

Looking at Figure 6 we see that the "stagflation" period starts with the "oil price shocks" (to simplify the analysis the actual two shocks were concentrated in one "hypothetical" shock) in t = 12. The Phillips curve becomes positive and exhibits a counter-clockwise cycle, a phenomenon which corresponds to the actual evolution of Phillips curves in the four countries around the oil price shock years 1973/74 and 1979/80 (see Figures 2 and 3).

Phase III is characterized by the adjustment process of the industrial economies to the oil price shocks. The combination of technological change (structural change into a high-tech society, also in order to regain competitiveness against the NICs on the world market), the substitution of oil intensive techniques as well as energy conservation (a development which led to an oil glut and a near breakdown of the OPEC cartel), the real wage adjustment (the importance of "wage moderation" in order to overcome stagflation stress in particular BRUNO-SACHS, 1985) as well as the policy regime changes (from Keynes to Friedman) induced a process of "disinflation".
In this process the slack capacity or excess supply in the economy ($u_t > u^*_t$) causes the actual rate of inflation to fall below the expected rate so as to induce a downward revision of the latter. In this context one can speak of a second "policy regime change" in the LUCASian sense.[1)]

---

1) Recent empirical studies which concentrated on the US-Phillips curve found only little change in the coefficients of the standard Phillips curve (no major shift) according to the DRI model (BLANCHARD, 1984) and time-series techniques (TAYLOR, 1984) after the monetary policy regime change in the United States in October 1979 (shift from interest rate to money stock targeting). But the policy change was probably only minor and the period (1980-83) to short for a serious test of the "policy regime change" hypothesis.

As a result, in nearly all industrialized countries the inflation rate fell to levels comparable to those of the mid-sixties (see Figures 2 and 3). This transition from high to low inflation rates ("disinflation" process) is also captured in Figure 6. We assumed that during this Phase III (from 6 = 18 to 24), the business cycle becomes gradually "normal" again, i.e. $\delta = \pi$.

In summing up the results of our generalized Phillips curve, one can draw the following tentative conclusions: The Phillips curve concept is in some version or another the core of modern macroeconomics. But the hope of the sixties that with the Phillips curve the economics profession finally found the long expected "economic constant" like the natural constants in physics, which would it make easier to understand and govern the "chaotic" business life (ROTHSCHILD, 1982, 184), faded as time passed by.

Our approach starts with the assumption that inflation and unemployment are two separate processes and can be welded into a Phillips curve-like relationship only under special conditions. If one takes this view one must conclude that the Phillips curve is nothing more than just a historical accident, brought about by some rare political constellations.

IV. The LUCAS Supply Function - A Special Case of the Generalized Phillips Curve

In this section it will be demonstrated that the so-called "LUCAS supply function" is a special case of our generalized Phillips curve. Furthermore it is shown that it is possible to derive the "invariance proposition" with our generalized Phillips curve, a proposition which belongs to the folklore of the "New Classical" macroeconomics.

A. Macro Model

In order to show this, the dynamic version of the generalized Phillips curve, developed in section III (equation 3.7), is embedded in an exceedingly simple macro model (similar to those of BEGG, 1982; SHEFFRIN, 1983, 40 ff.; MINFORD-PEEL, 1983, 15 ff.), such that one can find a rational expectations solution.
Fiscal policy will be assumed to be held constant, and monetary policy will be the only policy variable affecting the demand for output. The velocity of money will also be a constant (implicitly assuming that the demand for money is not responsive to the interest rate). Furthermore it is assumed that wages are indexed with price inflation. The labour market clears in a particular way. All workers participate in a bargain to determine the nominal wage contract for a single period at a time. At the beginning of each period, workers and firms agree on a nominal wage for the period. Since firms and workers care about real wages, this requires that they form expectations about the price level which will prevail over the duration of the nominal wage contract. At the end of the period the effects of any unforeseen change in prices may be absorbed into the nominal wage (see BEGG, 1982, 135).

With these assumptions, the aggregate demand for output can be written, in logs, as

Aggregate demand

(4.1) $m_t + \bar{v}_t = q_t + y_t$

where m_t = log of the money supply
$\bar{v}_t$ = log of the (constant) velocity of money
q_t = log of the (GDP) price level
y_t = log of real output (GDP).

In equation (4.1) it is assumed that the appropriate deflator (q_t) is the price of domestic output (GDP). In the Phillips curve, however, the usual variable is a deflator that is a weighted average of domestic and import prices, namely the private consumption deflator or the CPI. In such a formulation the definition of the "price level", p_t in logs, could be written as

Definition of CPI

(4.2) $p_t = \alpha q_t + (1-\alpha)(\overline{pw}_t + x_t)$

or alternatively for q_t

Definition of GDP deflator

(4.2') $q_t = (1/\alpha)p_t - ((1-\alpha)/\alpha)(\overline{pw}_t + x_t)$

where α and $(1-\alpha)$ are the expenditure shares of domestic goods and imports; $\overline{pw}_t$ = log of the (constant) world price level.

Following DORNBUSCH's overshooting model (1976, 1163) "x_t" is the expected rate of depreciation of the domestic currency, or the expected rate of increase of the domestic currency price of foreign exchange. Denoting the logarithms of the current and long-run rate by "e_t" and "$\bar{e}_t$", respectively, DORNBUSCH assumes that

Exchange rate expectation formation

$$(4.3) \qquad x_t = \theta(\bar{e}_t - e_t)$$

This equation states that the expected rate of depreciation of the spot rate is proportional to the discrepancy between the long-run rate and the current spot rate. θ is the coefficient of adjustment.
Analogous to a money supply (feedback) rule (see quation 4.6) one could conceive an exchange rate policy with a feedback character (such a formulation would for instance be adequate for Austria's "hard-currency policy").

After the transformation from percentage changes in the price level ($\dot{p}_t$) to the CPI price level ($\dot{p}_t = p_t - p_{t-1}$; p_t in logs) the generalized (dynamic) Phillips curve of equation (3.7) is used here to determine the price level

Generalized Phillips curve

(4.4) $$p_t - p_{t-1} = p^*_t - p^*_{t-1} + b\left[\left(\frac{u_t - u^*_t}{a}\right)\cos\delta \pm \sqrt{1 - \left(\frac{u_t - u^*_t}{a}\right)^2}\ \sin\delta\right]$$

where u_t = unemployment rate (in %, not in logs)
u^*_t = "natural" rate of unemployment (in %) ($u^*_t = u_o e^{g_u t}$)
p_t = log of the price level (CPI)

p^*_t can be interpreted as price expectations. Then:

p^*_t = ${}_{t-1}p^e_t$ = log of the price level (CPI) that the public expects will prevail in time t viewed from period t-1.
($p^*_t - p^*_{t-1} = \dot{p}^*_t = \dot{p}_o e^{g_p t}$ = "expected" rate of change of the price level).

$p_{t-1} \approx p^*_{t-1}$

b = $e^{g_{r2} t} r_2$ = fluctuations of the amplitude for p_t

a = $e^{g_{r1} t} r_1$ = fluctuations of the amplitude for u_t

D = shift dummy for δ (in equation 3.7), set equal to one.

In the standard rational expectations models one uses explicitly the "LUCAS supply function" in order to determine aggregate supply. In the approach chosen here, the "LUCAS supply function" is implied by the generalized Phillips curve.[1)]
The labour market and the goods market are connected here by a special version of OKUN's law in the following way

OKUN's law

(4.5) $u_t - u^*_t = \varrho(y_t - y^*_t)$

where y^*_t = log of full employment (or potential) output

ϱ = coefficient of adjustment ($\varrho < 0$).

---

1) If one assumes that $\delta = \pi$ (= "normal" business cycle), then $\sin\delta = 0$ and $\cos\delta = -1$ (for $\delta = 0$, the case of "stagflation, $\sin\delta = 0$ and $\cos\delta = +1$).
In the special case ($\delta = \pi$) the generalized Phillips curve (equation 4.4) results in a
"LUCAS supply equation" for the labour market

(4.4') $u_t = u^*_t + \beta_1(p_t - p^e_t)$

where $\beta_1 = -(a/b)$.

And one gets a
"LUCAS supply equation" for output by substituting equation (4.5) in (4.4)

(4.5') $y_t = y^*_t + \beta_2(p_g - p^e_t)$

where $\beta_2 = +(\beta_1/\varrho)$.
It is interesting to note that the LUCAS supply function can be derived as a special case from our generalized Phillips curve and, hence, is the same statistical construct as the Phillips curve. GRANDMONT (1985, 1033) criticizes the "LUCAS supply function" more fundamentally by pointing out that the relationship between output and price surprises involves equilibrium magnitudes and therefore cannot be interpreted as supply or demand functions.

In rational expectations models, price expectations are not fixed or predetermined but respond to anticipated movements in the money supply. It is assumed here that the policy authorities utilize the following

Money supply rule

(4.6) $m_t = \bar{m}_t + \epsilon_t$

where $E(\epsilon_t/I_{t-1}) = 0$

$\bar{m}_t$ = log of monetary target (known constant).

With rational expectations, the price expectations are determined within the model in light of future developments of the money supply. This is expressed as

Rational expectations

(4.7) ${}_{t-1}p^e_t = E(p_t/I_{t-1})$

Equation (4.7) asserts that people's expectations of the price level (${}_{t-1}p^e_t$) equal the mathematical expectations of the price level, given both the structure of the model and the information (I_{t-1}) available.

B. Rational Expectations Solution

Given the expectation hypothesis of equation (4.7) one can solve the macro model by substitution.

Substituting (4.5) in (4.4) and (4.4) in (4.2') as well as (4.2') and (4.6) in (4.1) gives

$$(4.8)\quad \bar{m}_t + \epsilon_t + \bar{v}_t = y_t + (1/\alpha)p^*_t + (1/\alpha)b\left[\left\{\rho\left(\frac{y_t - y^*_t}{a}\right)\right\}\cos\delta \pm \sqrt{1 - \left\{\rho\left(\frac{y_t - y^*_t}{a}\right)\right\}^2}\sin\delta\right] - ((1-\alpha)/\alpha)\left[\overline{pw}_t + \theta(\bar{e}_t - e_t)\right]$$

Taking the mathematical expectation on both sides of the equation as of time t-1 (and assuming: $p_{t-1} = p^*_{t-1}$; $p^*_t = p^e_t$; $y^*_t = y^e_t$, $\bar{e}_t = e^e_t$) gives

$$(4.9)\quad \bar{m}_t + \bar{v}_t = y^e_t + (1/\alpha)p^e_t - ((1-\alpha)/\alpha)\overline{pw}_t$$

or, solved for p^e_t

$$(4.9')\quad p^e_t = -\alpha y^e_t + (1-\alpha)\overline{pw}_t + \alpha\bar{m}_t + \alpha\bar{v}_t$$

Now one can <u>solve for output</u>, y_t, by subsituting equation (4.9') into equation (4.8). Simplifying (e.g. assuming that the expression $\left\{\rho\left(\frac{y_t - y^*_t}{a}\right)\right\}^2$ is approximately zero) gives the solution

$$(4.10)\quad y_t - y^*_t = \frac{\pm(1/\alpha)b\sin\delta}{\left[1 + \frac{(1/\alpha)b}{a}\rho\cos\delta\right]} + \frac{((1-\alpha)/\alpha)\theta(\bar{e}_t - e_t)}{\left[1 + \frac{(1/\alpha)b}{a}\rho\cos\delta\right]} + \frac{\epsilon_t}{\left[1 + \frac{(1/\alpha)b}{a}\rho\cos\delta\right]}$$

or

$$(4.10')\quad y_t = y^*_t + \left[\ (4.10)\ \right]$$

A simplification of (4.10) is possible for the special case of $\delta = \pi$. Then $\sin\delta = 0$ and $\cos\delta = -1$, and (4.10) yields

$$(4.10'') \quad y_t - y^*_t = \frac{((1-\alpha)/\alpha)\,\theta\,(\bar{e}_t - e_t)}{\left[1 - \frac{(1/\alpha)b}{a}\varrho\right]} + \frac{\epsilon_t}{\left[1 - \frac{(1/\alpha)b}{a}\varrho\right]}$$

If one assumes that "b" is approximately equal to "a", the first term in equation (4.10) is a constant.
Expectations of the exchange rate ($x_t = \theta(\bar{e}_t - e_t)$, in the second term of equation (4.10), also influence actual output only because of the special DORNBUSCH assumption for exchange rate changes.[1)]

In general, however, equations (4.10) and (4.10'), respectively, reflect the "invariance proposition" of the "New Classical" macroeconomics. Output fluctuates randomly around the full employment level, with the fluctuations due to unanticipated movements in the money stock (ϵ_t).[2)]

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1) With the use of only one deflator (e.g., p_t) in the equations (4.1) and (4.2) this term would vanish. Furthermore, the definitions of the price levels (equations 4.2 and 4.2') would be made superfluous by this simplification.
If instead of (4.3) a "hard-currency" exchange rate policy rule would have been formulated (e.g. like: $x_t = \bar{e}_t + \omega_t$; with $\bar{e}_t$ the exchange rate target and ω_t a stochastic term) equation (4.10) would include a second stochastic element (ω_t) in the second term.
2) In contrast to the proposition of the "New Classical" macroeconomics that only unanticipated inflation matters, GRANDMONT (1985, 1031 ff.) demonstrated with an endogenous competitive business cycle model that there is a systematic relationship between equilibrium levels of output and the real rate of interest (or inflation).

Substituting equation (4.10) into (4.5) and (4.4) respectively, one can <u>solve for the price level</u>, p_t

$$(4.11) \quad p_t = -\alpha y^e_t + (1-\alpha)\overline{pw}_t + \alpha \bar{m}_t + \alpha \bar{v}_t + $$
$$+ b\left[\left\{\frac{\varrho}{a}\left[4.10\right]\right\}\cos\delta \pm \sqrt{1-\left\{\frac{\varrho}{a}\left[4.10\right]\right\}^2}\sin\delta\right]$$

Hence, the behaviour of prices reflect the choice of the money supply (feedback) rule.

The gap between the actual price and the expected price is[1)] $(p^*_t = p^e_t)$

$$(4.12) \quad p_t - p^e_t = + b\left[\left\{\frac{\varrho}{a}\left[4.10\right]\right\}\cos\delta \pm \sqrt{1-\left\{\frac{\varrho}{a}\left[4.10\right]\right\}^2}\sin\delta\right]$$

This rather messy expression shows that - as postulated by the <u>rational expectations hypothesis</u> - there is no trade-off between inflation and unemployment (i.e. <u>no Phillips curve</u>). The rational expectations hypothesis implies that deviations in the unemployment rate from its natural rate can still occur when mistakes are made in predicting inflation, but these errors must be of a random nature.

Finally, a simplification of (4.12) is possible. If, for instance, $\delta = \pi$, then $\sin\delta = 0$ and $\cos\delta = -1$. In this special case, equation (4.12) yields

$$(4.12') \quad p_t - p^e_t = -\frac{b}{a}\varrho\left[\frac{((1-\alpha)/\alpha)\,\theta\,(\bar{e}_t - e_t)}{\left[1 - \frac{(1/\alpha)}{a} b\varrho\right]} + \frac{\epsilon_t}{\left[1 - \frac{(1/\alpha)}{a} b\varrho\right]}\right]$$

---

1) The solution for q_t would follow if (4.12) were substituted into the definition of q_t (equation 4.2'). The expression of (4.12) would be a little bit more messy but none of the qualitative results would be affected by this exercise.

To sum up the results of this section: in general it could be demonstrated that our generalized Phillips curve cannot only be interpreted along traditional lines (in particular the "expectations-augmented" version of the Phillips curve analysis) but it can also be given a rational expectations interpretation.

If the generalized Phillips curve is embedded in a simple macro model and it is assumed that expectations are formed rationally two main conclusions follow:
First, the "LUCAS supply function" is a special case of our generalized Phillips curve and hence, is practically the same statistical artifact as the Phillips curve.
Second, all the standard results which belong to the folklore of the "New Classical" macroeconomics can be obtained, in particular, the "invariance proposition" and the postulate that there is no (systematic) Phillips curve trade-off.

V. Conclusions

The prime motivation for writing this article was the deep frustration that the Phillips curve is a widely used concept in theoretical and empirical macroeconomics, but has no sound foundation. Depending on the theoretical point of view, the Phillips curve offers either a stable enduring trade-off or it offers no trade-off at all. Thus, the big variety of interpretations of one and the same phenomenon (the Phillips curve) sheds a significant light on economics as a science.

In order to overcome this shortcoming in one of the most prominent concepts in macroeconomics we try to formulate a generalized version of the Phillips curve.
For this purpose we start with the idea that the Phillips curve is basically the construct of two separate variables, inflation and unemployment. The "stylized facts" of modern business cycle analysis claim that in "normal" business cycles inflation moves procyclically and unemployment countercyclically. When putting both variables together in a "wave-analytic" approach, this results in a "normal" negatively sloped short-run Phillips curve. In a period of "stagflation", when prices and unemployment - due to external shocks - move countercyclically, one gets a positively sloped Phillips curve. But in general, the generalized short-run Phillips curve is an ellipsis.

In reality, however, the variables inflation and unemployment are not only characterized by a cycle but also by trends. Putting trends and cycles together and making again use of the "wave-analytic" approach, one gets a generalized Phillips curve, which is able to map all possible changes of the pattern of the empirical Phillips curve and, hence, is also open to all theoretical interpretations known so far. Therefore, our generalized curve is a true "Phillips possibility curve".

Thus, the generalized Phillips curve can either be interpreted by reference to the traditional "expectations-augmented" approach or the "rational expectations" approach. In both cases one can reproduce the postulated results. As an interesting side-result, it could be demonstrated that the so-called "LUCAS supply function" can be derived as a special case of our generalized Phillips curve and, hence, is the same statistical artifact as the Phillips curve itself.

Appendix: Difference and Differential Equations - Their Solutions as Basis for the Generalized Phillips Curve

1. When time is taken to be a discrete variable, the pattern of change of a variable y is described by difference equations. A homogeneous second-order linear difference equation takes the general form

(A.1) $$y_{t+2} + a_1 y_{t+1} + a_2 y_t = 0$$

or

$$y_t + a_1 y_{t-1} + a2y_{t-2} = 0$$

When $\frac{a_1^2}{4} < a_2$, the characteristic roots - thanks to De Moivre's theorem - will be conjugate complex of the general form (see CHIANG, 1974, 583 and STÖWE-HÄRTTER, 1967, 256)

(A.2) $$y_t = CR^t \sin(\omega t + \delta)$$

or

(A.2') $$y_t = CR^t \cos(\omega t + \delta) \quad \text{if } \delta = 90^0 + \delta^*$$

where the constants C and δ are an arbitrary amplitude and phase, respectively. ω is the angular frequency and R is given by $\sqrt{a_2}$.

The oscialltion of equation (A.2) can be damped ($a_2 < 1$), regular ($a_2 = 1$) or explosive ($a_2 > 1$).
It is assumed that the intertemporal equilibrium is stationary, i.e. oscillations around a constant trend.

2. In the continuous-time context, the techniques of differential equations is used to analyse the pattern of change of a variable y. A homogeneous second-order linear differential equation takes the general form (see CHIANG, 1974, 529 ff.)

$$\text{(A.3)} \qquad y''(t) \;+\; a_1 y'(t) \;+\; a_2 y \;=\; 0$$

where $y'(t) = \frac{dy}{dt}$ and $y''(t) = \frac{d^2y}{dt^2}$

for the complex-root case $\left(\frac{a_1^2}{4} < a_2\right)$, a general solution (see STÖWE-HÄRTTER; 1967, 288) is

$$\text{(A.4)} \qquad y(t) \;=\; Ce^{-gt}\sin(\omega t \;+\; \delta)$$

or

$$\text{(A.4')} \qquad y(t) \;=\; Ce^{-gt}\sin(\omega t \;+\; \delta) \quad \text{if } \delta = 90^0 + \delta^*$$

where $g = \frac{a_1}{2}$ and the constants C and are an arbitrary amplitude and phase. $\omega = \sqrt{a_2 - \frac{a_1^2}{4}}$ is the angular frequency.

The multiplicative term e^{-gt} influences the amplitude C in the following way:

1. If $-g < 0$, the time path tends to converge to equilibrium because each successive cycle has a smaller amplitude than the preceding one. This is the case of damped fluctuations.
2. If $-g = 0$, the amplitude is constant. It is a time path with regular (or uniform) fluctuations.
3. If $-g > 0$, this will produce a magnifying effect on the amplitude. The time path is characterized by explosive fluctuations.

In all three cases, the intertemporal equilibrium is assumed to be stationary. I.e. fluctuations (business cycles) around a constant trend (horizontal straight line).

3. Economic time series are often characterized by non-regular cyclical fluctuations and can therefore be described by stochastic difference equations or autoregressive relations of the general form (see for example the analysis of the HICKS business cycle model by BLATT, 1978, 296)

$$(A.5) \qquad y_t = a_1 y_{t-1} + a_1 y_{t-2} + \epsilon_t$$

where a_1 and a_2 are constants or estimated coefficients when using the OLS estimation technique, and ϵ_t is a stochastic term.
This type of equation is also called bandpass noise waveform (see BRACEWELL, 1978, 334 ff.).

A numerical example for the correspondence between the above difference equation and a differential equation is given by BRACEWELL (1978, 334, 335).
If one considers the case in which the term ϵ_t is not included and where t is not restricted to integral values, one gets a differential equation. Let y(t) be a function of t whose values of t are y_t. Then

$$(A.6) \qquad y(t) = \left[a_1 g(t-1) + a_2 g(t-2)\right] * y(t)$$

This equation is obeyed by a damped oscillation

$$(A.7) \qquad y(t) = Ce^{-gt}\cos(\omega t + \delta)$$

where C and δ are an arbitrary amplitude and phase, respectively, and the angular frequency ω and the damping constant g are (approximately) related to the (assumed or estimated) constants a_1 (= 1.84) and a_2 (= -0.9) by

(A.8) $a_1 = 2 - \omega^2 - g$

(A.9) $a_2 = -(1 - g)$.

The exact correspondence between the solution of a difference and a differential equation is given by $R^t = e^{-gt}$ (see the equations (A.2) and (A.4)) or by $\left(\sqrt{a_2}\right)^2 = e^{-(a_1/2)t}$.
If g converges to zero, e^{-g} becomes (1-g) and one gets the approximation of (A.9).

Inclusion of the stochastic term yields the realistic picture of the bandpass noise. Its general character is seen to consist of an oscillation centered on the midfrequency of the band and having an envelope amplitude (C) that varies considerably. Also the phase (δ) of the oscillation drifts at a rate connected with the rate at which the amplitude of the envelope varies.

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