The Impact of Uncertainty on the Optimal Decision of Risk Neutral Firms

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1) The aim of the paper

This paper wants do demonstrate under what conditions risk theory is able to give an unambiguous answer to the question whether firms will produce more, the same or less under uncertainty as compared to certainty.

In chapter two we offer four alternative sufficient conditions under which an unambiguous answer to the question is available. The first of these conditions refers to the risk attitude of the entrepreneurs, it is well known in literature and is reported only as a sort of reminder. We will not follow this path since it is to my opinion this very preoccupation of economic literature with the risk attitude (in the sense of risk aversion, neutrality, -loving, followed by absolute and relative degrees of risk aversion, finally discussing differences in the degree of relative risk aversion between two relevant economic agents) that prevented the literature to investigate more objective (technically) reasons for the influence of uncertainty. Our second condition refers to this economic or technological influence on the decision under certainty for risk neutral firms ("technological concavity"). The third condition refers to the case where price stickiness prevents an equilibrium at least in the short run ("disequilibrium models"), the fourth condition refers to a two stage decision process, where a fine tuning of the decision variable is possible after the veil of uncertainty has lifted ("ex-post-flexibility").

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In chapter 3 we present an overview on the scattered literature of the firm's behavior under uncertainty, showing how it fits our four conditions. In chapter 4 we try to find which empirical circumstances are important for a final assessment of the influence of uncertainty on a modern industry In chapter 5 we present empirical evidence on these crucial facts, to evaluate the most likely influence of uncertainty on a "real economy" in contrast to a normative system of general equilibrium.

In appendix 1 we represent the results of a questionnaire on the impact of uncertainty as it is seen from the point of view of Austrian manufacturing industry, in appendix 2 we relate the optimal behavior of firms to the Rational Expectation Hypothesis.

2. General Conditions

In this chapter we investigate under which general conditions the optimal value of a decision variable under uncertainty will be lower (larger) than under certainty.

We assume a utility function (1) in which utility U depends on the variable Z (which can be unterstood as profits): Z itself depends on two variables X and Y, (which usually are quantity produced and price). In the world of certainty X_o is known and there exists an optimal solution Y^* for the decision variable Y, under which profits are maximized (the second order condition is assumed to hold).

(1) $U \left[Z\ (X_o,\ Y)\right] \rightarrow \text{Max } Y^*$ (certainty maximum)

Under uncertainty we assume maximization of expected utility (Neumann-Morgenstern Utility Maximization). Uncertainty exists about the varible X for which a probability density function f(x) is known1), its expected value is assumed to be the same as the fixed value X_o under certainty (mean preserving introduction of risk).

(2) $E\ U \left[Z\ (X,Y)\right] \rightarrow \text{Max } \hat{Y}$ (uncertainty maximum)

The optimal value of the decision variable labelled $\hat{Y}$ under uncertainty can be shown to be smaller (equal, larger) than the optimal value unter certainty Y^*, if U_{YXX} is smaller (equal or larger) than zero 2). (Rothschild, Stiglitz 1971, Diamond, Stiglitz 1974 etc.).

(3) $\hat{Y} \lesseqgtr Y^*$ if $U_{YXX} \lesseqgtr 0$

Unfortunately this condition is not very useful, since U_{YXX} proves for most problems to be neither unambigously3) positive nor negative (see Hey 1981 or Kraus 1979).

2.1 Risk aversion

An unambiguous result is available (this is well known in the literature) if we assume that Z is linear in X ($Z_{XX} = 0$, let us call this "linear technology"). Linear technology (and the side condition that the optimal value of the decision reacts under certainty positively to the variable about which uncertainty may occur) leads to proposition 1.

Proposition 1: Linear technology ($Z_{XX} = 0$) plus dY /dX 0 yields the following sufficient condition

$$(4) \qquad U_{ZZ} \lesseqgtr 0 \rightarrow \hat{Y} \lesseqgtr Y^*$$

Proposition 1 implies that risk attitude is the main channel for effects of uncertainty.

In the economic literature the case of biased action4) in presence of risk aversion (or risk loving) is well known. But it seems that its impact on economic thinking is smaller, since people's attitudes toward risk are unknown. Furthermore, there are arguments that firms should behave as if they were risk neutral and that entrepreneurs should be at least less risk averse (if not risk loving) than consumers.

2.2 Technological concavity (convexity)

Other channels of uncertainty like costs or demand conditions are not unknown, but less popular. It is characteristic that Lippman and McCall (1982, p212) in their survey on the economics of uncertainty have to recall that "though in many circumstances, risk aversion is a fact ..., much economic behavior is a direct consequence of uncertainty and is independent of risk aversion". Nevertheless they do not offer any general condition for an influence of costs or market condition (nor does Hey 1981 in his excellent book on uncertainty), though such a condition is available as a quite simple application of the Rothschild-Stiglitz condition (equation 3) for risk neutral agents.

Proposition 2: A linear utility function ($U_{ZZ} = 0$) and technological concavity, neutrality, convexity ($Z_{YXX} < 0$, $Z_{YXX} = 0$, $Z_{YXX} > 0$) yield the following sufficient condition

$$(5) \qquad Z_{YXX} \lesseqgtr 0 \Rightarrow \hat{Y} \lesseqgtr Y^*$$

The proof needs only Jensen's inequality and the second condition for optimal choice under certainly ($Z_{YY} < 0$)5).

This channel for an effect of uncertainty ("technological concavity") depends on demand or cost conditions, it is therefor "objective" in the sense that it does not depend on the subjective attitude towards risk. It does not contradict maximization of expected profit under infinite repetitions.

2.3 Marginal costs of uncertainty (disequilibria models)

There is a third channel trough which uncertainty influences the optimal decision if we give up the asumption that some ex post control (usually the price) does change fast enough to guarantee equilibrium.

In disequilibria models there are ex post either unsold goods or unsatisfied damand, ex ante we have to calculate the costs for both of them (potentially unsold production plus potentially unsatisified demand). Uncertainty therefor unambiguously increases expected costs and we have to equate expected marginal revenues to the sum of expected marginal costs of production and marginal cost of uncertainty. This reduces the optimal production under uncertainty (given convex costs under certainty). We want to label the extra costs of uncertainty as "marginal costs of uncertainty". These costs reduce the optimal production under uncertainty (given convex costs under certainty).

Assume a certainty model (equations 6-8), where revenue and costs depend on production (additively) and we get the well known first or second order conditions. For uncertainty where expected sales depend on the smaller of demand (x) or production (y) we get the conditions 1o and 11.

Certainty modell

$$(6)\quad \Pi = r\ (y) - c\ (y)$$

$$(7)\quad \Pi_y = r'\ (y) - c'\ (y)$$

$$(8)\quad \Pi_{yy} = r''\ (y) - c''\ (y) < 0$$

Uncertainty modell

$$(9)\quad E\Pi = \min\ \left[r(x),\ r(y) \right. - c\ (y)$$

$$(10)\quad \frac{\partial E\Pi}{\partial y} = \underbrace{r'(y)}_{\substack{\text{marginal}\\ \text{revenue}\\ \text{under}\\ \text{certainty}}} - \underbrace{F(y)\ .\ r'(y)}_{\substack{\text{marginal costs}\\ \text{of uncertainty}}} - \underbrace{c'(y) = 0}_{\substack{\text{marginal}\\ \text{costs}\\ \text{under}\\ \text{certainty}}}$$

$$(11)\quad \frac{\partial^2 E}{\partial y^2} = r''(y)\ \left[1 - F(y)\right] - r'(y) - c''(y) < 0$$

Proposition 3: Given a certainty model of the type 6 and a uncertainty disequilibrium model of the type 9, uncertainty adds an additional marginal cost component which is positive (since F (q) as well as r'(y) are positive), this yields for this type of model the unambiguouse result of equation 12 (recall that r"(y) is smaller than c"(y) by equation 8).

$$(12)\qquad \hat{Y} < Y^{*}$$

The most prominent special case of this model is where marginal revenue is constant (r' = p). The result of an unanbiguously lower production under uncertainty was presented by Hymans' model of "competition under demand uncertainty (Hyman 1966). It did not become very popular, since a fixed price is considered as contradictory to the idea of competition. Nevertheless a short run price stickiness and price taking firms all selling a certain proportion of their "normal" production in case of a negative demand shock, may not be so unrealistic (see chapter 5 for empirical support). The model of fixed prices with demand uncertainty is intensively used in the inventory literature (it is labelled newsboy model in its simplest form) without discussion of the market environment. In the last years it was rejuvenated by Mullineaux (198o), Benassy (1982) and Costrell (1983) as a fix-price uncertainty model with stochastic rationing, though none of them stressed the effect of this model of unambiguous decreasing production under uncertainty.

2.4 Asymetric ex post flexability

The fourth possibility for changing the optimal production is given if it is possible to make a preliminary decision about the decision variable; and then after the veil of uncertainty is lifted, to revise this decision at some cost. It is easy to show that if the cost of revising the decision upwards is larger than that of downward revision the optimal preliminary production will rise, in the other case it will fall. Downward irreversibility of gross investment is one related form of asymmetry.

Proposition 4: Suppose it is possible to make a preliminary decision Y and revise this upward (downward) at cost C_1 (C2) then

(13) $c_1 \gtrless c_2$ tends to imply $\hat{Y} \gtrless Y^*$

This asymmetry effect is rather unattractive from a theoretical point6) of view, from the practical point of view upward revisions seem often to be much easier (less costly) than downward revisions. Reselling production, getting rid of investment goods, laying off personnel in the short run (especially in business troughs or facing shocks effecting a whole industry), usually proves very difficult.

3. Presentation of some models

Instead of describing the host of models presented in the literature, they are summarized in table 1. We limit ourselves to the case of risk neutrality, since the effects of risk aversion are treated so prominently in the literature.

The competition model under output price uncertainty (model 1 on table 1) is a typical example of "linear technology" and uncertainty has therefor no influence on the optimal decision. The same is true for a monopoly (3.1), if the monopolist is a quantity setter (Q-ex-ante-model), but only if uncertainty is additive or of the type p = f (q).u. If however the uncertain demand function is of the type q = f(g).u (see model 3.2), then Z_{YXX} is not zero (since the uncertainty variable enters expected revenue), but under realistic conditions 6) it is negative. This fact has been overlooked in the literature to a surprising extent. Most authors write - following Leland 1972 - that uncertainty for a quantity setting monopoly will not change the decision. He assured this result by a Principle of Increasing risk, which implies a special setting of the uncertainty. Nickell 1978 has found the contrary result in a quite different context assuming the second type of multiplicative uncertainty, without discussing whether or why he assumes another type than Leland. There are no a priori reasons as to which type of multiplicative uncertainty is more realistic.

For the price setting monopolist (model 4) the results depend on the 3 rd cross derivative of the cost function, a higher price is able to insure the firm against the neccessity of very high costs of eventually necessary market clearing production.

The monopolist who has to set ex ante prices and quantities (P-Q-mode) faces a combination of arguments of the price setting monopolist and the fix-price-disequilibrium models (see model 5.1 and 5.2).The optimal price may be higher, equal or lower than under certainty on the one hand, the possibility of unsold production tends to decrease the quantity produced. In case of a higher optimal price production is forced down by two components (the higher price and the possibility of unsold products), in case of a lower optimal price the quantity under uncertainty is influenced by two opposing forces (the lower prices increases production, the possibility of unsold products decreases optimal production).

An unambiguous result (lower production) is given by the "competition model under demand uncertainty", which may be called fixed price disequilibrium model (model 2 on table 1) or which may be characteristic for some kind of fixed prices under oligopolistic behavior. In this case it is more important whether the conditions for the model are realistic than to discuss the result. Nevertheless we want to stress two issues. First the extension of the model to goodwill and holding costs, and to the value of unsold products (they can be sold in the next period) as well as to the value of backlogged orders. If these costs are symmetric they do not change the one period results. Symmetry of these "dynamic costs" means

- first that the goodwill loss of a firm accruing from not being able to satisfy demand is as large as the holding costs

- second that the proportion of unsatisfied demand which can be backlogged is the same as the proportion of unsold goods which can be stored.

In contrast to this argument inventory literature favours the result that the incorporation of a future sales value for stock increases the optimal stocks. This result stems from the implicit or explicit assumption that backlogging of demand is less likely than durability of goods. In some very influential articles (e.g. Veinott 1966, Johnson-Montgomery 1974) the costs of backlogs are included (if consumers wait, the products have to be produced in the next period), but the revenues are forgotten.

The second aspect to be discussed is that many disequilibrium models assume linear costs (for all cost components) in models with uncertain but stationary demand. The optimal post order stock (or production) in this case cannot be compared to a unique optimal production under certainty, but to expected demand. In this case the result, whether more or less than expected demand is produced, depends on the exact values of some parameters: high profits, goodwill costs, durability of goods increase optimal production, lower profits, large holdings costs etc. lead to lower optimal values as compared to expected demand.

The infinite horizon model for determining optimal post order (post production) stock is given by the recursion (13). The first term in each curled bracket represents the sales revenue as in the one period model. The h (   ) and g (   ) terms represent holding and goodwill costs. The V (   ) terms calculate the future expected revenues from an item stocked or a demand backlogged. In the first case the revenue is positive since an item stocked decreases future production costs (depending) on the discount parameter α and the durability parameter a. In the second case it is a cost since backlogged demand has to be produced in the next period (depending on the degree of backlogging b). The second term in the second line is the term sometimes forgotten in the literature, it represents the revenue of backlogged demand.

(14) $$V(I) = \max_{q \geq 0} \int_0^{q+I} \left\{ \left[px - h\,(y-x)\right] + \alpha V \left[a(-x+y)\right] \right\} f(x)dx +$$

$$+ \int_y^{\infty} \left\{ p\,y + \alpha\,bp\,(x-y) - g\,(x-y) + \alpha\,V \left[b(-x+y)\right] \right\} f(x)dx - c\,q$$

Using dynamic programming techniques the solution is (14). This rather complicated formula shows that production in the multiperiod model will be higher than in the one period model if goodwill costs exceed holding costs and if durability exceeds the feasibility of backlogging. If however these are symmetric then the one period model represents a fair approximation of the dynamic problem. If a = b = 1 and h = g = 0 the formula collapses into the newsboy result (15).

(15) $$F(y) = \frac{(p-x)\ \ (1-\alpha b)+g}{p+g+h-\alpha b(p-c)-\alpha\,ac}$$

(16) $$F(y) = \frac{p-c}{p} \qquad \text{if } a = b = 1 \qquad h = g = 0$$

If costs of backlogs are included, but their revenues are forgotten (Veinott 1966, Johnson-Montgomery 1974) we get (16), which implies the implausible result that the possibility of backlogging increases optimal post order inventory. If the goods are durable (a = 1), but backlogging is not feasible (b = o), holding and goodwill costs are negligible (g= h = o), we get (17), which implies that almost all potential demand should be satisfied however unprobable this is.

(17) $$F(y) = \frac{p-c+\alpha bc+g}{p+h+g+\alpha bc-\alpha ac} \Rightarrow \frac{\partial \hat{y}}{\partial b} > 0$$

(18) $$F(y) = \frac{p-c}{p-\alpha c} \qquad \text{for } \alpha \Rightarrow 1,\ \hat{y} \Rightarrow \infty$$

4. Crucial facts for the evaluation of the influence of uncertainty on the production decision

The models presented in table 1 - and very briefly dicussed in chapter 3 - allow for any influence of uncertainty on the optimal production decision. However, the results show some facts on which the direction of the impact depends.

The first is the well known effect of risk attitude (question 1). If people are risk averse then a downward pressure on the optimal decision is likely.

The influence of "technological concavity" hinges on a third cross derivative which is difficult to evaluate. The discussion of the monopoly model has shown that in case of price setting there will be no influence, in case of quantity setting either no influence or a somewhat larger probability of a downward bias (in case of multiplicative uncertainty of the type q = f(p).u, a linear or quadratic demand function or a demand elasticitiy smaller than -1 will suffice). If it can be shown that quantity setting (question 2) is prevalent, then chances exist for a downward bias of production under monopoly.

The existence of disequilibria as an empirical phenomenon would be the strongest channel for uncertainty to reduce optimal production. If a market clearing price (question 3) existed for example in the case of output price uncertainty, then uncertainty would not change the decision. If there were no market clearing price then there exists a heavy downward pressure.

This downward pressure can be lowered if backlogging (question 4) does not exist or if goodwill costs exceed holding costs considerably.

If it is easier to revise upward than downward (question 5), then optimal production under uncertainty will tend to be lower than under certainty.

5. Empirical evidence on the five crucial questions

In this chapter results are reported from a larger study on the impact of uncertainty on Austrian manufacturing industry (Aiginger 1983 B). Readers interested in the exact kind of the proof are referred to this book. Some of the results stem from econometric applications, some from a questionnaire among approximately 1ooo entrepreneurs (see appendix 1 for the questionnaire).

Question 1: <u>Are entrepreneurs risk averse?</u>

Many experimental studies have investigated this topic. We chose another way and asked entrepreneurs, whether they based their decisions on the expected return of investment projects alone, or whether they preferred among two investment projects one with high risks and considerable chances or one where they might forsake some mean expected profits for the certainty of the return.

We defined as small projects those which amount up to approximately one half of an annual investment programm, and as large those exceeding an annual programm considerably. For small risks risk-averters (36,82% of the firms) and risk lovers (33,66%) balanced each other out, less than one third was risk neutral. Risk aversion and risk loving increased for small firms, risk neutrality is predominant for firms with more than 1ooo employees (6o,78%).

For large risks risk aversion outperforms risk loving 53,15: 12,37. Approximately one third is risk neutral for large decisons. Again risk aversion and risk loving decreased with the size of the firm. Only 3,92% of the large firms are risk lovers (14% of the firms with less than 1oo employees).

Question 2: Price setting or quantity setting?

Using Hay's (197o, 1972) technique of testing whether in case of surprises firms changed their prices, their production, their inventories or their backlogs, a significant quantity reaction (o,65% for a demand shock of 1%), but no price reaction (o,o2%) was observed.

Investigating the determinants of price changes econometrically, long-term determinants like labor costs and energy prices outperformed short term demand influences in Austria.

Asked how they react to a short-term surprise, 2o% of the firms answered via a price change, 56% via a quantity change, 48% that they would change their inventory level.

Asked if in case of uncertainty they set price or the quantity, 27,64% of the firms said that they set the price (and adjust the quantity ex-post), 15,94% that they have to set price and quantity and 53% that they set the quantity.

Question 3: Market clearing price or disequilibria?

It has been said already that only 2o% of the firms labelled price to be the main response to a surprise, and that econometric results support this survey result.

Confronted with five different modes of bahavior under uncertainty (P-ex-ante, Q ex post; Q-ex-ante, market price ex post; Q-ex-ante, monopoly price ex post; p-q mode; ex post quantity flexibility) only 6,99% of the firms regarded the classical competition model (Q-ex-ante, market price-ex-post) as relevant (7,71% of the small firms). The fix-price competition model was not listed among the possible answers, in order to force the firms to remain within the framework of normatively appealing models. The disequilibrium monopoly model (P-Q mode: 22,49%) and the ex post flexibility model (preliminary q, than revision: 31,11%) together got more than 5o% of the answers.

Question 4: <u>Is backlogging feasible?</u>

Backlogged orders in modern industrial economies are usually two or three times as large as finished stocks inventories. In the US finished stocks amount to 14,2% of annual sales (1954-1982), backloggs to 28,9%, net inventories are therefor negative (-14,7%).

Theoretical considerations imply that net inventories, if they are negative, should decrease (becoming more negative) with increasing uncertainty. Empirical data for the US as well as for Austria comply with this forecast. Decreasing finished goods stocks in the period of slow and uncertain demand (198o/82) may be plausible, increased order books, however, have to be interpreted as a conscious shift from production on stock to production on orders as a typical optimal behavior under uncertainty.

Question 5: Are upward revisions less costly than downward revision

Entrepreneurs labelled more instruments as feasible in case of pessimistic forecasts, especially they are willing to let inventories run down, but will not allow them to build up, indicating less goodwill costs than inventory holding cost and/or easy backlogging).

Investment plans are more often revised upward than downward. For small firms this tendency occupies up to one third of their annual investment program.

Asked which errors are more costly, 21% of Austrian firms assessed upward and downward errors as equally costly, 63% reported optimistic forecasts as more costly, 16% overpessimistic forecasts. This asymmetry slightly increased with the size of the firms.

6. Summary

The empirical facts seem to indicate that in "real economies" the effect of uncertainty tends to decrease production. The limitations of empirical investigations presented should be stressed: they were perfomed mainly on an aggregate level, they mainly refer to Austrian manufacturing, they rely heavily on questionnaires. Above all empirical investigation will never be able to decide normative questions or to explain the behavior in the general equilibrium. Nevertheless in the short run, given all the rigidities and disequilibria which exist, uncertainty tends to lower optimal production even in absence of risk aversion. Risk aversion becomes important for large, for once-for-all decisions, but it is not the only channel through which uncertainty changes decisions.

"Technological concavity" created by concave marginal revenues or by convex marginal costs, marginal costs of uncertainty in disequilibria model or asymmetric costs of revisons of the preliminary decision are able to bias the decision downward in a real world economy without invoking subjective risk attitudes.

Table 1 A

Summary of the Theoretic Models

Mode	Literature	Main Result Proposition	Condition for Main Result	Economic Rational	Empirical Implikation (EI) Caveats (C) Critical Parameters (CP)
1) Competition - price uncertainty - q ex-ante mode					
Y : q X : p	Sandmo 1971	$\hat{q} = q^*$ $Z_{YXX} = 0$	-	$\underbrace{E(p)}_{p^*} = c'(q)$	EI: no involuntary inventories, no fluctuations of involuntary inventories, expected production = actual production, actual price differs randomly from expected one
2) Competition - demand uncertainty - q ex-ante mode					
Y : q X : x	Hymans 1966 "Newsboy-model"	1) $\hat{q} < q^*$ MCU 2) $\hat{q} < Ex$	$c''(q) > 0$ $p-c < c$ ($= p < 2c$)	expected unsold quantity can be reduced opportunity cost of unsatisfied demand smaller than sunk cost	C: is this a competitive model ? CI: disequlibria inventories, forecast errors even in competitive markets, fixed prices CP: profit share versus costs
3) Monopoly - demand uncertainty - q ex-ante mode					
Y : q X : p = f(q,u)	Leland 1972 3.1.	$\hat{q} = q^*$ $Z_{YXX} = 0$	additive uncertainty, multiplicative uncertainty + PIU	production can do nothing to reduce price uncertainty	EI: no disequlibria, expected production = actual production, actual price differ from expected one randomly
	Nickell 1978 3.2.	$\hat{q} < q^*$ $Z_{YXX} < 0$	multiplicative uncertainty $q=f(p).u \rightarrow p=f^{-1}(\frac{q}{u})$ and one of these conditions: $\epsilon < -1$, q_p linear or quadratic	indirect influence via demand curve (expected marginal revenue is concave)	C: depends on specific type of multiplicative uncertainty: is uncertainty additive or multiplicative (if the latter: is $p = g(q).u$ or $p = f^{-1}(\frac{q}{u})$?)
4) Monopoly - demand uncertainty - p ex-ante mode					
Y : p X : q = f(q,u)	Leland 1972	$\hat{p} \gtrless p^*$ $Z_{YXX} \gtrless 0$	if $c''' \gtrless 0$ PIU or additive demand	high price insures against eventually expensive production	EI: no disequilibria, expected price = actual price, actual production differs from expected one randomly
5) Monopoly - demand uncertainty - p and q ex-ante mode					
Y : p,q	additive uncertainty Karlin, Carr 1962 5.1.	$\hat{p} < p^*$ $\hat{q} \gtrless q^*$ MCU vs price elasticity	$c'(q) = c$	smaller price to reduce additive uncertainty, quantity smaller if p-c very small, but larger according to demand elasticity	EI: disequlibria inventories and backloggs, expected variables = realizations
	multiplicative uncertainty 5.2.	$\hat{p} > p^*$ q likely to be smaller than q^* MCU & price elasticity	$c'(q) = c$	higher price (chosen to reduce multiplicative uncertainty) and small profits reinforce each other to lower production	EI: disequlibria inventories and backloggs, expected variables = realizations C: assumption of constant costs

MCU = marginal costs of uncertainty

Table 1 B

Mode	Literature	Main Result Proposition	Condition for Main Result	Economic Rational	Empirical Implikation (EI) Caveats (C) Critical Parameters (CP)
6) Inventory models - demand uncertainty - fixed prices					
Y : q + I X : x	following the line of Veinott 1966	$\hat{q}$ + I likely to be smaller than E (x)	p fixed $c'(q) = c$	small p-c and symmetric holding and goodwill costs as well as symmetric backlogging and durability parameters	EI: disequlibria inventories and backloggs, net inventories negative, procyclical movement of planned net inventories, decreasing net inventories with increasing uncertainty CP: p - c : c; g : h; a : b
7) Factor demand models					
Y : K,L , q X : p	see e.g. Hey 1979 Hartman 1975 Barta, Ullah 1974	$\hat{K} = K^*$ $\hat{L} = L^*$ $Z_{YXX} = 0$	$\hat{q} = q^*$ "well behaved" production function	implication of the result for output	C: both factors under ex-ante controll, equilibrium model. $F_L F_{KK} - F_K F_{KL} < 0$ $F_K F_{LL} - F_L F_{KL} < 0$
8) Y : K,L X : x		$\hat{K} < K^*$ $\hat{L} < L^*$ MCU	$\hat{q} < q^*$ $c''(q) > 0$	implication of the result for output	C: the same as for the competitive model under demand uncertainty
9) Y : K,q^{max} X : x	Nickell 1978 p. 72 f	$\hat{K}, \hat{q} \gtrless E(x)$	$i \gtrless \frac{1}{2}(p-w)$	i fixes upper limit of production, foregone profits have to be balanced only against i, since wages can be saved if demand is low	C: constant prices, constant unit wage, investment cost EI: strong fluctuations of wages (and employment) disequilibria inventories
10) Y : K X : p	Hartmann 1976	$\hat{K}$ likely to be larger than K^* $Z_{YXX} > 0$	$\sigma < \frac{1}{1-\eta}$ competition ex-post controll	marginal product of labour rises with increasing rate	EI: uncertainty increase capital stock, fluctuations of wages (employment), investment anticipation = actual investment C: equilibrium model
1) Y : K X : p ex post decision: utilization rate	Kon 1983	$\hat{K}$ likely to be smaller than K^* $Z'_{YXX} < 0$	large utilization effect	possibility of unused capacity outweights advantages of large programs	C: disequilibrium in the sense of unused capacity
2) Y : K X : w	Nickell 1978 p. 80 f	$\hat{K}$ likely to be larger than K^* Z_{YXX} & MCU	$\sigma < \|\epsilon\|$ monopoly, L ex-post controll	low substitutability favours "reserve capacities"	EI: as in last case above
3) Asymmetric ex-post flexibilities					
Y : I^o or q^o X : x	e.g. Nickell 1978 p. 105 ff	$K^o < K^*$ $q^o < q^*$	upward revisions easier than downward revision	irreversibility of investment or production, cost differences in emergency reactions ($c_2 > c_1$)	C: easy results often dependent on constant price and cost assumptions EI: investment anticipation and production plans biased downward

Table 2

Indicators of Asymmetry in the Ex-Post-Flexibility

1) Differences in the first reaction to a demand shock within the planning period (1.100 firms, Austria)

	all firms: positive demand shock	all firms: negative demand shock	small firms: positive shock	small firms: negative shock	large firms: positive shock	large firms: negative shock
price	20	20	12	18	39	21
quantity	56	56	47	46	55	63
inventory	60	36	41	26	81	58
capacity	16	15	23	18	0	9

2) Differences in errors of same size and different sign (1.100 firms, Austria)

	all firms	small firms	large firms
costs are about the same	21 %	18 %	30 %
larger for optimistic sales forecast	62 %	58 %	64 %
larger for pessimistic sales forecast	17 %	24 %	61 %

small firms: less than 100 employees

Table 3

Risk Attitude of Austrian Entrepreneurs

1. Minor Decisions

	Risk aversion	Risk lovers	Risk neutrality
All firms (n = 918)	36,82	33,66	29,52
< 100 employees (n = 530)	40,75	38,68	20,57
100-499 employees (n =285)	32,98	31,58	35,44
500-999 employees (n = 52)	28,85	13,46	57,69
1.000 and more employees (n = 51)	25,49	13,73	60,78

2. Large Decisions

	Risk aversion	Risk lovers	Risk neutrality
All firms (n = 873)	53,15	12,37	34,48
< 100 employees (n = 494)	57,49	13,97	28,54
100-499 employees (n = 277)	51,99	11,19	36,82
500-999 employees (n = 51)	37,25	11,76	50,98
1.000 and more employees (n = 51)	33,33	3,92	62,75

Table 4

Adequate Description of Uncertainty

	Known density	Keynsian uncertainty	Descriptions inadequate
All firms (n = 935)	75,11	41,03	1,82
100 employees (n = 549)	45,72	52,09	2,19
100-499 employees (n = 282)	72,34	26,60	1,06
500-999 employees (n = 52)	71,15	28,85	0
1.000 and more employees (n = 52)	80,77	15,38	3,85

For the questionnaire see appendix 1

Notes

1) All theoretical models will assume maximization of expected utility as the chosen decision technique. Uncertainty is not differentiated from risk (in the sense of Knight 1921), in contrast we assume a that the decision-maker has a probability distribution about the uncertainty variable (which Knight would have called risk). The rationale for this assumption is not that I believe that the concept of expected utility maximization is the only feasible one. The Keynesian view that "true" uncertainty changes the very behavior of agents (for example favoring the use of more flexible production techniques, reducing the willingness of people to undergo long term commitments etc., see Rothschild 1981) is probably the more realistic description for the impact of uncertainty albeit less operational. The same may be true for Shackle's theory stressing the importance of "focus values" (1962), for Simon's concept of limited rationality (1955, 1978), or other critics of Expected Utility maximization like Kahneman & Tversky ("prospect theory, 1979), Radner (1975, "satisficing provessesses") or regret theory (Loomes, Sugden 1982). Though we use the technique of expected utility maximization our analyses and results may be regarded as "Keynesian" in spirit, since we allow for price stickiness and ex post adjustments in the decision parameters and then get the result that uncertainty changes the behavior and increases the importance of flexibility. This is a genuine Keynesian result proven with Von Neumann Morgenstern Utility Maximization

2) A suffix as usual denotes a partial derivative. All functions are assumed to be well-behaved in the usual sense (continuously differentable, integrals exist and are finite).

3) U_{XXY} is ambigous if Z_Y changes its sign. But, in any interesting problem Z_Y must change sign, since in a certain world $Z_Y \mp 0$ and $Z_{YY} < 0$ are the conditions for the optimal choice of Y (assuming $U' > 0$), see Hey 1981, p. 344.

4) Bias in the sense of a difference of the optimal action under uncertainty as compared to certainty.

5) The proof makes use of the assumption that the second order condition for maximization under certainty holds ($Z_{YY} < 0$) and of Jensen's Inequality for convex or concave functions. We prove the case of $\hat{Y} < Y^*$ defining Z_Y concave in X.

$$Z_Y (X_o, \hat{Y}) > EZ_Y (X,\hat{Y}) = 0$$

The inequality holds for any concave function (and so for Z_Y), the equality stems from the first order maximization under uncertainty.

It follows that Z_Y (X_o, Y) is positive, and using $Z_{YY} < 0$ this implies that $\hat{Y}$ is smaller than Y^* (where $Z_Y = 0$).

The result may also be derived from the Rothschild Stiglitz condition.

6) The expression "trends to" is used since the effect is somewhat complicated. For equilibria models with non linear costs the result depends on the interaction of "normal production costs" and emergency costs (a 3rd cross derivative, see Turnovsky 1973).
We can demonstrate the tendency for a fix-price model with linear normal production costs, with upward (downward) revision costs of c_1 (c_2) and feasibility parameters for upward (and downward) revisions a (resp.b).
Optimal production is given by

$$F(q) = \frac{p-c-bp+bc_1}{p+bc_1-bp-ac_2}$$

Optimal production is very likely to be smaller than expected demand, especially if upward revisions are more feasible and less costly than downward revisions.

7) See Nickell 1978. The demand function has to be linear or quadratic or the elasticity has be less than -1.

Appendix 1

Survey on Firm's Strategies Under Uncertainty (conducted among Austrian Entrepreneurs in January 1984, 1.000 firms)

Changes in the world economy, changes in the structure of demand, larger price fluctuations and the reactions of economic policy are all often subsumed under the heading "increased uncertainty". This questionnaire is designed to supplement the theoretical discussion with practical criteria.

1. Characterization of uncertainty: "Uncertainty" can have many different faces. Please check which of the following planning situations applies most closely to your firm. If possible check only one answer

   21.6 In our planning we determine the price at which we want to sell and adjust quantity produced depending on demand. Unintended stock fluctuations are rather unusual.

   7.0 We plan quantity to be produced during the next month or quarter. Market prices fluctuate in a way that we normally sell everything we produce. Unintended stock fluctuations are rather unusual.

   15.9 We plan quantity produced during the next month or quarter. There is no single market price, but we adjust our own price in such a way that we can sell our production. Unintended stock fluctuations are rather unusual.

   22.5 We have to plan quantity as well as price (or the price is sticky in the short run). Higher or lower demand is reflected in lower or higher stocks of finished goods.

   34.1 We plan an ex-ante (preliminary) production quantity, in the short run the price is fixed. When demand is higher we can adjust production upward (or downward in the reverse case).

2. When comparing two investment projects which amount to around 1/2 of your usual annual investment budget you can easily calculate the rate of return for project A. Project B has high chances of success but also considerable risks.

   The same alternative exists for strategic decisions, an i.e. investment program that amounts to a multiple of a usual annual investment program.

for "small" decisions		for "large" decisions
36.8	we prefer the project with known rate of return	53.2
33.7	we prefer the project with higher chances but also higher risks	12.4
29.5	we decide according to the higher rate of return which the alternatives have, assuming "normal" economic conditions	34.5

3. Which of the following descriptions fits the present situation better?

   51.1 The economic situation is characterized by imponderabilities. The analysis of past developments and market forecasts enable us, however, to estimate a "mean expected demand" as well as a rather pessimistic variant for which we can access an estimated probability and an optimistic variant which again will occur with a certain probability.

   41.0 Uncertainty is so strong that past developments do not offer any clues as to future sales possibilities, market forecasts are not really useful. The important thing is to be prepared for anything, to remain flexible and to act quickly on opportunities and risks.

   4.8 None of the above applies

4. Assuming that a firm notices that stocks of unsold products accumulate considerably. Given that slack demand is likely to persist for a while, why does such a firm not lower its price in such a situation?

39.5 because demand would not increase with the lower price
23.3 because competitors would also cut prices
27.9 short-term price cuts spoil the market
4.4 short-term price cuts are technically unfeasible
13.9 price cuts confuse the purchaser
4.5 to cut prices according to market conditions inhibit the production planning process
8.1 our firm normally adjusts prices so rapidly that stocks rarely accumulate
35.9 we produce only on order
30.8 we adjust quantity rapidly so that stocks do not accumulate
4.2 other .....

5. Which of the strategies mentioned do you think is able to overcome the uncertain economic situation?

69.1 opening up of new markets
16.2 concentration on few markets
1.4 increase in finished goods stocks
0.6 increase in raw material stocks
9.6 switch to production on order
11.4 longer-term production planning
27.6 more flexible production techniques
20.6 improved market information
49.5 higher marketing efforts
15.6 wider product variety
9.8 decrease in finished goods stocks
6.8 lower raw material stocks
21.5 specialty production
8.2 increase in desired profit margin
11.9 stricter selection of investment projects
13.0 rapid investment plan adjustments
27.8 more research and development

6. Frequently it is difficult to estimate the size of a product's market. If this should be possible for your major product ....

How large is your market share?

	domestic	foreign
less than 5 %	20.7	65.8
5 % - 20 %	30.3	16.1
20 % - 50 %	31.6	10.3
more than 50 %	17.5	7.8

How many competitiors do you have?

	domestic	foreign
only producer	4.8	1.1
up to 5 competitors	39.8	14.9
few large, many small ones	33.0	25.6
many competitors	22.5	58.5

Appendix 2: The optimal decision of firms under uncertainty and the usual operationalization of the Rational Expectations Hypothesis as a conditional expected mean.

One of the most popular hypotheses of economics and especially modelling the impact of monetary policy on economic behavior is the Rational Expectations Hypothesis. Originally stating that economic agents should use all available information including that about the working of the economy, this notion was operationalized as the conditional expected value (1).

It is well known that the concept of the conditional expected value is an optimal concept only for linear problems. If the objective function is not linear (because of risk aversion, of technological concavity, marginal costs of uncertainty or asymmetries) the optimal forecast will differ from the conditional expected value.

$$(1)\ \text{REH: } \hat{x} = \int_{-\infty}^{\infty} x\, f\,(x_t \mid I_{-1})\ dx = Ex \mid I_{t-1}$$

Let us assume the variable to be forecast to be future sales (x), then the utility maximizing agent will not use an expected value (Ex) but distinguish between a "true expectation about x", which is characterized by a density function and "planned optimal sales figure" $\hat{y}$, which is given by the maximization process (2). The planned sales will therefor differ from the expected value of the density function and from any optimal action derived by only using the expected value of the uncertainty variable.

$$(2)\ \hat{y} = \max_{y} \int_{-\infty}^{\infty} U\ [Z\ (x,y)]\quad f(x \mid I_{-1})\ dx = \max_{y}\ \underset{x}{E}\ U\ [Z\ (x,y)]$$

This concept of interpreting "sales anticipation" (the "expected sales") is not an innocent concept, since the planned sales are interpreted as actions in the terminology of decision theory and not as expectations (which should by definition be independent of tastes and consequences).

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List of Abbreviations

U utility

Z argument of objective function (e.g. profit)

X "random variable"; X_0 in world of certainty

Y "decision variable"

$\hat{Y}$, Y^* optimal decision in case of uncertainty resp. certainty

U_{XXY}, Z_{YXX} suffixes as usual denote partial derivatives

π profit

p price, ($\mathring{p} = p + g$)

$c(q)$ production costs (c' = marginal costs, c'' = second derivative), c = constant unit costs

c_1, c_2 cost of "emergency" production, production cutback

x, q quantity demanded, produced

$\bar{q} = q + I$ stock on hand (inventory after ordering, before demand); I starting stock before production.

E expectation operator

$F(.)$ distribution function, $f(.)$ density function (symmetric)

u additive or multiplicative error term in demand function

h, g cost of holding 1 unit of stocks respectively goodwill loss for 1 unit of unsatisfied demand

$\alpha = \frac{1}{1+r}$ discount factor (r = discount rate)

a, b durability parameter ($a = 1$ for durable goods, $a = 0$ for perishable goods) resp. backlogging parameter ($b = 1$ if no demand is lost, $b = 0$ if all demand unsatisfied today is lost)

w, i wages, investment (per unit of output)

$M(q,u)$ marginal revenue (for a given q and u)

ϵ price-elasticity of demand

K ($\hat{K}$, K^*) capital stock (optimal under uncertainty resp. certainty)

$\tilde{\sigma}$, η elasticity of substitution, scale parameter

S standard deviation

k asymmetry parameter